Jun. Raja Ram

INTERMEDIATE PHYSICS

VOLUME I

BY

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ELEVENTH EDITION

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PREFACE TO THE SIXTH EDITION

In the first place I express my thanks to all teachers of Physics for their wide appreciation of the book. The readers of this text, extend from Kashmir to Burma. I am grateful to the publishers for the efficient way in which they have been maintaining the supply over such wide distances. I should also appreciate the efforts of the printers who have worked under heavy strain.

On my part I can assure my fellow teachers that I have thoroughly revised the text. If any errors have still occurred, the author will greatly appreciate any notices sent to him pointing out the same and will take steps to rectify them.

Calcutta,
27. 6. 56.

D. B. Sinha

PREFACE TO THE SEVENTH EDITION

Important changes, both in contents and arrangement, have been made in Part I (General Physics) of this volume. Questions of interesting nature from recent examination papers of the various Universities in India have been also incorporated. The author thanks the Publishers and the Printers for their ungrudging co-operation at every stage.

Calcutta, July, 1958

D B Sinha

PREFACE TO THE EIGHTH EDITION

The contents remain the same in this edition. Slight changes in treatment here and there, and a thorough checking up throughout have been made. Topics for which references to the 'Additional Volume' of this book will be found are meant mostly for the Bombay and Rangoon Universities.

Calcutta, August, 1959

D B Sinha

PREFACE TO THE ELEVENTH EDITION

In this edition most of the old blocks have been replaced by new ones. The topics relevant to General Physics, Heat and Sound treated ...r 'Appendix' at the end of Vol II of this book have now been ...erred to this volume. But for all these the book otherwise has ...ed the same as in the last edition.

Calcutta, November, 1964

D B. Sinha

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ABBREVIATIONS

The following abbreviations have been used in the text in connection with examination questions

A. B—Ajmer Board
All.—Allahabad University
And. U—Andhra University
Anna U.—Annamalai University
Banaras (or B.H.U.)—Banaras Hindu University
Bihar—Bihar University
Bomb—Bombay University
C. U—Calcutta University
C. P.—Central Province University
Dac.—Dacca University
Del U. (or Del)—Delhi University
Del. H. S.—High School Board, Delhi
E. P. U (or East Punjab)—East Punjab University
G. U. (or Gau.)—Gauhati University
Guj U.—Gujrat University
M B B—Madhya Bharat Board
M U—Madras University
Mysore—Mysore University
Nag U.—Nagpur University
Pat. U—Patna University
Poo (or Poona)—Poona University
P. U—Punjab University
Rajputana (or R U)—Rajputana University
U. P. B—Uttar Pradesh Board
Utkal—Utkal University
Vis. U.—Viswavarati University.

PART I
GENERAL PHYSICS

CHAPTER I

INTRODUCTION

1. The Five Senses, and Knowledge:—We possess a number of bodily faculties called the senses, *viz.* the senses of *sight, hearing, smell, taste,* and *touch,* which give us ability to gain experiences in this universe. All that we know, collectively called *knowledge,* is derived from these experiences. In other words, **knowledge is based on the sense-perception.** The senses remain in a crude form in childhood and in normal cases develop more and more as the age advances. Maturity is thought to be attained when these senses begin to act properly or fully.

2. The Sciences, Basic and Subsidiary:—Literally, the word *science* means knowledge. By usage, however, any knowledge is not called science now-a-days. Science means to-day what is called *systematised and formulated knowledge.* It has been classified according to certain principles. In order to understand these principles we have to remember that our universe consists of matter and energy only, matter again partly consisting of living beings and partly of inanimate realities. These all are collectively called the *nature.* So what we call **natural science** or **natural philosophy** concerns with all the phenomena in nature, the phenomena being partly *biological* and partly *physical.* The **biological sciences** deal with the living beings and energy mainly, while the **physical sciences** deal more with inanimate matter and energy, though there is no sharp frontier dividing the two. The physical sciences consist of the two main divisions: **Physics** and **Chemistry.** These two sciences have grown more or less independently as if they belonged to two different schools of thought, though essentially their mission is the same.

The natural sciences are the *basic sciences* from which all other *subsidiary* sciences such as Engineering, Agriculture, Medicine, Astronomy, Aeronautics, Geography, Geology, etc. have sprung up. As time will pass, other branches of specialised subsidiary sciences are bound to come forward as the usefulness of the same for human cause will be more widely appreciated. In following up the natural sciences, both basic and subsidiary, the importance of another science namely **Mathematics,** which is the most basic of all sciences, cannot be overstated.

3. The Object of Physics:— The physical sciences, as already stated, deal primarily with inanimate *matter and energy* only. Inanimate matter and energy exist in different forms in the different parts of the universe. The object of Physics is to study the properties of both of them and their inter-relations. Their inter-relation is oftentimes very subtle and cannot be easily traced. There are a multitude of phenomena in nature which are still obscure, and our physical sciences, though they are considerably advanced to-day, have not yet been able to explain them. These obscurities in nature which remain to be clarified form what we often call the mystery of the universe. The *ultimate object* of Physics is to solve these mysteries and to reveal the nature. In what we have said above, we have not included the obscurities or mysteries of the domain of life.

4. Matter and Energy:—**Matter** is anything which exists in nature occupying some *bulk* (i.e volume) and can be perceived by one or more of our *senses*. As will be known later, after the consideration of Newton's Laws of Motion (Art 78), its **effect** is to offer resistance to causes which tend to produce a change in its position, configuration or motion. The water, the air and the vegetations are only some different kinds of matter. Ninety-odd different *elementary* kinds of matter have been recognised in our modern physical sciences and they, by combination, constitute the whole *material universe*. The quantity of matter in a given volume, called the *mass* of it, remains the same even if the volume, or shape is altered by external causes. That is, matter refers to the stuff and not to the volume or shape. For example, a piece of wool can be compressed to occupy a smaller volume but the mass of it remains the same as before. This shows that matter is capable of *extension* or *compression*. Ordinarily, matter has *weight*, but the weight is not an *inherent* property of matter. For the weight of a given piece of matter does really arise on account of its position with respect to the earth and when that position changes, the weight changes. We have even the possibility of the weight becoming zero at certain situations. Mathematically, one such situation is when a given piece of matter can be placed at the centre of the earth. But even then the mass of it will remain the same though it will lose all its weight. Thus, though the weight is a very common property of matter, it is not an *essential* property.

Energy, like matter, is something which exists in nature, though in different kinds. It pervades throughout this universe but has no bulk to be perceived by our senses. It has also no weight and knows no extension or compression. But what is to be remembered is that work, whatever be its nature, can never be produced without expenditure of energy. Energy is, therefore, defined as the cause of work. So energy and work are synonymous, i.e. what is energy is work and what is work is energy. That is the reason why energy is [illegible]itatively measured by work. As work may be of various types, the corresponding energies are differently named, depending on the

type of work. The main divisions of energy are: *mechanical* energy (energy possessed by matter on account of position, configuration or motion), *heat* energy, *sound* energy, *light* energy, *magnetic* energy, and *electrical* energy. Each one of them is transformable into any other form or forms and this shows the ultimate identicality between different kinds of energy.

5. Sub-divisions in Physics:—The subject of Physics is, for convenience, usually divided into the following branches:—

(1) General Physics, (2) Heat, (3) Sound, (4) Light, (5) Magnetism, and (6) Electricity.

Of these branches, 'General Physics' deals with the general properties of matter and energy, while the rest deal with the detailed study of energy in special forms.

6. Measurements:—The physical sciences are called exact sciences, for they give us accurate knowledge. This exactness or accuracy comes from what are called *measurements*. The study of Physics involves measurements of various types at every stage. So Physics is often called the science of measurements. The principles and techniques of measurements have grown as a very important branch of Physics. Precision measurements have revealed far-reaching results in our physical sciences and so stress is always very rightly laid on the precision or accuracy in measurements.

The keynote of progress everywhere and so in precision measurement also, is *exact comparison*. To enable comparisons, it is necessary to establish and maintain *concrete and exact standards* of measurement. The maintenance of exact standards is necessary for another reason too. Industries to-day cover a wide field of scientific applications, and some of them have attained a high degree of perfection. They have constantly to improve their products if they are to exist in a competitive market. As a result they make a constant demand on the scientists to provide them with more accurate tools or standards for checking the articles they manufacture.

7. Units and Standards:—In making a measurement of any physical quantity, some *definite* and *convenient* quantity of the *same kind* is taken as the standard, in terms of which the quantity as a whole is expressed. This conventional quantity used as the standard of measurement is called a **Unit.** The numerical measure of a given quantity is the *number* of times the unit for it is contained in it. Thus, when we say that a stick is 5 feet long, what is meant is that a certain length, called the foot, has been taken as the unit for measurement and the length of the stick is 5 times of it.

Every physical quantity requires a separate unit for its measurement and so the number of units we have to deal with is as many as there are physical quantities of different kinds.

The actual realisation of a unit for any physical quantity requires the establishment, construction, and maintenance under specified conditions of a prototpye of it, called its **primary standard** on which it is based. The unit may be equal to, a multiple, or a sub-multiple of the standard for practical reasons. The standards need not be as many as there are physical quantities, for all the physical quantities we have to deal with, are not independent quantities.

8. Fundamental and Derived Units :— The unit for any physical quantity can be derived ultimately from the units of *length, mass* and *time*. Moreover, these three units are independent of each other. So these three units are called the *fundamental units*, and the units of all other physical quantities, which are really based on these three units, are termed the *derived units*.

Derivation of other units from the fundamental units—

The unit of area is the area of a square each side of which is of unit length; and the unit of volume is the volume of a cube, each side of which is of unit length. So the unit of area, or that of volume, is derived from the unit of length which is a fundamental unit.

Again, a body has unit speed when it moves over unit length in unit time Hence the unit of speed is derived from the units of length and time. Similarly, the unit of force is derived from the units of length, mass, and time Thus the *units of area, volume, speed, force*, etc are all *derived units*. Not only all other mechanical units, but also the units of all non-mechanical quantities, magnetic, electric, thermal, optical, acoustical, can, with the help of some additional *notions*, ultimately be derived from the above three mechanical units This shows the true fundamental nature of these three units. The derived units ordinarily bear simple relation to the three fundamental units

9. Two Important Systems of Fundamental Units :—

(i) *The C G S. System (Metric System)*,

(ii) *The F P.S. System (British System)*

In the C G S system, *C* stands for Centimetre (cm) as the unit of length, *G* for Gramme (gm.) as the unit of mass, and *S* for Second (sec.) as the unit of time.

In the F.P S system, *F* stands for Foot (ft) as the unit of length, *P* for Pound (lb.) as the unit of mass, and *S* for Second (sec.) as the unit of time.

9(a). **Practical Units and Absolute Units :—** It is often found that some derived units are inconveniently large or inconveniently small . In such cases some sub-multiple (when the derived unit is too large) or some multiple (when the derived unit is too small), is used as a unit for the sake of convenience. Such units are called **Practical Units** whilst those derived directly from the centimetre, gram, and second (or the foot, pound, and second) are termed

Absolute Units, the system of measurements being called the absolute system.

10. Standard Notations :—

PREFIXES	MEANING	
Sub-multiples		
Micro	$\frac{1}{1,000,000}$	... 10^{-6}
Milli	$\frac{1}{1000}$	... 10^{-3}
Centi	$\frac{1}{100}$	... 10^{-2}
Deci	$\frac{1}{10}$	... 10^{-1}
Multiples		
Deca	$\frac{10}{1}$	... 10^{1}
Hecto	$\frac{100}{1}$	... 10^{2}
Kilo	$\frac{1000}{1}$	... 10^{3}
Mega	$\frac{1,000,000}{1}$	... 10^{6}

11. The Fundamental Units, their Multiples and Sub-multiples :—

The fundamental units are those of *length*, *mass* and *time*, i.e. (A) The Unit of length, (B) The Unit of mass, and (C) The Unit of time.

(A) The Unit of Length.—

(1) In the **C.G.S.** system, the unit of length is the centimetre (cm.) which is $\frac{1}{100}$ th part of a standard length, called the International Prototype Metre (m).* The Prototype Metre is preserved at the International Bureau of Weights and Measures at Sèvres, near Paris. The prototype metre is the distance at 0°C. temperature between two parallel lines engraved on the central flat portion of a platinum-iridium bar of special x-form cross-section (Fig. 1) supported in the horizontal plane.

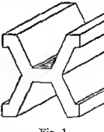


Fig. 1

* This is a copy of the Borda Platinum Standard—the *metre des archives*—the original standard, which was intended to be equal to 10^{-7} or one-ten-millionth of the distance (measured over the earth's surface along the meridian through Paris) from pole to equator. According to Clarke, the correct length of a quadrant of the earth $= 1{\cdot}0007 \times 10^{7}$ metres; the mean of the values obtained by Helmert and the U. S. Survey for the mean polar quadrant is $1{\cdot}00021 \times 10^{7}$ metres. The length of the prototype bar as constructed is an arbitrary standard.

(2) In the F.P.S. system, the fundamental *unit of mass* is the *Pound Avoirdupois*. It is the mass of a standard known as the Imperial Standard Pound (marked "P. S. 1844, 1 lb.") consisting of a platinum cylinder preserved at the Standards Office of the Board of Trade, Old Palace Yard, London

British Table of Mass

16 Drams (dr.)	=1 Ounce (oz)
16 Ounces	=1 Pound (lb)
28 Pounds	=1 Quarter (qr)
4 Quarters	=1 Hundred-weight (cwt.)
20 Hundred-weights	=1 Ton (T.)

1 Pound Avoirdupois (lb) =7000 grains

1 Pound Troy (Jewellers' or Apothecaries' weight) =5700 grains.

Conversion Table

1 grain	=64 8 mgm
1 ounce	=28 35 gm
1 pound (lb)	=453 6 gm =0·4536 Kgm
1 Kgm	=2 205 lb
1 Ton (T.)	$=20\times4\times28=2240$ lbs

The Indian *"tola"* has a weight of about 12 grams; so *"one seer"* or 80 tolas is equivalent to 960 grams, which is nearly equal to one Kilogram, of 1000 grams

(C) The Unit of Time.—The unit of time is the mean solar second in both the CGS and FPS systems It is based on the **mean solar day*** as the standard of time The mean solar day is divided into 24 *hours*, an hour into 60 *minutes*, and a minute into 60 *seconds* Therefore the mean solar day is equal to $24\times60\times60$ (=86,400) mean solar seconds That is, a mean solar second is 86,400th part of the mean solar day.

The sun appears to us to move across the sky because of the diurnal rotation of the earth about its polar axis. The meridian at a place is an imaginary *vertical plane* through it, and so the sun is said to be in the meridian when it attains the highest position in course of the apparent journey in the sky The interval of time between two successive transits of the centre of the sun's disc across the meridian at any place is called a solar day. The length of this solar day varies from day to day owing to very many reasons but it has the same cycle of variations repeated after a solar year which is

* Since the 1956 meeting of the International Committee of Weights and Measures, the *mean solar second*, the fundamental unit of time has been altered from being a fraction of the *mean solar day* to a fraction of the year, the accepted standard year being 1900

$365\frac{1}{4}$ days approximately. The mean value of the actual solar days averaged over a full year is called the mean solar day. An ordinary clock, watch or chronometer keeps the mean solar time, and is regulated against standard clocks and chronometers controlled under specific conditions.

The Sidereal Day.—The interval of time between two successive passages of any fixed star across the meridian at any place is a constant time and is known as a sidereal day. It is shorter than the mean solar day by about 4 mean solar minutes. The mean solar second is actually $\frac{1}{86,164.10}$ of a sidereal day.

12. M. K. S. Units :—In this system, the units for length, mass, and time are the metre, kilogram, and second, respectively.

13. Advantages of the Metric (C.G.S.) Sysetm :—(1) Each unit is exactly ten times the next smaller unit. Hence the reduction from one unit to another is effected simply by a proper shift of the decimal point. Thus 1·234 metres = 123·4 cm. = 1,234 mm.

But, in the British system, cumbersome multiplications and divisions are necessary in reducing one unit to another, *e.g.* from feet to inches, ounces to pounds, etc.

(2) The units of length, volume, and mass are conveniently related. Thus, knowing that the mass of one cubic centimetre of water at 4°C. is one gram, we can write down at once the volume of any amount of water in cubic centimetres, if we know its mass in grams, and *vice versa*.

For example, the mass of 10 litres or 10,000 cubic centimetres of water = 10,000 grams; and the volume of 10,000 grams of water = 10,000 cubic centimetres (or 10 litres). In the British (F.P.S.) system inconvenient constants have to be remembered, *viz.* the mass of 1 cubic foot of water = 62·5 pounds, 1 quart = 69·278 cubic inches, etc.

(3) This system has been adopted in all countries by scientific men.

14. Dimensions of Derived Units :—The relation of the unit of any physical quantity to the fundamental units (length, mass, and time) of any absolute system of measurement is indicated by what are known as the *dimensions* of the unit concerned. The dimensions do not represent any exact amount but only show the *nature* of the relationship.

A numerical quantity has no dimensions, for it is unrelated to the fundamental units. Because breadth or height is a length only, they have the dimension of length. A special kind of symbol is used to indicate the dimension of any physical quantity. Symbolically, the notation [...] stands for the unit of a physical quantity,

CHAPTER II

MEASUREMENTS

16. Measurement of Length:—The type of work and the accuracy necessary in it decide which appliances are to be used for the measurement of a length. The different types of appliances in use are, therefore, described below according to their suitability for particular work, namely (*a*) ***Field work***, (*b*) *Workshop practice*, and (*c*) ***Laboratory work*** There can, however, be no restriction on any of these appliances being used, according to necessity, for a type of work other than that under which it is placed below.

17. Different Types of Appliances for Measurement of Length:—

(a) Field Work.— In field work, such as survey work, etc. long distances, sometime along curved routes, are to be measured. For such work, the *chain* and the *tape* are generally used.

(i) The Chain.— Ordinarily it is of two kinds, either the **Gunter Chain** (which is 66 ft in length), or the **100 ft. chain.** Metre *chains* are also used in many countries. All *chains* are divided into 100 equal *links* so that each Gunter link is 0·66 ft. long, *i.e.* 7·92 inches, and each link of the 100 ft chain is 1 foot The '*making-up*' or folding of the chain, if done properly, gives the chain, when not in use, a neat appearance as shown in Fig 3. Moreover, proper making-up is necessary, for otherwise there may be bending of the links In order to mark the end of a chain length

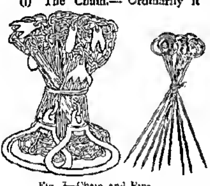


Fig 3—Chain and Pins

an *arrow* or *pin* is used. It is a stout wire pointed at one end for sticking into the ground and formed into a loop at the other. The total length of a pin is about 14". A bunch of them is also shown in Fig 3, right.

The chain is made of [illegible] iron or steel wire. It consists of links connected to each other. Each link has three small oval rings

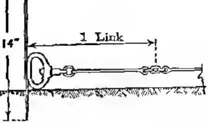


Fig. 3(a)—A Link

The centre of the middle ring is the end of the link as shown in

Fig. 3(*a*). The ends of the chain are formed with brass handles which are connected to the wire links by swivel joints. The first link is measured from the back of the handle as shown by the pin in Fig 3(*a*). As all links in the chain look alike, they are marked at each *tenth* link with a brass tag.

In measuring a length, the chain is placed along the route avoiding sag. The entire distance is measured chain after chain.

(ii) The Tape.—*Linen tapes* are also used in taking measurements of *main lines* but usually they are used for taking measurements from the chain line in any given direction, usually at right angles to it. Linen tapes are ordinarily made in 50 ft., 66 ft. and 100 ft. lengths. They are marked in feet and inches on one side and the 66 feet tape has also 100 parts marked on the reverse side. *Steel tapes* are also made in those sizes. Generally their graduations are correct at 62°F. They are often used for checking up the accuracy of linen tapes. Usually they are neatly rolled upon a spindle inside a flat-shaped circular leather box. The zero-end of the tape projects through an aperture in the side (Fig. 4) of the box and has a brass link attached which is too large to slip through the aperture. Any length of the tape is drawn out of the box, when necessary, by pulling at this link.

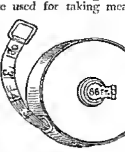


Fig. 4—Tape.

Comparison of Chain Lengths

1 Gunter chain $= 66$ feet$=22$ yards$=100$ links

1 Gunter link $= 7{\cdot}92$ inches.

10 Gunter chains $= 220$ yards.

80 Gunter chains $= 1$ mile.

From the above table it is clear that the lengths of athletic tracks, namely 220 yds. run, 440 yds. run, 880 yds. run, and the mile run, can be conveniently measured by the Gunter chain, being 10, 20, 40 and 80 chains respectively.

(iii) The Beam Compass.—In survey work, it frequently happens that a length to be represented on the map according to a given scale is too large to be dealt with an ordinary divider or a pair of compasses.

In such cases a *beam compass* (Fig. 5) is used. Here the length of the beam between the ends of the compasses can be adjusted and made as great as required. Either the pen (or pencil) end *A*, or the pointed end *B*, can be clamped anywhere on the beam and while one is left clamped, the other, kept slack, can be made to

slip easily along the beam to set it for a definite length. Some compasses are provided with a slow-motion screw to enable the pencil-piece, when clamped roughly to the correct length, to be moved a little this side or that side until the exact correct length is arrived at.

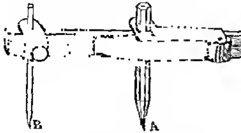


Fig 5—Beam Compass

(b) Workshop Practice.—The ordinary workers in workshops require handy instruments which may be used by them readily without the necessity of arithmetical calculations. For length measurements, *simple callipers* and *gauges* have proved to be suitable.

(i) The Simple Callipers.— Such an appliance consists of two similar pieces of metals hinged together at one end and suitably curved at the other end Fig. 6(*a*) shows one such instrument commonly used for the measurement of external diameters, and Fig. 6(*b*)

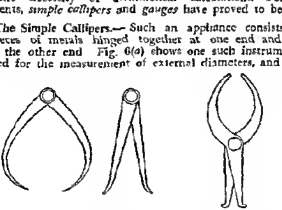

Outside Callipers (*a*) Inside Callipers (*b*) Combined Callipers (*c*)

Fig 6—Simple Callipers

depicts another such instrument used for internal diameters, while Fig. 6(*c*) represents a combined instrument, the upper part being used for external diameters and the lower part for internal diameters.

The method of use is to stretch out the free ends till their distance apart equals the length under measurement, whether the length is an external diameter, or internal diameter, or the length of any piece. *Then this measure taken by the callipers referred to some* ... *a gauge for comparison.* For turning and boring work in workshops, the standards of reference formerly consisted solely

of the cylindrical **External and Internal Gauges,** one pair of which is shown in Fig. 7.

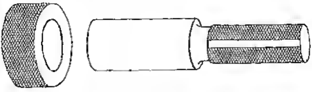


Fig. 7—Internal and External Gauges.

These gauges are manufactured true to $\frac{1}{10,000}$ inch. The workman sets his callipers to the standard gauge by his sense of touch and then transfers it to the job for comparison and makes the finish accordingly. The accuracy of the finished job depends on the skill and experience of the workman.

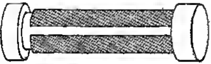


Fig. 7(a)—Internal Limit Gauge.

(ii) Limit Gauges.— Interchangeable machine parts are the growing demands of to-day. Such parts require to be machined to a definite degree of accuracy. To attain this accuracy *limit gauges* are used as *standards* of reference in modern practice. Fig. 7(a) shows an internal limit gauge. One end of it is slightly smaller than the other, the difference in the diameters being decided upon by the accuracy to which it is intended to work. The principle is that the smaller end must go in while the larger end must not, if an internal diameter has its proper value. The external gauge [Fig. 7(b)] is also similarly used for turning cylindrical pieces.

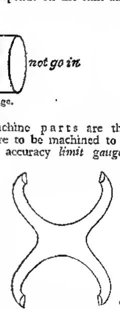


Fig. 7(b)—External Limit Gauge.

(c) Laboratory Work.—

(i) An ordinary Scale.— For ordinary measurements of lengths in the laboratory where a measurement correct up to a millimetre, or one-eighth, or one-sixteenth of an inch is sufficient, an ordinary straight scale is directly used. Such a scale is usually made of box-wood

or steel with one edge generally graduated in inches and the other in centimetres (Fig 8) An inch is again ordinarily sub-divided into 8 or 16, 10 equal parts and a centimetre into tenths, *i.e.* into ten parts, each being a millimetre.

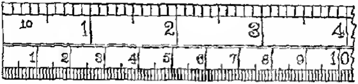


Fig 8—An Ordinary Scale

The ends of the scale should not be used in measuring a length, for they are liable to wear out with use In making a measurement, the scale is to be placed alongside the length under measurement, one of the graduations of the scale being made to coincide with one end of it and the length is then to be read off from the graduation of the scale coinciding with the other end. If this end does not correspond to any mark of the scale exactly, the fraction of a scale division is ascertained by *eye-estimation*.

Steel scales are usually one foot long while a metric scale is a **metre scale or a half-metre scale.**

Diagonal Scales and Vernier Scales.—The accuracy of a reading is liable to vary from person to person if eye-estimation is used to read the fraction of a division Again, in eye-estimating the fraction of a sub-division, a quantity less than half or one quarter of one sub-division is difficult to be ascertained without unduly straining the eye Yet in our physical measurements such fractions often require to be determined accurately. Two devices have been made available to us for such measurements without sub-dividing the small divisions of a scale further, one is the *Diagonal Scale* and the other a *Vernier Scale*. By them the measurement of the fractional part of a sub-division is mechanically made at a fixed accuracy

(ii) Diagonal Scale.—The advantage of this scale (Fig. 9) is that if the smallest division marked on the scale reads up to, say, $\frac{1}{10}$th unit, it is possible with the help of *dividers* to read dimensions up to $\frac{1}{100}$th unit without further sub-dividing the smallest units. The arrangement is as follows: One extra unit length is extended to the left and is divided into 10 equal parts at the top edge and also at the bottom edge. If the smallest sub-division of the scale is 0·1 unit, to read 0·01 unit with this scale, the zero mark of the extra *unit length is joined by an oblique line to the 1* mark of the top-edge

and the 1 mark of the bottom-edge to the 2 mark of the top-edge and so on successively. The width of the scale is divided into ten equal spaces by lines drawn horizontally; these parallel lines are cut

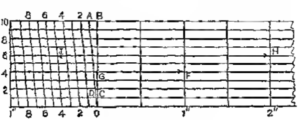


Fig. 9—Diagonal Scale.

perpendicularly by the lines of unit divisions such as 1″, 2″, etc. The principle of measurement is as follows:—

Consider the $\triangle OBA$. The distance OC is $\frac{1}{10}$ of OB. As OA and OB are straight lines, the distance CD must equal $\frac{1}{10}$ of BA, from the property of a triangle. But BA is 0·1 and therefore CD is 0·01. The lengths on the scale are read off by figures on the bottom horizontal line and hundredths by the figures on the vertical line at the left end of the scale. For example, any length like 1·04″ will be obtained by putting the point of one limb of a pair of dividers at the intersection of the vertical through the mark 1″ with the fourth parallel (shown by the point F) and the point of the other limb at the intersection of the 4th parallel with the zero diagonal, *i.e.* the diagonal OA (shown by the point G). Similarly, a length 2·46″ will be obtained by putting the points of the two limbs of the divider at H and J.

Note.—As already pointed out, $CD = BA \times \frac{OC}{OB}$. By making the ratio $\frac{OC}{OB}$ as small as we like, we can make CD any small portion of BA.

(iii) The Vernier.— The device carries the name of its inventor, Pierre Vernier, a Belgian Mathematician. It is a short scale by the help of which the fractional part of a main scale division can be determined mechanically at a fixed accuracy. This auxiliary short scale is placed in contact alongside the main scale and can be slided along it.

Verniers may be straight or angular as desired and the method of their use is the same.

Fig. 10 shows a straight scale the upper edge of which is graduated in millimetres and is provided with a sliding auxiliary scale representing a straight vernier. Readings up to one-tenth of a scale division may be taken with the help of this vernier. The lower edge is divided in inches and each inch is sub-divided again into 8 equal parts. A straight vernier slides along it. Readings up to $\frac{1}{64}$ inch may be taken with the help of this vernier.

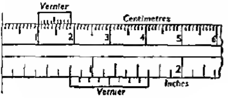


Fig 10—Straight Vernier

Fig 11 shows an angular scale with an angular vernier sliding along it, as is found in a spectrometer, sextant, etc. The main scale

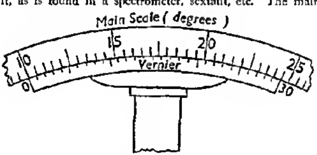


Fig 11—Angular Vernier

is graduated in degrees and each degree is again sub-divided into 2 parts Here the vernier has 30 divisions, and they coincide with 29 divisions of the main angular scale. Readings up to 1 minute may be taken with the help of this vernier

General Theory.—The vernier is so divided that a certain number n of its divisions is equal to $(n-1)$ or $(n+1)$ principal scale divisions

If v=value of one vernier division, s=value of one scale division, we have, $(n \mp 1)s = nv$; or, $v = \frac{n \mp 1}{n} s$

$\therefore$ *The least count=Diff of s & v=1/n×s.*

So the vernier is said to read 1/nth of a scale division.

N.B. The **least count (or vernier constant)** of a straight vernier is expressed as a decimal millimetre or centimetre but the vernier constant of an *angular vernier* is expressed in *minutes* or *seconds* and not in decimals

How to use a Vernier.—

Vernier Type (1).— (*i*) Find the *value* in fraction of an inch, or centimetre, or degree in case it is an angular vernier, of *the smallest division of the principal scale.* Let it be 1 mm., *i.e.* 0·1 cm. in Fig. 12 (Type 1).

(*ii*) Count the number of divisions on the vernier, and slide the vernier to one end (*i.e.* to the zero position) of the main scale in order to find the number of scale divisions to which these are equal. In Fig. 12 (Type 1), 10 vernier divisions = 9 scale divisions.

(*iii*) Calculate the difference in length between one scale division and one vernier division. This is the smallest amount—called the **least count** (or **vernier constant**)—which can be read with the help of the instrument.

Here, 10 vernier divisions = 9 scale divisions.

$\therefore$ 1 vernier division $= \frac{9}{10}$ scale division.

$\therefore$ Least count = 1 sc. div.—1 ver. div.

$$= \left(1-\frac{9}{10}\right)\text{ sc. div.} = \frac{1}{10}\text{ sc. div.}$$

$$= \frac{1}{10}\times\frac{1}{10}\text{cm.} = 0{\cdot}01\text{ cm.}\quad (\because 1\text{ sc. div.} = 1\text{ mm.})$$

(*iv*) Now put the object AB to be measured on the scale, one of its ends A being at zero. The vernier is then pushed along the scale until its zero just touches the opposite end B of the object. Read the principal scale just before the zero of the vernier. It is 6 in Fig. 12 (Type 1). Then the length of the object AB is greater than 0·6 cm. (but less than 0·7 cm.) by the distance between the 6th division of the principal scale and the vernier zero. To get this length—

(*v*) Look along the vernier to see which of its divisions coincides with a scale division. The 2nd verner division coincides with a scale division. *Multiply this number by the least count and add this to the reading obtained from the principal scale.* This is called a **forward reading** (or **positive**) **vernier** or an **ordinary vernier** and is the most common form of vernier.

The value of the fraction of the scale division between the 6th and the vernier zero = (2 × 0·01) cm. = 0·02 cm.

$\therefore$ The length of the object = 0·6 + 0·02 cm. = 0·62 cm.

Verify thus—

The length of the object AB (Fig. 12) = 8 sc. divs. − 2 ver. divs. = 8 mm. − $\left(2\times\frac{9}{10}\right)$sc. divs. (for 1 ver. div. $=\frac{9}{10}$ sc. div.) = 8 mm. $-\frac{9}{5}$mm. $=\frac{31}{5}$ mm. = 0·62 cm.

Vernier Type (2).— In Type 1, a vernier division is smaller than a scale division, but sometimes, though very rarely, the vernier division may be larger than the scale division such that $(n+1)\ s = nv$.

In the second form (Fig. 12, Type 2) we have 10 ver divs $=11$ sc. divs.

$\therefore$ 1 ver. div. $=1\frac{1}{10}$ sc. div.

$\therefore$ Least count$=$1 ver. div. $-$ 1 sc. div.$=\frac{1}{10}$ sc. div.$=$0·1 mm.$=$0·01 cm.

A vernier division, in this case, is $\frac{11}{10}$ of a scale division, *while the numbering of the vernier divisions runs opposite to the main scale.*

In measuring a length *AC*, one end *A* is put at the zero of the scale as in the case of the ordinary vernier, and to the other end *C*, the *zero of the vernier* is brought. To know the fraction of the scale division, by which the *zero of* the vernier is ahead of the 26th mark of the scale, the point of coincidence of any mark of the vernier

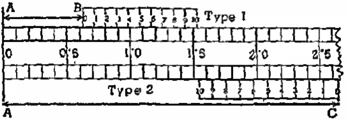


Fig. 12—A combined positive and negative Vernier

with a mark of the scale is noted and in the figure this point of coincidence is the 4th mark of the vernier. The 3rd mark of the vernier thus is ahead of the preceding mark of the scale by 0·1 unit and the 2nd by 0·2 unit, and so on till the zero mark of the vernier is reached which must be ahead of the 26th mark of the scale by 0·4 unit This is the fraction wanted. So the total length *AC* is 26·4 mm.$=$2·64 cm.

N.B. What is to be remembered here is that the numbering in this second type of vernier must run in a direction opposite to the run of the main scale, for if it did otherwise, *i.e.* had the zero of the vernier been on the same side as the zero of the main scale, the unknown fraction of distance between its zero mark in this condition and the preceding mark of the main scale would have gone on increasing by a length equal to the least count at the end of each progressive division of the vernier on account of a division of the vernier being greater than a main scale division.

Vernier of Type 2 are called **Backward Reading** or **Negative** ..., or **Barometric** verniers When they had been invented, ... were first fitted to scales attached to barometers and this is why

they are often referred to as barometric verniers. Such verniers are now-a-days very seldom used.

It will be clear from Fig. 12 (Type 2) that an ordinary vernier would have been useless if used to measure a long length like AC, for a greater part of the vernier in that case would have gone outside the main scale. *Thus, lengths long enough to run up to the end of a given scale cannot be measured with an ordinary vernier.* While short lengths like AB cannot be measured with a bakward-reading vernier, for the vernier will, in that case, go out of the zero of the main scale and become useless. The backward-reading vernier was fitted to barometers, for it could work up to the very end of the main scale.

Note.—All verniers are not exactly the same as the one described, but by adopting the same rules, as given above, any vernier can be read.

18. Measurement of Small Lengths:—In the laboratory the following three instruments, namely (*a*) Slide Callipers, (*b*) Screw-gauge, and (*c*) Spherometer, are commonly used for measuring the fractional part of a main scale division in measurements of small lengths. They have their own fields of application depending on the suitability of the instrument and the convenience of measurement.

(a) The Slide Callipers.—The principle of the vernier is applied to a number of instruments of which the simplest is the slide callipers. Fig. 13 shows the arrangements of the appliance. The main scale

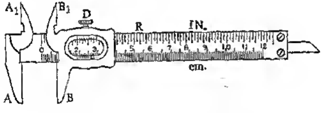


Fig. 13—Slide Callipers.

has been drawn on a steel frame R at right angles to which there are two steel jaws one of which AA_1 is fixed at one end. The other BB_1 is provided with a vernier and a fixing nut D and is slided along the main scale in making a measurement.

The measurements are in inches when the scale in the upper edge is used and are in centimetres with that in the lower edge. The object under measurement is put between the jaws (lower jaws

for *external* diameters or lengths, and upper jaws for *internal* diameters or lengths of small gaps) and the sliding jaw is adjusted till the material is held between them with the minimum pressure; in this position the sliding jaw is fixed up by the fixing nut *D*. The position of the zero of the vernier is then found with the help of the main scale and the vernier as usual.

Instrumental Error.—*When the jaws are in contact*, the zero of the vernier should coincide with the zero of the main scale. The error that arises, if the jaws do not coincide, is called the *zero-error* or *instrumental error*. This error, therefore, will be equal to the distance of the zero of the vernier from the zero of the main scale. It is regarded as *negative* if the zero of the vernier is on the left of the zero of the main scale, *i.e.* towards the fixed jaw AA_1, *when the two jaws are just in touch* and the correction is positive, *i.e.* it is to be added to the observed reading; if the zero of the vernier is on the right of the zero of the main scale, the error is *positive* and the correction is negative, *i.e.* it is to be subtracted from the observed reading. Hence the **correct length = observed length ± instrumental error.**

Screw and Nut principle.— The principle is that when a screw works in a fixed nut perfectly, the linear distance through which the point of the screw moves is directly proportional to the rotation given to the screw-head. In other words, the linear distance travelled through by the point of the screw for one complete rotation is constant. This constant is called the **pitch of the screw** which is evidently the distance between corresponding points on two consecutive turns of the thread as shown by p in Fig. 14.

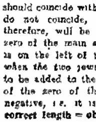


Fig. 14—*p shows* the pitch of the screw.

The principle of the screw and nut is applied to some common laboratory appliances like the screw-gauge, the spherometer, etc. The pitch of the screw in these instruments is usually 1 mm. or ½ mm.

Micrometer Screw.—A micrometer screw is a common laboratory appliance used for the determination of small lengths at a fixed accuracy. The arrangement in it is simple. There is a circular scale, called the **micrometer-head**, of large diameter, fitted to the screw and also a linear scale arranged parallel to the axis of the screw. The linear scale is ordinarily graduated in millimetres and the circular scale is divided into 100 or 50 equal divisions.

Least Count.— If the circular scale on the screw-head is divided into n divisions, and the pitch of the screw is p, then the distance $\frac{p}{n}$ travelled by the screw-point for a rotation of the screw-head through one circular division is the smallest length that can be determined accurately and is called the **least count** of the instrument.

Back-Lash Error.—This is an error which is associated, more or less, with all instruments working on the screw and nut principle. And instruments, perfect when new, may develop this error with use due to wear and tear. Due to looseness between the screw and the nut, equal amount of rotation of the screw-head in opposite directions may be found to produce unequal linear motions of the point of the screw. Error due to this uncertainty is called **back-lash error.** This error may be avoided, if the screw is turned always in the same direction, when a adjustment is made while taking a measurement.

(b) The Screw-Gauge.—The screw-gauge (also called the *Micrometer Screw-Gauge*) is used for measuring accurately the dimensions of small objects, such as the diameter of a wire, the thickness of a metal plate, etc. It consists of a fixed rod A (Fig. 15) having a plane end and a moveable rod B having also a plane end facing A. The rod B has a screw cut on it and the screw works inside a hollow cylinder, called the **hub** having a straight scale L **(linear scale)** etched on it along a reference line R. This scale is used to indicate the number of complete turns of the screw. The rod A and the hub are firmly held co-axially at the two ends of a strong metal bar bent in the U-form. The screw is worked by means of a large milled screw-head H which moves over the outside of the hub. Any fine adjustment of the screw-head is made by turning a head, called the *friction clutch* (not shown in the figure) with which all modern instruments are fitted. It should be turned with gentle uniform pressure. On being rotated, it automatically slips as soon as A and B touch each other. The

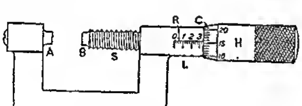


Fig. 15—A Screw-Gauge.

levelled edge of the screw-head has etched on it a circular scale C, called the **head-scale** which is divided into a number of equal parts, usually 50 or 100, and is used for the determination of the fraction of one complete rotation. One complete turn of the head-scale moves the end of the screw through a distance equal to its **pitch,** which is the distance p between the consecutive threads of the screw (Fig. 14). *So pitch is the amount by which the gap between A and B* (Fig. 15) *is opened or closed by one complete rotation of the head-scale.*

the case, read the number of divisions between the zero of the disc-scale and the edge of S, which is the **zero error**. Repeat the observation several times and take the mean of the readings as the *zero error*. *This quantity must be subtracted algebraically from all readings taken with the instrument.*

Note.—If the **zero of the disc-scale** is **above** the zero of the vertical scale, the difference of the positions of these two zero marks —which is the zero error—is taken as **positive**, and the quantity is to be *subtracted* from the total reading. If the zero of the disc-scale is **below** the zero of the vertical scale, the zero error is negative, and it should be *added* to the total reading.

For example, let the error be 3 divisions of the disc-scale behind its zero, *i.e.* below the zero of the vertical scale, then the value of the zero error is $-(3 \times 0.005) = -0.015$ mm (taking 0.005 mm. as the least count). If now the reading taken with the instrument be, say, 1.27 mm., the corrected reading will be $\{1.27-(-0.015)\} = 1.285$ mm.

Had the error been 3 divisions of D above the zero of the vertical scale, the value of the zero error would have been $+0.015$ mm., and in that case the corrected reading would have been $\{1.27-(+0.015)\} = 1.255$ mm.

(1) To Measure the Thickness of a Plate of Glass (by Spherometer).—

(a) Pitch-Scale Method.— First determine the zero error of the spherometer placing it on a base plate. Now raise the screw and place the test plate on the base plate underneath the screw point, and then take the readings of the vertical scale S, and the disc D, when the screw point just touches the top of the plate, while the other three feet of the instrument still stand on the base plate. Repeat the observation several times at different places on the surface of the test plate and take the mean reading. The difference between this and the zero error gives the average thickness of the plate.

(b) Rotation Method.— It is found with most of the spherometers that two complete turns of the disc are necessary to move the screw-point through one division of the vertical scale.

At the time of taking any reading with such an instrument it is often found difficult to judge whether the reading indicated on the disc-scale is a fraction of the first or the second revolution after passing the last division of the vertical scale. For this, and also *to avoid the zero error*, it is convenient not to take any account of the vertical scale reading. Instead of this, the movement of the screw-point should be stated in terms of the rotation of the circular divisions only. That is, placing the test plate on a base plate,

(i) first note which division of the circular scale is against the zero of the vertical scale when the screw-point touches the top of the test plate while the outer three legs of the spherometer stand on the top of the plate and then, on removing the test plate,

(*ii*) count from this, the whole and fractional turns of the circular scale until the screw-point again just touches the base plate. If, for example, 2 whole turns and 56 small divisions of the third turn are necessary for this adjustment, the thickness of the plate=2 whole turns+56=256 divisions=256×0·005 mm. (∵ 0·005 mm. is the least count)=1·28 mm.

(2) To Measure the Radius of Curvature of a Spherical Surface (by Spherometer) :—

Pitch-Scale Method.— (*i*) Place the spherometer with the fixed legs resting on the curved surface, and adjust the screw until its point just touches the surface. Read the scales. Repeat the observation several times placing the instrument in different positions of the curved surface. Calculate the mean of the readings.

(*ii*) Place the instrument on a plane glass plate and adjust the screw until its point touches the surface. Read the scale. Repeat it several times, and take the mean reading.

(*iii*) Find the difference h between the two mean readings. This gives the vertical distance traversed by the screw-point.

(*iv*) Measure the distance d between any two of the three legs. To do this, place the instrument on a piece of paper and press gently so as to mark the positions of the three legs D, E, F (Fig. 17).

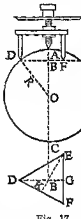


Fig. 17

Now measure carefully the mean distance d between these marks. Then the radius of curvature R is given by,

$$R = \frac{d^2}{6h} + \frac{h}{2}.$$

Note.—(1) As d enters as a square in the result, the measurement of d should be made very carefully, otherwise, any small error in this measurement will be magnified in the final result.

(2) *Do not forget to express both d and h in the same unit.*

(3) Here also the *rotation method* of measurement, as described above [see p. 26], may be applied. That is, calculate the value of h from the readings of the *circular scale only* without taking any account of the vertical scale.

(4) When using a spherometer, it should be noted whether there is any slackness between the nut and the screw, because any such slackness will permit of appreciable rotation of the disc without producing any corresponding movement of the screw along its axis. This error, due to lost motion, called the back-lash error, can be avoided by always turning the screw in the same direction while taking any reading.

(5) The term $h/2$ can often be neglected in comparison with $d^2/6h$.

Proof of the Formula.—The diagram (Fig. 17) represents a side view of a spherometer resting on a spherical surface. The central leg A and two of the fixed legs of the spherometer are visible. AB represents the vertical distance h through which the central leg must be raised (or lowered) so that it may just touch the curved surface. DB (S) is the distance between any of the fixed legs and the central leg, when they are all resting on a plane surface. If R be the radius of curvature, we have $DO^2=DB^2+BO^2=DB^2+(AO-AB)^2$,

$$\text{or, } R^2=S^2+(R-h)^2;\ \text{ or, } R=\frac{S^2+h^2}{2h}=\frac{S^2}{2h}+\frac{h}{2} \qquad \ldots\ (1)$$

The formula (1) can be put into *another form*. When the central leg just touches the plane of the other three legs, let B be the position of the central leg, and D, E, F, the positions of the other three legs which form an equilateral triangle (Fig 17, lower). The angle GDF is 30°, and K is the middle of DF, the length of which, say, is d.

$\therefore\ DK=DB\ \cos 30°=S\sqrt{3}/2$, or, $d/2=S\sqrt{3}/2$, or, $d^2=3S^2$.

Substituting the value of S^2 in (1) we have,

$$\mathbf{R}=\frac{\mathbf{d^2}}{\mathbf{6h}}+\frac{\mathbf{h}}{\mathbf{2}}.$$

N.B. The method of measurement is the same for both convex and concave surfaces.

19. Measurement of Area:— To find the length of a straight line, it is necessary to take only one measurement. So length is said to have one dimension. But in order to measure an area two linear measurements are necessary. Thus for the area of a rectangle, two lengths, length and breadth, must be measured. That is, **an area has two dimensions.**

Units of Area.—The unit of area in the Metric system is the *square centimetre*, and that in the British system is the *square foot*.

Metric Table of Area	British Table of Area
100 sq. millimetres = 1 sq. centimetre	144 sq. inches = 1 sq. foot
100 sq. centimeters = 1 sq. decimetre	9 sq. ft. = 1 sq. yard
100 sq. decimetres = 1 sq. metre.	4840 sq. yds. = 1 acre
	640 acres = 1 sq. mile

(a) Areas of Regular Figures.— In order to measure the areas of regular geometric figures, two linear measurements, as involved in the following relations, are to be taken:—

Area of rectangle = length × breadth.

,, ,, parallelogram = base × perpendicular height.

,, ,, triangle = ½ × base × altitude.

,, ,, trapezium = ½ × sum of parallel sides × perpendicular distance between them.

,, ,, circle = π × (radius)^2.

,, ,, ellipse = π × semi major axis × semi minor axis.

Area of the curved surface of cylinder = circumference of cross-section $\times$ length.

Area of the surface of a sphere = area of the curved surface of the circumscribing cylinder $= 2\pi r \times 2r = 4\pi r^2$,

(where r = radius of the sphere, r is also the radius of the base of the circumscribing cylinder, and the height of the cylinder $= 2r$).

(b) Areas of Irregular Figures.—The area of an irregular figure can be measured, by dividing the figure into suitable regular figures like triangles, rectangles, circles, etc., and then adding up the areas of these parts. Besides, it can be measured.

(i) By squared paper.—*Draw an outline of the figure on a squared paper.* The boundary of the figure (Fig. 18) passes through a number of small squares on the paper. Now count the total number of complete squares and then count round the boundary line, *ignoring those which are less than half a square, but counting as whole squares those that appear to be more than half squares.* In the case of exact halves, take two to count as one square.

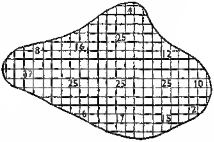


Fig. 18

N.B. This is only a rough method, and the result will not be very accurate. This method, however, can readily be applied to find the area of a country by drawing to scale, on squared paper, **a map of its boundary.** If the above figure represents the boundary of the map of a country, then the area of the country can be calculated as follows :—

Scale of map, 80 miles = 1 in. or, 1 sq. in. = 6400 sq. miles.

Hence, area of the country = 6400 $\times$ area on graph paper in sq. inches (sq. miles)

(ii) By Planimeter.—The area of any plane figure can be directly recorded by this instrument. The two arms AB, BC are freely joined at B. A graduated wheel D revolves in the elbow at B round an axis parallel to the arm BC.

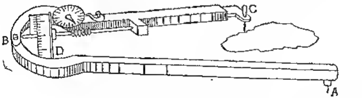


Fig. 19—A Planimeter.

In use, the lowest part of the wheel D, the spikes A and C touch the plane of the figure, the point A being kept fixed in position somewhere at a convenient position outside the figure while C is

slowly moved along the boundary line of the figure in such a way th it returns to its initial position finally. The difference between th readings of the wheel before and after the spike C goes round the figure gives the area

(iii) By weighing.—Draw the figure on a thin sheet of cardboard, or a thin metal plate, whose thickness should be as uniform as possible. Cut the figure out of it, and weigh it accurately. From the same sheet cut an area the shape of which may conveniently be a rectangle, or a triangle, and find its weight. Calculate the area of the rectangle or the triangle, from its linear dimensions. Then calculate the area of the figure from the relation,

$$\frac{\text{area of figure}}{\text{area of rectangle}} = \frac{\text{weight of figure}}{\text{weight of rectangle}}.$$

20. Measurement of Volume :—The space occupied by a body is called its volume. In order to measure the volume of a body *three lengths, i.e. length, breadth, and height or thickness,* should be considered. Therefore, **a volume has three dimensions.**

Unit of Volume.—The unit of volume in the metric system is the *cubic centimetre* (c.c), and that in the British system is the *cubic foot* (cu. ft.).

A common unit of volume for liquids in the British system is one **gallon** (1540 c.c.) which is equal to the volume of 10 lb. (avoir) of pure water at 62°F., while that in the C.G.S. system is one **litre**, which is equal to the volume of 1 kilogram or 1000 c.c. of pure water. So one millilitre (ml.) means 1 c.c.

METRIC TABLE OF VOLUME

1000 cubic millimetres = 1 cubic centimetre (c.c.)
1000 cubic centimetres = 1 cubic decimetre (1 litre)
1000 cubic decimetres = 1 cubic metre.

BRITISH TABLE OF VOLUME

1 cubic foot = 1728 cubic inches (cu. in.)
1 cubic yard = 27 cubic foot (cu. ft.)

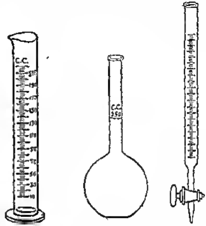
Remember the following :—

The litre is the volume of 1 kilogram of cold water.
One gram of cold water fills one cubic centimetre.
One fluid-ounce equals 28·35 cubic centimetres.
One cubic foot equals 28·31 litres.

One cubic foot of cold water weighs 62·5 lbs.
The gallon is the volume of 10 pounds of cold water.

One gallon equals 4·54 litres.
One fluid-ounce is the volume of 1 ounce of cold water.

(a) Measurement of Volume of a Liquid.—The volume of a liquid is readily measured by pouring it into a graduated vessel. These graduated vessels are available in various forms. Fig. 20(*a*) represents a graduated cylinder, the markings in it being usually in c.c. Fig. 20(*b*) shows a measuring flask whose capacity is *ordinarily marked* on its body and is used when a definite volume of a liquid equal to its capacity is taken initially. Fig. 20(*c*) shows a burette in which a liquid is taken when a measured volume of it is to be poured into a vessel.



(*a*) Measuring Cylinder (*b*) Measuring Flask (*c*) Burette

Fig. 20

(b) Volumes of Regular Solids.—To calculate the volume of a solid which has a regular geometrical figure.

Remember the following :—

Volume of rectangular solid (cuboid)	= length × breadth × height
,, ,, cylinder	= area of base × height.
,, ,, pyramid, or cone	= $\frac{1}{3}$ × area of base × height.
,, ,, sphere	= $\frac{4}{3}\pi$ × (radius)^3.

Volume of a Sphere = $\frac{4}{3}\pi r^3$.

Proof.—The surface of a sphere can be imagined to be divided into an infinite number of small figures [Fig. 20(*d*)], each of which is practically a plane surface and may be considered to form the base of a pyramid having a height equal to the radius *r* of the sphere, *i.e.* with its top at the centre of the sphere. The sum of the bases of all the pyramids is the whole surface of the sphere, and the sum of all these small pyramids is the volume of the sphere.

The volume of a small pyramid $= \frac{1}{3} \times$ area of base $\times$ height

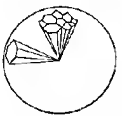


Fig. 20(*d*)

$\therefore$ The volume of the sphere $= \frac{1}{3} \times$ sum of the area of the bases of all the pyramids $\times$ height.

$= \frac{1}{3} \times$ surface area of the sphere $\times$ radius

$= \frac{1}{3} \times 4\pi r^2 \times r = \frac{4}{3}\pi r^3$;

[$\because$ surface area of a sphere $= 4\pi r^2$.]

(c) Volumes of Irregular Solids.— The volume of a small piece of an irregular solid, or that of a regular one can be determined,

(i) By displacement of water.—The volume of a small solid may be directly obtained by lowering it carefully into water contained in a graduated vessel, say, a graduated cylinder as depicted in Fig. 20(*a*) and noting the rise of the surface of the water. The rise of the surface, *i.e.*, the difference between the first and second positions of the meniscus, gives the volume of water displaced by the solid; and, as a body immersed in a liquid displaces its own volume of the liquid, the difference between the two positions of the meniscus gives the volume of the solid

When the body is too big to go inside the measuring vessel, secure a fairly large vessel and attach a narrow piece of gummed paper vertically to the side of it Put a horizontal pencil mark at a level which will be well above the top of the immersed solid Pour water in the vessel until its surface is in level with the pencil mark.

If now the solid is introduced, an equal volume of water will be displaced or pushed above Put another mark corresponding to the surface of water again. Then take out carefully by a pipette the amount of displaced water, *i.e.* the amount of water between the two pencil marks, and measure it by a graduated vessel This will give the volume of the solid

Note.—(1) If the solid floats in water, push it by a needle fixed to the end of a wooden pen holder until the solid is completely immersed

(2) If the solid is **soluble in water**, use instead of water, some other liquid, say, alcohol or kerosene, in which it is not soluble

(ii) By weighing.—Knowing that at ordinary temperatures *one cubic centimetre of water weighs one gram*, the volume of a small solid can be accurately determined by weighing the amount of water displaced by it.

If the weight of the displaced water is, say, 10 gms, then the volume of displaced water is 10 c.c. (because the volume of 1 gm. of water is equal to 1 c.c.), and so the volume of the solid is also 10 c.c. So, the weight in grams of the displaced water is numerically equal to the volume of the body in cubic centimetres. If the solid is

soluble in water, a liquid, in which the solid is insoluble, is to be taken. To know the volume in this case, the weight of the displaced liquid in grams is to be divided by the density of the liquid (*vide* Art. 260). To measure volume by weighing, another method based on the Archimedes' Principle (*vide* Chapter X) is available.

21. Measurement of Mass :— The mass of a body is ordinarily measured by means of a common balance (*vide* Art. 190). It can also be measured by a spring balance after calibrating it (*vide* Art. 199).

22. Measurement of Time :— Any process which repeats itself after a regular interval of time can be used to measure time. Depending on this principle the ancient people devised various devices, like the sun-dial, the hour-glass, the water-clock, etc. for measuring time.

(a) The Sun-dial.— This instrument (Fig. 21) was universally used by the ancient people. It consists of a horizontal circular board

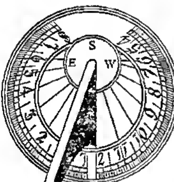


Fig. 21—The Sun-dial.

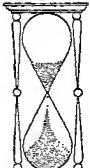


Fig. 22—The Sand-glass.

which has graduations from 1 to 12 as those on a clock. A triangular plate of metal fixed on the board vertically in a north-south direction serves as an obstacle to the rays of the sun. Any particular period of the day is indicated by the position of its shadow cast by the sun on the graduated board at that time. The shadow is longest when the sun is on the horizon, *i.e.* at the time of the sunrise or the sunset. After the sunrise as the sun rises up in the sky, the length of the shadow shortens and finally at noon when the sun is at the zenith, the shadow vanishes. After the noon it changes side and the shadow lengthens again as the sun declines.

The sun-dial can be used only on a sunny day and cannot be used at night or on a cloudy day.

(b) The Hour-glass (or Sand-glass).— This consists of two conical flasks joined neck to neck (Fig. 22) having an inter-communi-

cating narrow opening in the middle. A measured quantity of dry sand is taken in the upper flask and the principle involved in the measurement is that a definite interval of time is necessary for the passing of the sand from the upper flask into the lower.

(c) Clocks and Watches.—Clocks and watches are now-a-days universally used for the measurement of time and have practically superseded all primitive time-measuring devices. Their construction has been possible after the discovery of the laws of pendulum (*vide* Chapter V) in 1583 and it was left to a Dutch Physicist, Huygens, to use a pendulum afterwards for measuring time. In 1658 a clock fitted with a pendulum was first used by him to measure time. Since then, however, vast improvements in the mechanism of clocks have been made by later workers.

The length of a seconds pendulum (*vide* Art. 123) can be so chosen at any given place as to take one second to swing from one extreme position to the other. The motion of the pendulum can be communicated at the end of every swing to the hands of a clock by means of suitable mechanism. The hands move over a dial graduated in hours, minutes, and seconds. The energy of the pendulum is taken from a wound spring which runs it. The spring requires to be wound after regular intervals.

(i) The Watch.—The principle of a watch (pocket watch or wrist watch) is the same as that of the clock except that the pendulum is replaced here by a balance wheel controlled by a hair spring. The balance wheel oscillates, the necessary energy being supplied by a wound spring as in the case of the pendulum clock.

A Chronometer is a specially constructed watch which gives time with the greatest precision and is generally used for comparison purpose in regulating ordinary clocks or watches.

(ii) The Stop-watch or Stop-clock.—It is used when a small interval of time during an event or between two events is to be recorded.

Fig. 23—A Stop watch

The Stop-watch.—It is a specially constructed watch (Fig. 23) having a long second-hand which moves over a circular dial with 60 equal divisions, each division representing a second. Each such division is usually subdivided into *fifths* or *tenths*. At the beginning of an event when the knob at the top is pressed, the second-hand *starts* and it *stops* when pressed for the second time at the end of the event. There is a small minute-hand which moves over a small circular dial graduated into 60 divisions, each representing a minute so that one complete rotation of the minute-hand through

360° means an interval of one hour. The time recorded by the minute and second hands, when the hands stop as the knob is pressed for the second time, *i.e.* at the end of an event, gives the interval of time during an event. The hands fly back to zero positions when pressed for the third time and the watch becomes ready again for new observations.

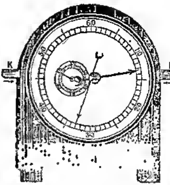


Fig. 23(*a*)—A Stop-clock.

(iii) The Stop-clock.— It is a table-clock run on the same principle as that of a stop-watch. The difference in mechanism is that a straight rod *KK* projecting out of the clock both ways at the sides is used to start or stop the clock [Fig. 23(*a*)]. When the right end of the projecting rod is pushed to the left, the clock starts and when pushed to the right from the left it stops. There is usually a third hand which can be set from outside over the second hand and it stays there indicating the starting time, when the minute and second hands are on the move.

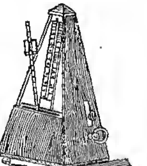


Fig. 24– A Metronome.

(iv) The Metronome.— This instrument (Fig. 24) is used to mark time. It has a mechanism (run by clock-work) to move the pendulum, by which ticks can be heard at the end of each swing. The ticking time can be altered by adjusting the position of a sliding-weight on the pendulum rod.

Questions

1. Give the construction and working of a spherometer. How would you determine the focal length of a lens with its help ? (U. P. B. 1948)

2. How would you measure the curvature of a spherical surface by a spherometer ? (M. B. B. 1951)

3. Give the principle of a vernier and explain its working. Each division of a main scale is 0·5 mm. 9 divisions of the main scale are equal to 10 divisions of the vernier. Length of a cylinder is measured. The readings are : 78 divisions of the main scale; and the 6th division of the vernier coincides with a division of the main scale. Calculate the length of the cylinder. (Del. H. S. 1953)

[*Ans.* 3·93 cm.]

4. The fixed legs of a spherometer are at the corners of an equilateral triangle of 4 cm. side. When adjusted on the surface of a spherical mirror, the instrument reads 1.500 mm. Find the radius of curvature of the mirror, taking zero error of the instrument to be zero. Prove the formula you use.
[*Ans.* 17.84 cm.] (U. P. B. 1931)

5. Assuming the earth to be spherical, calculate its surface area in square miles, taking its diameter to be about 8000 miles.
[*Ans.* 2.0114×10^{8} sq. miles.]

6. A circular ring is enclosed between two concentric circles whose radii are 119 ft. and 167 ft. long respectively. Find the length of the radius of a third concentric circle which will divide the ring into two rings whose areas shall be equal to one another.
[*Ans.* 145 ft.]

7. How would you measure the area of an irregular figure drawn on a sheet of paper?

8. Calculate the volume of gas in cubic feet contained in a cylindrical gasometer having a height of 150 ft. and diameter 150 ft.
[*Ans.* 2,651785.7 cu. ft.]

9. How will you find the volume of a solid of irregular shape?
(C. U. 1917, '29, Dec. 1932)

CHAPTER III

STATICS AND DYNAMICS

23. Body:—A body is a portion of matter limited in every direction. It occupies some definite space and has a definite size and shape.

A body is said to be **rigid,** if its parts always preserve invariable positions with respect to one another. Actually all bodies yield more or less under the action of forces. For our investigations, a body will be considered rigid unless otherwise stated.

24. Particle:—If a portion of matter is so small in size that for the purpose of investigations the distances between its different parts may be neglected, it is said to be a particle. *It is a material point occupying some position but having no dimension.* Rotation or spin has no meaning for it. Any motion of it only signifies a transference of position from one point in space to another.

25. Mechanics:—It is that branch of science which deals with the conditions of rest or motion of bodies around us.* It has two subdivisions, **statics** and **dynamics.** Statics is that branch of mechanics which deals with the science of forces balancing one another.

* The term *mechanics* was first used by Newton for "the science of machines and the art of making them." Subsequent writers, however, adopted this term as a branch of science which treats of the conditions of rest or motion of bodies around us.

The forces considered may act at a point or on a solid, a liquid, or a gas. The branch of statics which considers the relations between forces acting on a liquid *at rest* has a special name, **hydrostatics** and the branch which considers the equilibrium of a gas has another special name, **pneumatics.** *Dynamics* is that branch of mechanics which treats of the science of bodies *in motion*. It is divided into **Kinematics** and **Kinetics.** *Kinematics* deals with motion *without* reference to its cause. According to some writers this is a branch of pure mathematics. *Kinetics* is the science of motion *with* reference to its cause, *i.e.* it is the science of unbalanced forces or the relations between motion and forces. In **hydrodynamics** the relations between motion and force in fluids are considered. **Hydraulics** deals with the applications of the principles of hydrostatics and hydrodynamics to Engineering.

26. Position of a Point or body :—The position of a point or body lying on a plane can be determined in various ways of which the commonest is by finding the distances of it from two mutually intersecting straight lines (called the *axes of reference*) in the same plane measured along lines drawn from it parallel to the axes. These distances are called its *co-ordinates with reference to the axes.* The point of intersection of the axes is called the *origin*, its co-ordinates being 0,0. This is a standard or reference point taken apparently as fixed. The axes of reference may be mutually perpendicular to each other, when they are called *rectangular axes,* or they may be inclined to each other at an angle other than a right angle when they are called *oblique axes.* The rectangular axes are more convenient and are most commonly used. The co-ordinates referred to either rectangular or oblique axes are called *Cartesian co-ordinates* in honour of Rene Descartes (1596-1650) of Touraine, France.

27. The Rectangular Co-ordinates :— The horizontal and vertical lines XX' and YY' (Fig. 25) represent two rectangular axes having origin O. The co-ordinates of any point P referred to the axes XX' and YY' are respectively given by x and y, the former being called *the abscissa* and the latter, the *ordinate.* When the co-ordinates of a point with reference to a given pair of axes are given, the process of marking the position of the point on the plane is called *plotting the point.* A detailed study of how points are plotted on a graph paper using rectangular co-ordinates, *i.e.* how *graphs are drawn* is given in Appendix (B) at the end of the book.

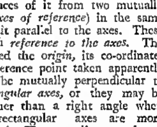


Fig. 25

Just as the position of any point on a given plane can be found when its co-ordinates with reference to two given axes in the plane

are given, the position can as well be traced if the *distance* of the point from the origin and the *angle* by which the line joining the point with the orgin is inclined to either of the given axes of reference are given. Both the above methods are used in our daily life. In geographical survey, generally, the observer himself or a very well-known object is taken as the origin and the geographical East-West and North-South lines passing through the origin are used as the axes of reference

For example, if it is stated that the playground of a college is a quarter mile to the South-East of the college premises, to arrive at the ground one has only to walk a quarter mile from the college premises along a direction equally inclined to the South and the East or, in the alternative, to walk $440 \times \cos 45°$ yds, *i e* 310·2 yds due South and then to walk further 310·2 yds due East It is to be noted, however, that in order to find the position of a point in *space*, i e when it is not sufficient to know its position in a given plane, its co-ordinates referred to three mutually perpendicular axes meeting at a common origin *apparently* fixed in space are to be known

28. Rest and Motion :— A body is said to be at rest when it does not change its position with time, it is said to be in motion when with time it changes its position

Absolute rest is unknown.— To know if the position of an object changes with time or not, a point *absolutely fixed* in space is required to be known No such fixed or stationary point is known in this universe When you say that a ball is at rest on the ground, the ground is considered stationary and the ball does not change its position with respect to it As a matter of fact, the ground, *i e* the earth, is not stationary; it is always in motion It moves round the sun and it also rotates about its own polar axis. The sun is also never at rest, with the planets bound to it it is in constant whirling motion amongst the galaxy of stars and the latter also are always in motion with respect to each other The ball, being on the earth, is sharing such motion and cannot be at absolute rest *So absolute rest is a term which has not meaning in reality* By stating that the ball is at rest on the ground, what is meant, as stated above, is that it is not changing its position with respect to the earth That is, rest here means relative rest *A body, therefore, is at relative rest with respect to another when it does not change its position relative to the latter.* A passenger seated in a running train is at relative rest with respect to the inmates of the train while actually he is moving with respect to the objects on the roadside Birds flying in the sky in a formation are at relative rest with respect to each other while they are in continuous motion

All Motion is relative.—As the motion of a body involves a change of its position, to measure motion a point fixed in position called the reference point is necessary, from which the change of

position is to be known. As already explained, no such fixed point is realisable in nature. So when we say that a body is in motion, the idea behind is that it is changing its position with respect to some known object, *i.e.* the body is in relative motion, with respect to the known object. It has been customary to refer the motion of all terrestrial bodies with respect to the earth.

29. Kinds of Motion (Translatory and Rotatory) :— The motion of a body may be either **translatory** or **rotatory** or both. The translatory motion may again be subdivided into *rectilinear* and *curvilinear motions.* A body is said to be in **translatory motion** when it moves in such a way that its constituent parts have such identical motion that the line joining any two points of the body always moves parallel to itself when the body is in motion. Fig. 26 illustrates the motion of body when the line joining any two points a, b is parallel and equal to the line $a'b'$ which joins up the same two points in a new position occupied by the body in course of its motion. So the motion is translatory. Moreover, the path of motion aa' of any point a, or bb' of any other point b is a straight line. Therefore this translatory motion is rectilinear too. When a stone *freely falls* from a height, when a train runs on *straight* rails, etc. a translatory rectilinear motion is produced.

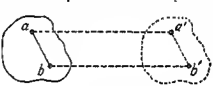


Fig. 26

When the motion of translation aa' takes place along a curved path it is called a curvilinear motion.

When a body turns about a fixed point or axis, it is said to be **in rotatory motion.**

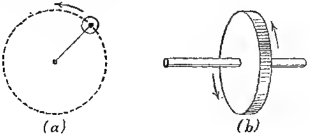


Fig. 27

A stone tied at the end of a string held in the hand and whirled round [Fig. 27(*a*)], the motion of a flywheel about a shaft [Fig. 27(*b*)], etc. are typical examples of rotatory motion.

A motion, besides being simply translatory or rotatory as already explained, may often be complex in nature resulting from a combination of a rotation and a translation. These both kinds of motion are involved when a body rolls down an inclined plane, or a rupee rolls on the floor. All sorts of complex motions may be produced by the suitable combination of the above two simple motions.

30. Terms connected with Motion :—

Displacement.—The displacement of a moving body in a given time is its change of position in a particular direction in that time, the change of position being found by joining the initial position to the final position by a straight line, whatever, might be the nature of the path actually traversed by the body in that time. The displacement is thus a quantity which has a magnitude as well as a direction, the length of the straight line giving the magnitude, the direction being given by that of the line in a *sense* pointing from the initial to the final position.

Suppose, a body starts from O, the origin of the rectangular axes XX' and YY' (Fig 28) and travelling along OCP in the plane of XX' and YY', reaches P in a given time Let x and y be the co-ordinates of P. Then the displacement of the body will be given by the dotted line OP whose magnitude is equal to $\sqrt{(x^2+y^2)}$ and is directed from O to P along the straight line OP which is inclined to the X-axis by an angle $\theta = \tan^{-1}\frac{y}{x}$.

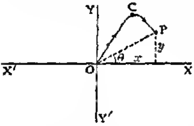


Fig. 28

Speed.—*The rate at which a moving body describes its path is called its speed.* It is measured by the distance travelled by the body along a straigth or curved path in unit time. It is a quantity which gives the idea of a magnitude only and has no reference to any direction.

It is said to be **uniform** when it passes over equal lengths of its path in equal intervals of time, however small these equal time intervals may be. It is *non-uniform* or variable when the body traces out unequal lengths of its path in equal intervals of time at different points of the path. When the speed is variable, often it becomes necessary to know the *speed at any instant* or *at any particular point of the path.* This is given by the actual distance passed over by the body in an indefinitely small interval of time around the instant in question divided by the time interval When the speed is variable, often it is very helpful in practice to know simply its **average speed.** The average speed of a moving body in

any given interval of time during its motion is given by the length of the traversed path divided by the time taken. If a length of path s is described by a body in time t, under variable motion, the average speed during that time is given by s/t. The body would have passed over the same length of the path in that time had it moved with a uniform speed s/t. Average speed may be taken, it should be noted, only if the variation of speed is small.

Velocity.—*The velocity of a moving body is its rate of displacement.* So it may also be defined as the *change of the position* of a moving body in a *definite direction* per *unit time*, or as the *distance traversed* by the body in a *definite direction* in *unit time*. To specify velocity, therefore, its *magnitude* as well as *direction* must be stated. Changes in either magnitude or direction or both change the velocity of a body.

Uniform Velocity.—The velocity of a moving body is said to be uniform when it always moves along the same straight line in the same sense describing equal distances in equal intervals of time, however small these intervals may be. In Fig. 29, a, b, c, d represent successive positions of a body having a uniform velocity of 10 ft. per sec.

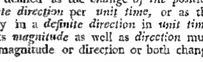


Fig. 29—Uniform Velocity.

Velocity at a point.—When the velocity of a body is *variable*, its value at any point of the path is given by the distance passed over by the body involving the point of the path in question in an indefinitely small interval of time divided by the time interval. If the path of motion is not a straight line, the direction of the velocity will be that of the tangent drawn to the curved path at the point in question pointing to the direction of motion. Thus, if a body moves along a curve OCP (Fig. 30) in the direction shown by the arrows, the velocity at any point C in the path will be in the direction CQ, which is tangential to the curve at the point C in question.

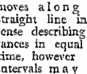


Fig. 30

Average Velocity.—When velocity of a body is non-uniform but taking place in the same direction, its *average velocity* is given by the total distance passed over by the body in a given interval of time divided by the time interval.

31. Distinction between Velocity and Speed:—(*a*) The velocity of a moving body is the distance traversed by it in a definite direction in unit time.

(*b*) The speed of a moving body is simply the distance traversed by it in unit time, where the distance *may not be in a definite direction*.

So, to specify a velocity completely, its *magnitude* as well as its *direction* must be stated; but to specify a speed completely, it is necessary to state only its magnitude. Hence *velocity is speed in some particular direction.*

To understand more clearly the difference between speed and velocity, take, for example, the case of a motor bicycle travelling round a circular track at a constant rate. In this case the speed of the bicycle is constant, but its velocity is constantly changing.

32. Units of Velocity or Speed :— A body has unit velocity or speed when it traverses unit distance in unit time.

The **C.G.S.** unit is	The **F.P.S.** unit is
one centimetre per second.	**one foot per second.**

33. Acceleration :— *The Acceleration of a body under motion is the rate of change of its velocity.* Acceleration is *uniform* when equal changes in velocity occur in equal intervals of time, however small the time interval may be. In other cases, it is *variable.*

Acceleration has both magnitude and direction, and so any change in either of them will change the acceleration of a body under motion.

Suppose the velocity of a body at the beginning of an interval of time is 9 ft. per sec., and at the end of the first second the velocity becomes 11 ft. per second (Fig. 31), then during the interval of one second the velocity of the body has increased by 2 ft. per second, the average velocity during the interval being 10 ft. per sec. If again, at the end of next successive seconds, the velocity becomes 13, 15, 17, etc. ft. per sec., then the change of velocity of the body is uniform, and is effected at the rate of 2 ft. per sec. in each second, the corresponding average velocities being 12, 14, 16, etc. ft. per sec., so the rate of change of velocity, *i.e.* the acceleration of the body, is 2 ft. per sec. per sec. In Fig. 31, *a* represents the position of the body at the beginning and *b,c,d,e*, the successive positions at an interval of 1 second. In this case, the velocity is increased by equal amounts in equal intervals of time. So, it is a case of *uniform* acceleration.

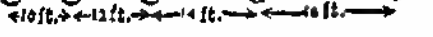


Fig. 31—Uniform Acceleration

In acceleration, the **unit of time** comes **twice,** because it involves a change of velocity, and also a time in which the change occurs. A falling stone gradually increases in velocity vertically downwards by 32 ft. per second in every second, so the acceleration of the stone will be expressed as 32 ft. per second per second [or 981 cms. per sec. per sec. (or cms. per sec.^{2})].

34. The Units of Acceleration :— A body has unit acceleration, if its velocity changes by unity in unit time.

The **C.G.S.** unit of acceleration is	The **F.P.S.** unit of acceleration is
one centimetre per sec. per sec.	**one foot per sec. per sec.**

35. Retardation :— When a moving body gradually slows down, its velocity diminishes, and the rate of diminution is known as retardation. *A retardation is a negative acceleration.* A stone thrown vertically upwards has negative acceleration, *i.e.* retardation, till it attains the maximum height. If the velocity of a train approaching a station decreases 2 ft. per sec. in a second, we say its acceleration is -2 ft. per sec. per sec., or retardation is 2 ft./sec.^{2}. Like acceleration, retardation may also be uniform or variable.

36. Angular Velocity :— *When a body moves on a plane, its angular velocity about any fixed point in that plane is given by the angle that may be imagined to be described per second by the line joining the body to the point, as the body moves.* It is said to be uniform, if equal angles are described in equal times, however small the time interval may be.

If in a time, t, the angle uniformly described be θ (pronounced "*theta*"), then the uniform angular velocity ω (pronounced "*omega*") is given by, $\omega = \theta/t$ degrees per second.

But the angular velocity is generally expressed in circular measure, i.e. *radians** per second.

In one complete revolution, four right angles are described and the circular measure of four right angles is 2π radians where $\pi = \frac{22}{7} = 3{\cdot}14$ approximately. Hence, if t be the time for n revolutions, $\omega t = 2\pi n$, or, $\omega = 2\pi n/t$ radians per sec.

If a body makes *n revolutions per minute* (R.P.M.), the number of revolutions per sec. (R.P.S.) is $n/60$.

$\therefore$ The angular velocity of the body,

$$\omega = 2\pi \times n/60 = \pi n/30 \text{ radians per sec.}$$

37. Relation between Linear and Angular Velocity in Uniform Circular Motion :— Let ω be the uniform angular velocity of a particle moving round the circumference of a circle of radius r (Fig. 32). If t seconds be the time for one complete revolution,

$t = 2\pi/\omega$ sec. ($\because$ the angle turned through is 2π radians).

Again, if v be the linear velocity of the particle,

$$t = \frac{\text{circumference}}{v} = \frac{2\pi r}{v} \text{ sec.}$$

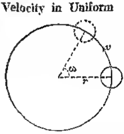


Fig. 32

Hence, $2\pi r/v = 2\pi/\omega$, $\quad$ or, $v = \omega r$ $\quad$ ... ... (1)

Thus, the linear velocity of any particle of the body rotating about a fixed axis is directly proportional to its distance from the axis of

* One *radian* is the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle. 1 radian$=57^\circ\ 17'\ 44{\cdot}8''$.

rotation and is obtained by the product of the angular velocity and the distance.

Examples. *(1) A circus horse trots round a circular path at a speed of 8 miles an hour, being held by a rope 20 ft. long. Find the angular velocity of the rope.*

8 miles an hour $= \frac{5280}{60 \times 60} \times 8$ ft. per sec. [$\because$ 1 mile $=5280$ ft.]

$$\text{Here } v = \frac{5280}{60 \times 60} \times 8, \; r = 20;$$

$$\therefore \frac{5280}{60 \times 60} \times 8 = 20\,\omega, \text{ from eq. (1), Art. 37}$$

or, $\omega = 0.58$ radian per sec.

(2) A flywheel rotates about a fixed axis at the rate of 150 revolutions per minute; find the angular velocity of any point on the wheel. What is the linear velocity, if the radius of the wheel is $3\frac{1}{2}$ ft.?

R.P.M. of wheel $=150$. $\therefore$ Angle described per minute $=150 \times 2\pi$ radians, when $\pi = 22/7$ $\therefore$ Angular velocity, $\omega = \frac{150 \times 2\pi}{60} = 5\pi$ radians per sec.

$\therefore$ Linear velocity $r\omega = 3\frac{1}{2} \times 5\pi = 55$ ft./sec.

38. Uniform Motion in a Straight Line:—

Distance traversed in *t* secs. by a body moving with Uniform Velocity v.

If the body moves with a uniform velocity v, then by definition, v is the distance traversed by the body in each unit of time.

Hence, in 2 units of time the total distance traversed is $2v$;

„ 3 „ „ „ „ „ $3v$,

and so „ t „ „ „ „ „ tv.

Therefore, if s be the distance traversed in time t,

$$s = vt.$$

Example. *A train moves at the rate of 60 miles an hour. Express its velocity in feet per second.*

1 mile $=5280$ ft., $\therefore$ 60 miles $=60 \times 5280$ ft.; and 1 hour $=(60 \times 60)$ sec.

So the train moves (60×5280) ft. in (60×60) secs.

or, $v = \frac{60 \times 5280}{60 \times 60} = 88$ ft. per sec.

Remember that 60 miles per hour = 88 feet per second

„ „ 40 „ „ „ $= \frac{2}{3} \times 88$ „ „

„ „ 30 „ „ „ $= \frac{1}{2} \times 88$ „ „

39. Rectilinear Motion with Uniform Acceleration:—When a body moves in a straight line with uniform acceleration, the relations between distance, time, velocity and acceleration can be expressed by simple equations first pointed out by Galileo. These equations are called the **Equations of Motion** which can be stated as follows:—

If a body moves along a straight line with uniform acceleration **f** *and if* **u** *and* **v** *be its velocities at the beginning and end of any inter-*

val of time **t** *considered during the motion, and* **s** *the distance traversed by it during that time, then,*

(*i*) $\mathbf{v} = \mathbf{u}+\mathbf{ft}$.

(*ii*) $\mathbf{s} = \mathbf{u}.\mathbf{t}+\frac{1}{2}\mathbf{ft}^2$.

(*iii*) $\mathbf{v}^2=\mathbf{u}^2+2\mathbf{fs}$.

(i) Velocity v acquired in time t secs. by a body moving with a uniform acceleration of f ft. per sec. per sec.

Suppose u is the velocity at the beginning of an interval of time t. Since the acceleration of the body is f, the velocity of the body is increased in each second by a velocity of f ft. per sec.

At the end of 1 *sec. the velocity is* $u+f$;

" " 2 *secs.* " " $u+2f$;

" " 3 " " " $u+3f$;

and so, " " t " " " $u+tf$.

Hence, $$\mathbf{v}=\mathbf{u}+\mathbf{ft} \qquad \ldots \qquad \ldots \quad (1)$$

or, $v-u=f\times t$.

or, *Increase of velocity=acceleration × time*

and $f=\dfrac{v-u}{t}$.

or, *Acceleration=increase of velocity ÷ time.*

Examples. (*1*) *A body starts from rest and acquires a velocity of 8 kilometres in 2 minutes. What is its acceleration?*

8 kilometres per sec.=800000 cms. per sec.; 2 minutes=120 secs.

Here $v=800000$; $u=0$; $t=120$; $f=$?

$v=u+ft$.; or, $800000=0+f.\ 120$;

or, $f=6666{\cdot}66$ cms. per sec. per sec.

(*2*) *A body has a velocity of 144 ft. per sec. at an instant and is subject to a retardation of 32 ft. per sec.² What is the velocity after 10 seconds?*

Here $u=144$; $f=-32$; $t=10$; $v=$?

We have $v=u+ft.=144+(-32)\times 10=144-320=-176$.

Here the body is moving with a velocity of 176 ft. per sec. in the opposite direction to that in which it started.

(ii) Distance traversed in t secs. by a body moving with a uniform acceleration of f ft. per sec. per sec.

Let the body move along a straight line with uniform acceleration f, and let u and v be its velocities at the beginning and end of any interval t during its motion, s being the distance traversed.

As the acceleration is uniform, and the velocity gradually changes from u to v, the average velocity during the time should therefore be something intermediate between u and v. Let V denote its velocity at the middle of the time considered, *i.e.* at time $\dfrac{t}{2}$, so that, $V=u+f\times\dfrac{t}{2}$ from (i), Art. 39.

Now, x seconds before this middle instant the velocity is $V-fx$ and in an extremely small interval of time T there, the distance travelled by the body is practically $(V-fx)T$. In the same small interval, x seconds later than the middle instant, the distance travelled by the body will be $(V+fx)T$. The total distance covered by the body during these two equal small intervals T, T is therefore,

$$(V-fx)T+(V+fx)T=2VT,$$

which is the same as if the body moved with the velocity V during both these intervals. The whole time interval t can be imagined to be divided into such pairs of equal small intervals equidistant from the middle instant and as for each pair the above reasoning holds, the actual distance S to be travelled during time t will be the same as if the body moved with a uniform velocity V from beginning to end. In other words, V represents the true average velocity of the body.

$$\text{Hence}\quad S=Vt=\left(u+f.\frac{t}{2}\right)t$$

$$=ut+\tfrac{1}{2}f.t^2 \qquad . \qquad (2)$$

Example. *Calculate the initial velocity of a train which runs down 325 feet of incline in 10 seconds with a uniform acceleration of 2 ft. per sec. per sec.*

Here $s=325$, $t=10$, $f=2$, $u=?$

$\therefore$ $325=10u+\frac{1}{2}\times2\times10^2=10u+100$;

or, $10u=325-100=225$. Hence $u=22.5$ ft. per sec.

(iii) Velocity of a body acquired in a distance s under acceleration f.

From Eq. in (1), $v^2=(u+ft)^2=u^2+2uft+f^2t^2=u^2+2f(ut+\frac{1}{2}ft^2)$
$=u^2+2f.s$ (3), from Eq. (2).

Examples. *(1) A train runs at a speed of 30 miles per hour. The brakes are then applied so as to produce a uniform acceleration of -2 ft./sec.² Find how far the train will go before it is brought to rest.*

30 miles per hour = 44 ft./sec.

Here $u=44$ ft./sec., $v=0$, $f=-2$ ft./sec.²; $s=?$

We have, $v^2=u^2+2fs$; or, $0=(44)^2+2(-2)\times s=(44)^2-4s$,

$\therefore$ $s=\dfrac{44\times44}{4}=484$ ft.

(2) A bullet moving at the rate of 200 ft./sec. is fired into the trunk of a tree into which it penetrates 9 inches. If the bullet moving with the same velocity were fired into a similar piece of wood 5 inches thick, with what velocity would it emerge, supposing the resistance to be uniform? (H. U. 1953)

In striking the trunk the initial velocity, $u=200$ ft./sec., and the final velocity, $v=0$ after penetrating 9 inches $\left(\dfrac{3}{4}\text{ ft.}\right)$ of wood, i.e. $s=\dfrac{3}{4}$ ft. The average retardation f is to be calculated from the equation, $v^2=u^2-2fs$

$\therefore$ $0^2=200^2-2f\times\dfrac{3}{4}$, whence $f=\dfrac{8}{3}\times10^4$ ft./sec.²

In the second case, the retardation is the same, the wood being of similar kind. The final velocity v after passing through 5 inches, i.e. $s=\frac{5}{12}$ ft. of wood, will be given by ($u=200$ ft./sec.)

$$v^2=200^2-2\times\frac{8}{3}\times10^4\times\frac{5}{12}, \text{ whence } v=133\cdot3 \text{ ft./sec.}$$

40. Special cases:—If the velocity at the beginning of the time is zero, we have $u=0$, and the above formulae take the following simple forms:—

$$(i)\ v=ft;\quad (ii)\ s=\tfrac{1}{2}ft^2=\tfrac{1}{2}vt;\quad (iii)\ v^2=2fs.$$

Example. *A body starting from rest, travels 150 ft. in the 8th second. Calculate the acceleration assuming it to be uniform.* (P. U. 1953)

Let space covered in 7 seconds and 8 seconds be respectively, S_7 and S_8.

Here $u=0$, $f=$constant; $S_8-S_7=150$ ft.

$$S_8=\frac{1}{2}f\times8^2=32\ f.;\ S_7=\frac{1}{2}f\times7^2=\frac{49}{2}f.$$

$$S_8-S_7=150 \text{ ft.}=32f-\frac{49}{2}f, \text{ whence } f=20 \text{ ft./sec.}^2$$

41. To calculate the Distance traversed in any particular Second:—

The distance traversed in the nth sec.$=$the distance traversed in n seconds $-$ the distance traversed in $(n-1)$ seconds$=$

$$(un+\tfrac{1}{2}fn^2)-\{u(n-1)+\tfrac{1}{2}f(n-1)^2\}\ldots\text{from Eq. (2)}, =u+\frac{2n-1}{2}f.$$

42. General Hints:—In working out problems,

(*i*) Set down all the values of the given quantities and the symbol for the quantity required, and then consider which equation, out of those given above, connects them. From this equation, find the unknown quantity.

(*ii*) Remember that all the symbols involved in the above equations are algebraic, i.e. *they may represent either positive or negative quantities.*

Examples. (*1*) *A body is thrown up with a velocity of 32 feet per second. Find how high it will rise.*

The body will rise till its velocity is zero after which it begins to fall and its velocity becomes negative.

Here $u=32$ ft./sec.; $v=0$; $f=g=$accel. due to gravity$=-32$ ft./sec.^{2}; $s=$?

We have $\quad v^2=u^2+2fs$, or, $0=(32)^2+2\times(-32)\ s$;

$$\therefore\ s=\frac{32\times32}{2\times32}=16.\quad\therefore \text{ The body will rise 16 ft.}$$

(*2*) *A body travels 100 feet in the first two seconds and 104 feet in the next four seconds. How far will it move in the next four seconds, if the acceleration is uniform?*

Here $\quad s=100$ feet; $t=2$ secs.; $u=$?; $f=$?

We have $s=ut+\frac{1}{2}ft.^2$; or, $100=2u+\frac{1}{2}f\times4$; or, $u+f=50$ ... (1)

Motion during the first six seconds—

$s=100+104=204$ feet; $t=6$ sec.; $u=$?; $f=$?

$204=6u+\frac{1}{2}f\times36$; or, $204=6u+18f$, or, $34=u+3f$ ... (2)

From (1) and (2), $f=-8$ ft./sec², $u=58$ ft./sec.
Considering the motion during the total time (10 secs),
$u=58$ ft./sec.; $t=10$ sec.; $f=-8$ ft./sec²; $s=?$

$s=ut+\frac{1}{2}ft^2$ $\therefore$ $s=58\times10+\frac{1}{2}(-8)\times10^2=580-400=180$ feet

Thus the distance travelled in the last four seconds
$=180-100-104=-24$ ft., i.e. it travels 24 ft. in the opposite direction.

43. Force:—*A force is that which acting on a body changes or tends to change the state of rest, or of uniform motion, of the body.*

(a) Representation of a Force by a Straight Line.—Every force has a certain magnitude and acts in a certain direction. A force is completely known if we know its (i) *point of application*, i.e. the point at which the force acts; (ii) *direction*; and (iii) *magnitude*.

All these can be represented by a straight line provided that,

(i) the line is drawn from the point of application of the force;

(ii) the line is drawn pointing in the direction of the force;

(iii) the length of the line is proportional to the magnitude of the force.

(b) Equilibrium.—*When two or more forces acting upon a body are so arranged that the body remains at rest, the forces are said to be in equilibrium.*

If at any point of a rigid body, two equal and opposite forces are applied, they will have no effect on the equilibrium of the body; similarly, two equal and opposite forces acting at a point in the body may be removed without disturbing the equilibrium of the body.

44. Principle of Transmissibility of Force:—A force acting at a point in a rigid body may be considered to act at any other point along its line of action provided that the latter point is rigidly connected with the body.

45. Composition and Resolution of Forces:—

(a) Resultant and Components.—*When two or more forces P, Q, S, etc. act upon a rigid body and a single force R can be found whose whole effect upon the body is the same as that of the forces, P, Q, S, etc., this single force R is called the resultant of the other forces and the forces P, Q, S, etc. are called the components of R.* The process of finding out the resultant is known as the composition of forces.

(b) Resultant of Forces acting along the same Straight Line.—If two collinear forces *P*, *Q*, act on a body in the same direction, their resultant is the sum of the two forces, $(P+Q)$, acting in their common direction of action.

If two collinear forces *P*, *Q*, act on a body in opposite directions, their resultant is equal to their difference and acts in the direction in which the greater of the two forces acts.

(c) Resultant of two Forces acting at a point of a rigid Body in different Directions.—When two forces act simultaneously at a point of a rigid body in different directions, their resultant can be obtained, both in magnitude and direction, by a law, known as the law of parallelogram of forces. This law is of utmost use in our sciences.

46. The Law of Parallelogram of Forces:—*If a particle is acted on simultaneously by two forces, represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, these forces are equivalent to a single resultant force, represented in magnitude and direction by the diagonal of the parallelogram passing through the same point.*

Let the sides OA and OB of the parallelogram $OACB$ (Fig. 33) represent two forces P and Q in magnitude and direction inclined at an acute angle BOA and let the diagonal OC represent their resultant R in magnitude and direction. Produce OA to D, and drop CD perpendicular on OD.

Let $\angle BOA = \theta = \angle CAD$. Then we have,

$OC^2 = (OA + AD)^2 + DC^2$

$= OA^2 + AD^2 + 2OA.AD + DC^2$

$= OA^2 + AC^2 + 2OA.AD$ ($\because AC^2 = AD^2 + DC^2$)

$= OA^2 + AC^2 + 2OA.AC \cos\theta$ ($\because AD = AC. \cos\theta$).

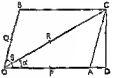


Fig. 33

or, $\mathbf{R^2 = P^2 + Q^2 + 2PQ \cos\theta}$

If $\theta = 90°$, $R^2 = P^2 + Q^2$, ($\because \cos 90° = 0$).

The direction of the resultant is obtained as follows:—

Let the resultant R make an angle α with one of the component forces, say OA. Then, $\tan\alpha = \dfrac{CD}{OD} = \dfrac{CD}{OA + AD} = \dfrac{Q\sin\theta}{P + Q\cos\theta}$.

Note.—If the angle θ be obtuse, D falls between O and A, but the expression for R^2 remains unaltered.

47. Experimental Verification:—Take a wooden board fitted with two frictionless pulleys [Fig. 33(*a*)], and fix it vertically. Fasten a sheet of paper on the board. Take three strings and knot them together in a point O, and to their ends attach three weights P (=3 lbs.), Q (=4 lbs.), and R (=5 lbs.), any two of which are together greater than the third. Pass the two strings carrying the weights P and Q over the pulleys and allow the third to hang vertically downwards with its weight R. Now the point O is in equilibrium under the action of these three forces.

Mark on the paper, by means of your pencil point, the direction of the forces, and, taking a convenient scale, say, an inch per pound-weight, measure off along OK and OH, lengths OA and OB, containing 3 and 4 units to represent P and Q respectively. Complete the parallelogram $OACB$ and join OC. It will be observed that, (i) the diagonal OC is vertical, and that (ii) OC is in the same straight line with OR, and contains R units (*i.e.* 5 units) of length in the same scale.

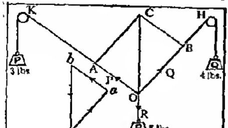


Fig 33(*a*)—Verification of Parallelogram of Forces.

Conclusion.—The knot O is in equilibrium under the action of three forces P, Q, and R. So the resultant of P and Q is equal and opposite to the force R (i.e. 5 lbs.), acting vertically upwards. But OC is vertical and it contains R units (*i.e.* 5 units) of length. Therefore OC represents the resultant in magnitude and direction of the forces P and Q represented by OA and OB respectively. This proves the law of parallelogram of forces.

N.B. (*i*) The downward force R represented by OR at O, which is equal and opposite to the resultant of the forces represented by OA and OB and by which the system is kept in equilibrium, is called the **equilibrant** of those two forces.

(*ii*) The above experiment will be found to be true whatever be the relative magnitudes of P, Q, and R, provided that any one of them is not greater than the sum of the other two.

48. Illustrations:—(*i*) If a boat O is pulled by two tugs in two different directions, and the forces exerted on the tugs are represented, in magnitude and direction, by two lines, OA and OB respectively [Fig. 33(*b*)], then the boat, instead of moving in the direction of either of the forces OA or OB, will move along OD, the diagonal of the parallelogram constructed with OA and OB as adjacent sides. OD represents the resultant of those two forces.

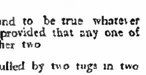


Fig 33(*b*)

(*ii*) If a man walks across the floor of a compartment of a railway train with a velocity represented by OA [Fig. 33(*b*)] while the train itself is running with a velocity OB, the resultant velocity OD of the man can be obtained graphically in the same way.

49. Triangle of Forces:— *If three forces acting at a point be represented in magnitude and direction by the sides of a triangle taken in order, they will be in equilibrium.*

[N.B. The forces here act at a point and not along the sides of the triangle. They are only represented in magnitude and direction by the sides of a triangle *taken in order*; the last expression means that the direction of the forces must be taken the same way round, *i.e.* they must go round the side of a triangle all in the same directions, either clockwise or anti-clockwise.]

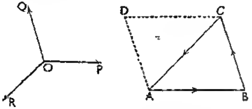


 Fig. 34

Suppose the forces P, Q, and R, acting at O are such that they can be represented both in magnitude and direction by the sides AB, BC, and CA respectively of the triangle ABC (Fig. 34); the theorem states that they shall be in equilibrium.

Proof.—Complete the parallelogram $ABCD$. BC and AD being equal and parallel, the forces represented by BC and AD are the same. By the parallelogram of forces, the resultant of the forces AB and AD is represented by AC, both in magnitude and direction. Hence the resultant of the forces AB, BC and CA is equal to the resultant of forces AC and CA and is thus zero. Hence the forces P, Q, and R, are in equilibrium.

(a) Converse of the Triangle of Forces.— The converse of the triangle of forces is also true. This can be stated as follows: *"If three forces acting at a point be in equilibrium, they can be represented in magnitude and direction by the three sides of a triangle taken in order."*

[N.B. The corresponding sides of the triangle representing the forces (which will be proportional to the respective forces) may be drawn parallel to the respective forces or respectively perpendicular to them or at any equal angles with them, taken the same way round.

(b) Experimental Proof.—On the same sheet of paper used for the experimental verification of the law of parallelogram of forces [Fig. 33(*a*)] draw a line parallel to the force P and from this, measure off a length ab to represent, to a convenient scale, the magnitude of P. From b draw bc parallel to the force R and make its length represent, to the same scale, the magnitude of R. In this way draw from C another line parallel to the force Q and containing Q units of length. If the whole work is accurately done, the end of the last line will coincide with the starting point a, and this line closes the triangle abc. Mark, by means of arrowheads, the directions of the forces on the sides of the triangle, and it will be found that the arrows go round the sides of the triangle in order.

(c) Practical Problem: A Hanging Picture.—In Fig. 35 a picture is suspended by the same string ACB from a nail C round which the string passes. It is in equilibrium under the action of the following forces: (i) the weight W of the picture, (ii) the tension T_1 of the string along AC, and (iii) the tension T_2 along BC. As the same chord passes round C, $T_1 = T_2$. The wt. W of the picture acts vertically downwards through the centre of gravity of the picture which is vertically below C, i.e. W passes through C. The three forces therefore, meet in the point C, and are, moreover, in equilibrium. So, by the principle of the converse of the triangle of forces, draw three lines ab, bc and ca representing in direction and magnitude the three forces W, T_1, and T_2 respectively. [It may be noted that if the value of W is known, the values of T_1 and T_2, which are equal, and which are represented by the lengths bc and ca, are also known, because they are drawn to the same scale.]

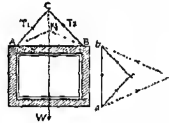


Fig. 35—A Hanging Picture

If the string is shortened as shown by the dotted line ANB, it will be seen, by applying the same principle, that tensions T_1 and T_2 of the string now will be represented by the sides bc_1 and c_1a which will be greater than bc and ca respectively. That is, the tensions are increased. It is clear from this that if *the string is shortened too much*, it is likely to break.

50. Lami's Theorem:—*If three forces acting at a point be in equilibrium, then each is proportional to the sine of the angle between the other two.*

Suppose the three forces P, Q, and R acting at O are in equilibrium (Fig. 36). Then according to this theorem,

$$\frac{P}{\sin(Q,R)}=\frac{Q}{\sin(R,P)}=\frac{R}{\sin(P,Q)}$$

That is, $\frac{P}{\sin\alpha}=\frac{Q}{\sin\beta}=\frac{R}{\sin\{360-(\alpha+\beta)\}}$.

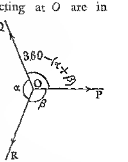


Fig. 36

The converse of the Lami's theorem is also true. That is, if *three forces acting at a point be such that each is proportional to the sine of the angle between the other two, they must be in equilibrium.*

51. Polygon of Forces:—*If any number of forces, acting at a point, be such that they can be represented, in magnitude and direction, by the sides of a closed polygon, taken in order, they shall be in equilibrium.*

Suppose the forces P, Q, R, S, and T acting at a point O are such that they can be respectively represented, both in magnitude and direction, by the sides AB, BC, CD, DE, and EA of the closed

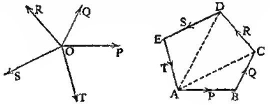


Fig. 37

polygon $ABCDEA$ (Fig. 37). Then the forces P, Q, R, S, and T shall be in equilibrium.

Join AC and AD. The resultant of forces AB and BC is, by the law of parallelogram of forces, given by AC. Similarly, the resultant of AC and CD, by AD, the resultant of AD and DE, by AE. Hence the resultant of all the forces is equal to the resultant of AE and EA, i.e. the resultant vanishes. In other words, the forces will be in equilibrium. The above construction applies to any number of forces.

The converse of the polygon of forces is not true.

52. Resolution of Forces:—We have seen above that two forces acting at a point in different directions can be compounded by the parallelogram law into a single resultant force. Conversely, a

single force acting at a point can be resolved into two components by constructing a parallelogram with the given single force as diagonal when the two adjacent sides of the parallelogram meeting at the point of application of the single given force give the two components of it. But as an infinite number of parallelograms can be drawn with a given diagonal, an infinite number of pairs of components can be obtained unless the directions of the components are specified.

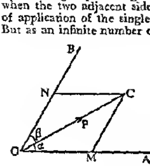


Fig. 38

53. Components of a single Force in two assigned Directions:— Suppose P is a given single force acting at O along OC (Fig. 38). Its components along the two assigned directions OA and OB will be given by the adjacent sides OM and ON of the parallelogram $OMCN$. If P makes an angle α with OA and β with OB,

$$OM=\frac{P\sin\beta}{\sin(\alpha+\beta)} \text{ and } ON=\frac{P\sin\alpha}{\sin(\alpha+\beta)}.$$

54. Resolution of a Force into two Components at Right Angles to each other:— This is in practice the most important case of the resolution of a force into two components.

Suppose OC (Fig 39) represents a force P to be resolved into two components one of which is, suppose, in the direction OA making an

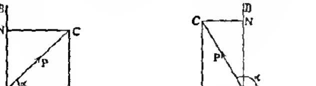


Fig. 39

angle α with OC and the other is perpendicular to OA. In both the above figures, the adjacent sides OM and ON of the parallelogram $OMCN$ give the desired components, of which

$$OM = P\cos\alpha, \text{ and } ON = P\sin\alpha.$$

55. Resolved part of a given Force in a given Direction:— The resolved part of a given force P in a given direction OA is the component OM in the given direction which together with a component ON in a direction at right angles to the given direction is equivalent to the given force (Fig. 39). Thus, the resolved part of P

along $OA = OM = P \cos \alpha$, *i.e.* it is obtained by multiplying the given force by the cosine of the angle between the given force and the given direction.

The resolved part of a given force in a given direction represents the whole effect of the force in the given direction. It follows, therefore, that a force cannot produce any effect in a direction perpendicular to its own line of action, for the resolved part ($P \cos \alpha$) of the force P in a direction perpendicular to its own line of action is *zero*, α being equal to 90°.

56. To find the resultant of a Number of Coplanar Forces acting at a point:—Let P_1, P_2, P_3 denote several coplanar forces acting at any point O (Fig. 40). Take any direction OX in the plane of the forces, and draw OY, perpendicular to OX.

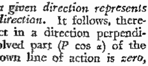


Fig. 40

Resolve each force into two components, one along the direction OX, and the other along OY.

Let the components of P_1, P_2, etc., along OX be X_1, X_2, etc., and components along OY be Y_1, Y_2, etc.

Now, if X be the resultant of all the forces along OX,

$X = X_1 + X_2 + X_3 + \ldots\ldots\ldots\ldots$

Similarly, if Y be the resultant of all the forces along OY,

$Y = Y_1 + Y_2 + Y_3 + \ldots\ldots\ldots\ldots$

The whole system of forces is then reduced to two forces, X and Y; and, if R be the resultant and if the resultant R makes an angle α, say, with the direction of X, $R \cos \alpha = X$, and $R \sin \alpha = Y$; by squaring and adding we have,

$R^2 = X^2 + Y^2 = (X_1 + X_2 + X_3 + \ldots\ldots)^2 + (Y_1 + Y_2 + Y_3 + \ldots\ldots)^2$,

Also, $\tan \alpha = Y/X$.

57. Conditions of Equilibrium of any Number of Forces acting at a point:—If two forces acting at a point are in equilibrium, they must be equal and opposite. If any number of forces P_1, P_2, P_3, P_4, etc. acting at a point O (Fig. 41) be in equilibrium, then, according to Art. 56,

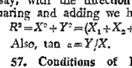


Fig. 41

$R^2 = X^2 + Y^2 = 0$, where R is the resultant of the forces, and X, Y are the algebraic sum of the resolved parts of the forces in the two mutually perp. directions OX and OY. Now the sum of the squares of two real quantities X, Y cannot be zero unless each is separately zero;

$$\therefore \quad X = 0, \text{ and } Y = 0.$$

Then, the necessary conditions for the equilibrium of concurrent forces may be obtained as follows:—

(1) Equate to zero the algebraic sum of the resolved parts of all the forces in some fixed direction.

(2) Equate to zero the algebraic sum of the resolved parts of all the forces in a direction perpendicular to the former.

The above two conditions are *necessary* and can also be shown to be just sufficient. **Conversely**, if the algebraic sum of the resolved parts of all the forces in two mutually perpendicular directions be each separately zero, the forces acting at the point shall be in equilibrium.

58. Some Practical Problems :—

(i) Why it is easier to pull a Lawn-roller on soft Turf than to push it.—When *pulling* the roller by the handle, the force OA (Fig 42), representing the force exerted by the hand, may be resolved into two components, one OB, acting horizontally, is effective in pulling the roller, and the other OC, which is vertically upwards, acts in a direction opposite to the weight of the roller, and thus reduces the pressure exerted on the ground, and so the normal reaction Consequently, the force of friction (between roller and turf) opposing the motion is also reduced [*vide* Chapter VII] and it becomes easier to pull the roller

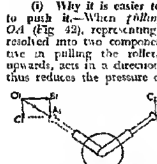


Fig 42

When *pushing* the roller, the force O_1A_1 is resolved into two components O_1B_1 and O_1C_1 of which O_1B_1 is effective in pushing the roller forward and O_1C_1, acting downwards, adds to the weight of the roller, and so increases its pressure on the ground. Consequently, the force of friction (between roller and turf) opposing the motion is also increased and it becomes more difficult to move the roller forward

(ii) The sailing of a boat against Wind.— Let the line PL represent the sail and let the force due to the wind be represented in direction and magnitude by WK. Resolve the force WK in two components, one LK parallel to, and the other NK perpendicular to the surface of the sail (Fig 43).

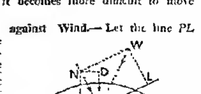


Fig 43

The force LK acting along the surface of the sail is ineffective and the effective component of the wind pressure is measured by NK.

Now resolve NK into MK along, and DK perpendicular to, the length AB of the boat. The component MK drives the boat forward

while the component DK tends to make the boat move at right angles to its length, *i.e.* sideways.

It should be noted, however, that the component DK moves the boat very slowly at right angles to its length, the resistance to motion in that direction being very great. A rudder is usually applied at A to neutralise this component.

(iii) The Effective Pressure of the Foot on a Bicycle Crank.—In cycling, the effect of the pressure applied by the foot on the crank changes according to the position of the crank. In Fig. 44 when the pressure of the foot is applied vertically downwards, with a force represented by OC, the component OB of it along the crank is lost, and the component OA, acting perpendicularly to the crank, is *only effective* in driving the cycle. It is evident that when the pressure of the foot acts perpendicularly to the crank, the pedalling becomes most effective because in that position no component of it is lost.

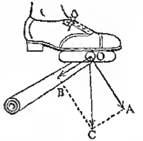


Fig. 44

(iv) Flying of a Kite.—Let AB be the surface of the kite [Fig. 45(*a*)]. Though the wind pressure acts on all parts on the under-surface of the flying kite, the total effect of it may be taken to be equivalent to a single force CO acting at a point O. The force CO may be resolved into two components, one OD acting along the surface, and the other OE acting at right angles to it. For the

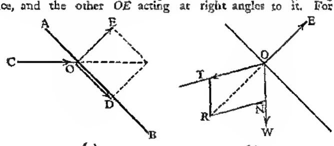


Fig. 45

steadiness of the kite, OD is not effective, and the component OE is the effective part of the wind pressure. Besides the force OE due to the wind pressure, there are two other forces, the tension T (represented by OT) of the string, and the weight W (represented by ON) of the kite acting vertically downwards [Fig. 45(*b*)]. The kite is in equilibrium under the action of these three forces. For the kite to be at rest, OE must be equal and opposite to the resultant of OT and ON, which is represented by OR. If OE increases, the kite will

rise until OR is again equal and opposite to OE. The weight W being constant, OR may increase due to any increase in the tension T. Again when the kite rises, the angle between T and W becomes less, and due to this also, their resultant OR may increase. Similarly, when the wind pressure decreases, the kite will fall. When the magnitude or direction of the wind pressure suddenly changes, the kite moves irregularly, but this is diminished by attaching a tail to the kite, which under the action of wind pressure, checks the sudden irregular movements.

(v) Flying of an Aeroplane.—It is seen, in the case of the flying of a kite, that wind pressure is an absolute factor. For this reason, when, at starting, the kite is near the ground, where there may be very little wind, the boy trying to fly the kite has got to run fast by holding the string. This produces sufficient wind pressure upon the surface, due to which the kite may rise to a considerable height where there may be sufficient air current and the running may no longer be necessary. The faster the running the better the flight. In other words, there must be sufficient wind pressure on the surface of the kite to make it rise to a considerable height into the air. This may be obtained by the movement of the air or the movement of the kite.

If the string of a flying kite breaks, the equilibrium of the forces is destroyed, and the kite either trembles, or glides down to earth backwards. If it were possible to attach to the kite just at the moment of rupture, a weightless engine and propeller, exerting a pull equal to the tension of the string, the kite would remain stationary. Besides this, if the wind pressure drops suddenly and the engine gives the kite a motion in a suitable direction so as to produce the wind pressure equivalent to the original one, the kite will again be stationary. If the magnitude of this wind pressure be increased by faster motion of the engine, the component OE (Fig. 46) will increase and the kite will move forward, and act as an aeroplane, *i.e.* a self-supporting **heavier-than-air machine.** The boy, running with his kite in order to produce a sufficient wind pressure on the surface of the kite, resembles very closely an aeroplane in which an engine and propeller take the place of the boy, and like the action of the boy, the action of the engine and propeller produces sufficient wind pressure on the wings of the aeroplane.

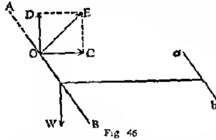


Fig. 46

An aeroplane must have a minimum velocity of 60 miles an hour in order to maintain its flight in the air, and if, any how, this speed be lost the machine cannot be controlled and the journey becomes highly dangerous.

Let AB represent the surface of the main wing of an aeroplane and OE the total wind pressure acting at O (Fig. 46). OE may be resolved into two components, one OC acting horizontally and the other OD acting vertically upwards. Besides these two component forces, the weight W of the aeroplane acts vertically downwards at the centre of gravity of the aeroplane. At the time of starting, the engine makes the propeller rotate swiftly due to the action of which the aeroplane runs forward on the ground, and, when the speed of the aeroplane becomes rapid enough to make the vertical component of the wind pressure, namely OD (Fig. 46), slightly greater than the weight W, the aeroplane leaves the ground and rises. The forward motion of the aeroplane, besides creating wind pressure on its wings, as described above, also overcomes the horizontal component OC.

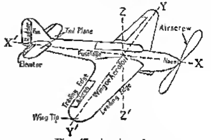


Fig. 47—An Aeroplane.

Now the action of the two forces OD and W (Fig. 46) would tend to turn the wing into a vertical position to prevent which there is a tail-plane ab, like the tail of the kite. The wind pressure acting on the tail-plane, the angle of which is controlled by the pilot, keeps the inclination of the wing constant.

The movable parts of the tail of the aeroplane modify the wind pressure so that the machine can ascend or descend according to the will of the pilot. The pilot also controls the rudder (Fig. 47) which is attached to the tail, and which works exactly like the rudder of a boat.

59. Composition of Velocities and Accelerations :— The parallelogram law of finding the resultant as explained in Art. 46 in connection with two forces acting at a point, applies equally also to the case of a moving point having two simultaneous velocities or accelerations. Hence, if a moving point has two velocities or accelerations given by u and v inclined at an angle θ and if w be their resultant passing through the same point inclined at an angle α with the direction of u, then

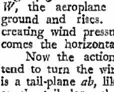


Fig. 48

$$w^2 = u^2 + v^2 + 2uv \cos\theta \qquad \ldots \quad (1), \text{ and,}$$

$$\tan\alpha = \frac{v \sin\theta}{u + v \cos\theta} \qquad \ldots \quad (2).$$

Example. *The wind blows from a point intermediate between north and east. The southerly component of velocity is 5 m.p.h. and the westerly component is 12 m.p.h. What is the velocity with which the wind blows?*

(*C. U. 1934; Del. 1948*)

(vide Art 59) Use the equation, $w^2=u^2+v^2+2uv\cos\theta$, where $u=5$, $v=12$, $\theta=90°$, $w=?$ The velocity of the wind, $w=\sqrt{(5^2+12^2)}=13$ m p h

60. Resolution of Velocity or Acceleration :— The principle of resolution as explained in the case of a force in Arts 53 and 54, along any two assigned directions and two mutually perp directions, applies wholly also to the case of velocity or acceleration.

Fig 49

61. Triangle of Velocities :— If a moving point possesses simultaneously velocities represented by the two sides, AB and BC of a triangle *taken in order*, they are equivalent to a velocity given both in magnitude and direction by AC (Fig 49).

62. Polygon of Velocities :— If a moving point possesses simultaneously velocities represented by the sides AB, BC, CD and DE of a polygon, the resultant velocity will be given by AE (Fig 50)

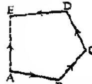


Fig 50

63. Relative Velocity :— The velocity of a body is usually given with respect to some object which may be regarded as fixed. For example, the velocity of a body on or near the earth's surface is usually given with respect to some object fixed on the earth But sometimes it becomes necessary to know the velocity of one body with respect to another when both of them are in motion Such velocity is called relative velocity and may be stated as follows:—

When the distance between two bodies is altering, either in direction or in magnitude or in both, then either body is said to have a velocity relative to the other, the relative velocity of one body B with respect to a second body A is obtained by compounding with the velocity of B a velocity which is equal and opposite to that of A

When those two bodies (A, B) are travelling in the same direction with uniform velocities u and v respectively, the velocity of B relative to A is thus $(v-u)$; and so the relative velocity will be zero when they travel with equal velocity. If they are travelling in opposite directions the relative velocity is $v-(-u)$, i e $(v+u)$

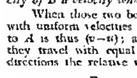


Fig. 51

If two bodies do not move on parallel lines but on lines inclined to each other, proceed as follows·—

Let the first body A move along OC with a velocity u whilst the second body along O_1D at an angle θ to OC with a velocity v (Fig. 51) Resolve v parallel to OC and perpendicular to OC, the resolved parts being respectively $v\cos\theta$ and $v\sin\theta$ So according to definition, the velocity of B relative to A, is $(v\cos\theta-u)$

parallel to OC, and $(v \sin\theta - 0)$ perpendicular to OC, for u has no component in that direction. Thus, the velocity of B relative to A has, in this case, two components $(v \cos\theta - u)$, parallel to OC and $v \sin\theta$, perpendicular to OC. The resultant of these two components gives the velocity of B relative to A.

Example. *A ship steams due east at 5 Knots* and another due north at 12 Knots. Find the velocity of the first ship relative to the second.*

In this case the observer is in the second ship and so the relative velocity R is obtained by compounding the velocity of the first ship, i.e. 5 Knots due east with a velocity equal and opposite to that of the second, *i.e.* 12 Knots due south (Fig. 52).

$\therefore \quad R = \sqrt{(5^2 + 12^2)}$ Knots $= 13$ Knots.

This relative velocity is inclined to the south at an angle θ given by, $\tan\theta = \frac{5}{12}$; or, $\theta = \tan^{-1}\frac{5}{12}$

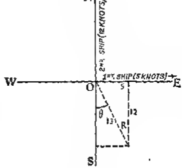


Fig. 52

Moments

64. Moment of Mass :— The moment of a mass about a given point or plane is the product of the mass and the distance of the mass from the point (or plane).

Centre of Mass.— The centre of mass of a given body or a system of bodies rigidly connected together, is a point such that if a plane is passed through it, the *mass-moments* (moments of masses) on one side of the plane is equal to the mass-moments on the other side. The centre of mass of all regularly shaped bodies lies at their geometrical centres.

65. Moment of a Force about a given Point :— The moment of a force about a given point is the product of the force and the length of the perpendicular drawn from the given point upon the line of action of the force. The length of the perpendicular drawn from the given point upon the line of action of the force is called the *arm of the moment.* The moment, therefore, never vanishes unless (*a*) either the force vanishes, or (*b*) the arm of the moment is zero, *i.e.* the line of action of the force passes through the point about which the moment is taken.

66. Effect of a Force applied to a Body :— From Newton's first law of motion it follows that the effect of a force acting on a body is to make it move if it is at rest, or change its motion if it is already in uniform motion. Now motion may be either translatory or rotatory. The question then arises whether a force externally

* Knot = a speed of 1 *sea-mile* per hour. A sea-mile is that arc of the earth's surface which makes an angle of 1 minute at the earth's centre. The British Admiralty counts this distance to be 6080 feet.

impressed on a body will produce translatory or rotatory motion or both. The nature of the resulting motion depends *on the position or point of application of the force to the body and on the condition in which the body is placed.* If the body is free and the line of action of the force passes through the centre of mass of the body, the resulting motion will be translatory. If the line of action of the force does not pass through the centre of mass of the body, the force produces translation of the body accompanied by rotation.

To illustrate this last point, let us consider a plane lamina (a body of small thickness, e.g. a piece of sheet-tin) whose centre of mass, suppose, is at C (Fig. 53). Let a force P act on the body in the direction shown in the figure and CN be the perpendicular drawn from C upon the line of action of P. To find the effect of P upon the body, imagine two equal and opposite forces P_1, P_2 acting in the same line applied at the point C, each being equal to P and parallel to the line of action of P. These two self-neutralising forces P_1, P_2 do not in any way alter the conditions under which P was applied, P_1 acting along CP_1, causes translation of the body in its own direction, whereas P and P_2 together rotate the body in an *anti-clockwise fashion.** So in considering the effect of a force upon a body, not only the magnitude and direction of the force are important, as pointed out by Newton's second law of motion, but the *position or point of application of the force to the body is also important.*

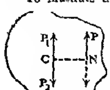


Fig 53

(a) Physical Meaning of the Moment of a Force about a Point or Axis.—If a body is restrained or fixed at a given point of it or about a line, no translatory motion of the body is possible. Let the plane lamina shown in Fig. 54, resting on a smooth table and fixed at the point O by means of a nail or hinge, represent such a body.

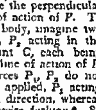


Fig 54

The effect of a force P acting on the body as shown in the figure would be to cause it to turn about the point O as centre and this effect would not be zero unless (1) the force P were zero, or (2) the force P passed through O, when ON would be zero. The magnitude of the **turning effect,** or *moment,* will depend on (a) the magnitude of P, and (b) the length of the perpendicular ON drawn from O upon the line of action of P. The turning action will be proportional to P, when the arm ON is and proportional to the arm ON when P is constant and so perpendicular ON drawn from O upon the line of action of P. out O, and is taken as a fit measure of the tendency of P to turn the body about O. The moment is also called torque.

* The *sense* in which the hands of a clock rotate is called the *clockwise* rection and the *opposite sense* is called the *anti-clockwise* direction.

(b) Positive and Negative Moments.—The moment of a force about a point or axis is a vector quantity. In Fig. 54, the *moment* of the force P about O, represents *a turning effect* tending to rotate the body about O in an anti-clockwise direction. Such anti-clockwise (or contra-clockwise) moments are, by convention, called *positive moments*. The moment of P' about the same point O is such that it tends to rotate the body in the clockwise direction. Such a moment tending to cause clockwise turning effect is called a *negative moment*.

(c) Algebraic Sum of Moments.—The algebraic sum of the moments of a set of forces about a given point is the sum of the moments of the forces, each moment being given its proper sign, positive or negative, as defined above, prefixed to it.

67. Principle of Moments :— If some forces in one plane acting on a rigid body have a resultant, the algebraic sum of their moments about any point in their plane is equal to the moment of their resultant. If the body is **at rest** under the action of several forces in the same plane, the algebraic sum of the moments of the forces about any point in their plane is zero. That is, the sum of the contra-clockwise moments is equal to the sum of the clockwise moments.

68. Moment of Inertia (or Rotational Inertia) :—The part played by the *mass* of a body in linear motion is played by the *moment of inertia* of the body in rotational motion. In studying rotational motion, the moment of inertia and the angular velocity are to be used corresponding to *mass* and *linear velocity* in translatory motion.

69. Kinetic Energy of a Rotating Particle :—Consider a *particle* of mass m (Fig. 55) rotating about O as axis in a circle of radius r with a constant angular speed w. Its $K.E. = \frac{1}{2}m \times (\text{linear velocity})^2$, at any instant $= \frac{1}{2}m \times (wr)^2 = \frac{1}{2}mr^2w^2$. If $mr^2 = I$, we have $K.E. = \frac{1}{2}Iw^2$. The quantities I and w in rotational motion thus play the same part as mass (m) and velocity (v) in linear motion. The quantity I ($= mr^2$) is called the moment of inertia of the particle of mass m about the axis O, the distance of the particle from O being r.

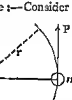


Fig. 55

70. Moment of Inertia of a body about an Axis :—Consider a body EFG rotating round the fixed axis AB with constant angular velocity w (Fig. 56). The body may be supposed to be built up of innumerable particles of masses, m_1, m_2, m_3, etc. Let them be distant r_1, r_2, r_3 etc. respectively from the axis AB. But each of these particles has the same angular velocity w, though their linear velocities will vary depending on their distances from the

axis AB. Kinetic energy of the body will be equal to the sum of the kinetic energies of these particles.

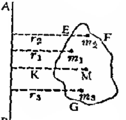


Fig 56

$$\therefore\ K.E. = K.E. \text{ of } m_1 + K.E. \text{ of } m_2 + K.E. \text{ of } m_3 + \ldots\ldots$$
$$= \tfrac{1}{2}m_1(wr_1)^2 + \tfrac{1}{2}m_2(wr_2)^2 + \tfrac{1}{2}m_3(wr_3)^2 + \ldots\ldots$$
$$= \tfrac{1}{2}\{m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots\ \} \times w^2$$

The summation within the second bracket is usually written as $\Sigma\, mr^2$.

$\therefore\ K.E. = \frac{1}{2}\,\Sigma\, mr^2 \times w^2$ The quantity $\Sigma\, mr^2$ is called the moment of inertia of the body and is the sum-total effect of the product of the mass of each particle and the square of its distance from the axis of rotation. That is $I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 +$

[$KE = \frac{1}{2} \times I$ (moment of inertia) $\times w^2$ (sq of angular velocity)]

Angular Momentum $= I\,w$

$$= \Sigma\, mr^2 \times w$$
$$= m_1(r_1^2 \times w) + m_2(r_2^2 \times w) + m_3(r_3^2 \times w) +$$
$$= (m_1r_1 \times w)r_1 + (m_2r_2 \times w)r_2 +$$
$$= (m_1v_1)r_1 + (m_2v_2)r_2 +$$

= sum of the moments of the linear momenta of the particles constituting the body.

= **Moment of Momentum.**

71. The Radius of Gyration :— If the whole mass M of a body (Fig 56) be supposed to be concentrated at a point such that the KE of this concentrated mass rotating about an axis AB is equal to $K.E.$ of the body with distributed mass rotating about the same axis AB, then the distance K of this concentrated mass M from the axis of rotation is called the radius of gyration of the body about the axis Thus,

$I_{AB} = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \quad = MK^2$, where K is the radius of gyration of the body

72. Parallel Forces :— *Forces* whose lines of action are parallel are called *parallel forces*. They are said to be like parallel forces when they act in the same direction and are said to be unlike parallel forces when they act in opposite directions.

RULES FOR PARALLEL FORCES ACTING UPON A RIGID BODY

(a) Like Parallel forces.— They always have a resultant The direction of the resultant is parallel to the direction of the forces. To find the magnitude and the point of application of the resultant of two like parallel forces, say, P and Q (Fig 57), at any distance apart, in any line AB perpendicular to the lines of action of the forces, en the resultant force R will act through C on AB such that $\times AC = Q \times CB$;

or, $\frac{AC}{CB} = \frac{Q}{P}$. That is, the point C divides the line AB internally in the inverse ratio of the forces.

From above, $\frac{AC}{AC+CB} = \frac{Q}{Q+P}$,

or, $\frac{AC}{AB} = \frac{Q}{Q+P}$;

or, $AC = \frac{Q}{Q+P} \times AB$ ... (1)

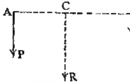


Fig. 57

Equation (1) gives the position of C when P, Q and AB are known. The magnitude of $R = P + Q$......(2). This resultant R and a third like force may be combined as above and a new resultant may be determined. Proceeding in this way, the ultimate resultant for any number of like parallel forces may be found both in magnitude and position of action.

(b) Unlike parallel forces.— If two unlike parallel forces are **unequal,** they have a resultant force. The case of two *unlike* and *equal* parallel forces is discussed afterwards under the *couple.*

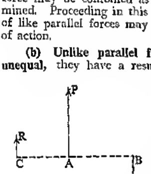


Fig. 58

Draw any line AB (Fig. 58) perp. to the lines of action of the unlike parallel forces P and Q, $(P > Q)$, and produce it to C such that $CA.P = CB.Q$......(1). That is, C divides the line AB externally in the inverse ratio of the forces. The point C gives the position through which the resultant force acts, its direction being the same as that of the greater force P, and the magnitude $R = P - Q$ ... (2).

73. The Couple :— The *equal* unlike parallel forces, whose lines of action are not the same, form a *couple.* The perpendicular distance between the lines of action of the two forces forming a couple is called the **arm** of the couple. The **moment** of a couple is the product of one of the two forces forming the couple and the arm of the couple. *A couple acting on a body exerts a turning effect on it* and the moment of the couple, known also as **torque,** measures this turning effect. An anti-clockwise moment is conventionally taken as positive and a clockwise moment as negative. Thus in Fig. 59, the couple (P, P) having arm AB tends to produce rotation of a body, in the clockwise direction and thus illustrates a negative couple.

Fig. 59—A Clockwise Couple.

74. Theorems on Couples :— The algebraic sum of the moments of the two forces forming a couple about any point in their plane is constant and equal to the moment of the couple. The effect of a couple on a rigid body is unaltered, if it be transferred to any plane parallel to its own, the arm remaining parallel to its original direction Any number of couples in the same plane acting upon a rigid body are equivalent to a single couple whose moment is equal to the algebraic sum of the moments of the couples A single force and a couple acting in the same plane upon a rigid body cannot produce equilibrium *To balance a couple, a couple of equal and opposite moment acting in the same plane or in a parallel plane is necessary*

75. Action upon a Rigid Body :—

(i) *Case of three Coplanar forces producing equilibrium* — If three forces, acting in one plane upon a rigid body, be such as to keep it in equilibrium, they must either pass through a common point or lie parallel

(ii) *Case of any number of Coplanar forces* —Any system of forces acting in one plane upon a rigid body can be reduced to either a single force or a single couple

(a) Conditions of Equilibrium of a Rigid Body.—Necessary and sufficient conditions for the equilibrium of a rigid body acted on by a system of *coplanar forces* may be obtained as follows —

Here both *translation* and *rotation* are to be taken into account For no translation to take place, the resultant must be zero, for no rotation, the algebraic sum of the moments of all the forces round any point in their plane must be zero If all the forces pass through any one point of the body, they cannot produce rotation, and the conditions of equilibrium are the same as those for a particle (*vide* Art 57) If they all do not pass through the same point, proceed as below—

1. Equate to zero the algebraic sum of the resolved parts of all the forces in some fixed direction.

2. " in a direction perpendicular to the former

3. Equate to zero the algebraic sum of the moments of all the forces about any point in their plane

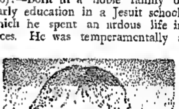
76. Vector and Scalar Quantities :— Any physical quantity, which requires *both magnitude and direction* for its complete specification, is called a vector quantity, and other quantities having magni- only are called **scalar quantities**. Displacement, velocity, ·ration, force, etc. which involve the idea of magnitude as well as .tion, are examples of vector quantities; while speed, time, mass,

volume, density, etc. which have magnitudes alone and no direction, are scalar quantities.

In *representing* any vector quantity, the following *three* things, (*i*) point of application, (*ii*) direction and (*iii*) magnitude, have to be considered, as pointed out already in the case of a force [*vide* Art. 43(*a*)]. Remembering the above a suitable straight line can be drawn to represent any vector.

Scalar quantities can be, as evident from their nature, added or subtracted arithmetically, but in dealing with vector quantities, the parallelogram law as already explained has to be applied. The method of finding the resultant of a number of vectors is called **vector addition** or **composition of vectors.**

77. Rene Descartes (1596—1650):—Born in a noble family of Touraine in France, and received early education in a Jesuit school. He was placed in the army in which he spent an ardous life in Dutch, Bavarian, and Austrian services. He was temperamentally a person who did not accept the ancient beliefs without putting them to systematic and deductive tests. According to the church mandates prevailing at that time, the ancient beliefs were too holy to be put to tests and any such tests were unlawful. At the age of twenty-three, so he went to Holland where he published his two famous books *Discourse on Method* and *Meditations.* Their contents antagonised the church and he was compelled to shift to Sweden in 1649.


Reno Descartes

Geometry advanced little, after Euclid (330 ?—280 B.C.), till Descartes took it up again about two thousand years later. His mathematical gifts truly rank him as the founder of analytical geometry. The method of representing lines and curves with equations is due to him and he is the originator of rectangular co-ordinates. The 'Cartesian' co-ordinates are so called after him. The Cartesian diver, a hydrostatic toy is also named after him. His invaluable direct contribution to science is his successful application of Snell's law of refraction to the formation of the primary and secondary rainbows. Though he calculated the *semi-vertical* angles correctly, the colours were left unexplained. This Newton did subsequently. He died in Stockholm but his coffin was carried to Paris where it was lain.

Questions

1. Explain the terms 'absolute motion' and 'relative motion'. Which of them is more important to man, and why? (Pat 1932)

2. Calculate the angular velocity in radians per second, of a particle that makes 300 r.p.m. What is the linear velocity if the radius is 4 ft.?
[Ans. 31·4 radians/sec.; 125·6 ft./sec.]

3. Explain what is meant by acceleration of a point moving in a straight line. Show that, when a body moves with a uniformly accelerated velocity in a straight line, the velocities at the ends of successive seconds are in arithmetic progression. (Pat. 1927)

4. Derive the relation, $S = ut + \frac{1}{2}ft^2$
(Del. H. S., 1949; Anna. U., 1950)

A train starting from a station is uniformly gaining speed until after 2 minutes it acquires the maximum uniform speed of 60 m.p.h. What is the distance passed over by the train during the variable state of its speed?
[Ans. 5280 ft.] (C. U. 1957)

5. A stone is thrown vertically upwards with a velocity of 160 ft. per second from the top of a cliff 120 ft. high. How high will the stone rise above the cliff, and after how long will it fall to the foot of the cliff? What will be the velocity of the stone when it is 80 ft. above the point of projection?

[Ans. (i) 400 ft., (ii) 10·70 secs. from the instant of throwing, (iii) 143·1 ft. per sec.]

6. A velocity of one foot per second is changed uniformly in one minute to a velocity of one mile per hour. Express numerically the acceleration when a yard and a minute are the units of space and time. (Pat. 1923)
[Ans. 9⅓ yds. per min.²]

7. Explain the rule known as the parallelogram of forces and show how it can be tested experimentally.
(Utkal, 1947; Anna. U. 1950, And. U. 1950, M. U. 1951, Pat. 1955)

8. (a) Define the terms 'resultant' and 'equilibrant' of forces. Explain each by means of an example. (b) State the law of triangle of forces and describe an experiment to verify it. (c) Three forces of 4, 5 and 6 gms. weight respectively act at a point and are in equilibrium. What are the angles between their lines of action?

[Ans. Angle between 4 and 5, 97°10′, between 5 and 6, 138°36′, between 6 and 4, 124°14′.]

9. Enunciate and give theoretical and experimental verification of the proposition known as the Triangle of forces. (Pat. 1932; '34, Nag. U. 1952)

10. The following forces act at a point: 18 lbs.-wt. due East, 16 lbs. wt. 60° North of East, 25 lbs.-wt. North-west, 40 lbs. wt. 75° South of West. Find graphically the resultant force at the point.
[Ans. 7·37 lbs.-wt. about 16° West of South.]

11. Explain with the aid of a diagram the flight of a kite. (Pat. 1927, '31)

12. Explain why it is easier to pull a lawn-roller than to push it.
(U. P. B. 1941, Pat. 1941; '54)

13. State and prove the law of parallelogram of velocities.
(Utkal, 1954; Del. 1943)

14. A swimmer can swim in still water at the rate of 4 miles per hour. He wishes to cross a river flowing along a straight course at the rate of 2 miles per hour so as to reach the directly opposite point on the other bank. In what direction should he attempt to swim?
[Ans. At an angle of 120° to the direction of the current.]

15. What is meant by relative velocity? Show how it is determined. Give examples to illustrate your answer. (Pat. 1946; *cf.* Utkal, 1951, '54)

A man walking on a road with a velocity of 3 miles per hour encounters rain falling vertically with a velocity of 22 ft./sec. At what angle should he hold his umbrella now in order to protect himself from the rain? (Pat. 1946)
[*Ans.* $\tan^{-1}\frac{1}{5}$ with the vertical.]

16. To a man walking at the rate of 2 miles an hour the rain appears to fall vertically; when he increases his speed to 4 miles an hour, it appears to meet him at an angle of 45°; find the real direction and speed of the rain.
[*Ans.* 45°; $2\sqrt{2}$ miles per hour.] (Pat, 1951; Utkal, 1951)

17. A railway passenger observes that rain appears to him to be falling vertically when the train is at rest, but that when the train is in motion the rain-splashes on the window are not vertical. Explain this and show how the relative velocities of the train and the rain-drops may be determined. Explain also why a passenger is thrown forward in the direction of motion of a train, when the velocity of the train is suddenly reduced.

18. A man in a boat rows at 2 m.p.h. relative to the water at right-angles to the direction of the current of a river flowing at 2 m.p.h. Another man starting from the same point walks along the bank upstream at 3 m.p.h. How far apart will the two men be after six minutes?
[*Ans.* 0·5385 mile.]

19. A man walks across the compartment of a railway carriage at right angles to the direction of motion of the train, when the train is travelling at 10 m.p.h., and walks back, with the same velocity relative to the train, when the train is travelling at 21 m.p.h. His resultant velocity in the latter case is twice that in the former case. Prove that this velocity relative to the train is very nearly 3·7 m.p.h.

20. When a train is at rest the rain-splashes on the window make an angle of 60° with the horizontal. When the train has a velocity of 25 m.p.h., the splashes make an angle of 30° with the horizontal. Find the velocity of the rain. (Utkal, 1948)
[*Ans.* 12·5 m.p.h.]

21. Define moment of a force and that of a couple. (Nag. U. 1952; P. U. 1950)

22. Define moment of inertia and explain its physical significance. (Poona, 1953)

23. Write notes on moment of inertia and radius of gyration. (G. U. 1951; Bomb. 1954)

CHAPTER IV

NEWTON'S LAWS OF MOTION: FORCE

78. Newton's Laws of Motion :—

The following three fundamental laws of motion were enunciated by Sir Isaac Newton in 1686. They constitute the very basis of the science of Dynamics and so also of the science of Astronomy. These laws are almost axiomatic, but nevertheless, the exactness with which the positions and motions of all *earthly* and *celestial* bodies can be predicted from calculations based on them, lends the strongest support to the truth of these laws.

(i) The First Law.— *Every body continues in its state of rest or of uniform motion in a straight line, except in so far as it be compelled by any external impressed force to change that state*

(ii) The Second Law.—*The 'change of motions', i.e. the rate of change of momentum is proportional to the impressed force, and takes place in the direction in which the force acts*

(iii) The Third Law.— *To every action there is an equal and opposite reaction*

79. The First Law of Motion :—

The law embodies *two* aspects·

(1) The first aspect of the law provides us with the fundamental law of inert material bodies which may be called the **Law of Inertia**, according to which, inert bodies have no tendency of *their own* to alter their states whether the state be a state of rest or a state of uniform motion in a straight line The former tendency is referred to as *inertia of rest*, and the latter, *inertia of motion*

Illustrations of the First Law.—

(a) Inertia of Rest.— (*i*) A rider on horseback experiences the effect of inertia, if the horse suddenly starts galloping, when the upper part of his body leans backwards This is because the lower part of the body moves forward with the horse, while the upper part tends to continue in its position of rest due to *inertia of rest* (*ii*) Due to the same reason a passenger standing or sitting loosely in a car falls backwards when a train or a tram car suddenly starts (*iii*) The dust particles lodged between the threads of a woollen coat fall off when beaten by a stick, because when the coat is suddenly set in motion, the particles tend to remain at rest (*iv*) When a stone is thrown at a window pane, the pane is smashed but a high speed bullet fired against the pane makes a clean hole because the glass surface near the hole cannot share the very quick motion of the bullet and so remains undisturbed as before, whereas in the first case the shock is felt on the whole glass surface

A simple experiment on the Inertia of Rest.—Take a ball and put it on a card just above a hollow cup fixed on a vertical stand (Fig. 60). A strip of metal acting as a stiff spring is fixed vertically on the base and its upper end (which is at least in level with the card) is drawn to a side and clamped. When the spring is released from the clamp, it jumps back and strikes the card. The card is thrown away by the impact, but the ball on it, owing to inertia of rest, falls down on the cup.

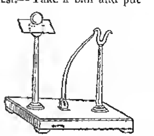


Fig. 60

(b) Inertia of Motion.—(*i*) A person alighting, without precaution, from a moving tram, is thrown forward. (*ii*) When at full speed if the horse stops suddenly, the rider on it will be thrown over the head of the horse. In each of the above cases, the lower part of the person comes to rest suddenly, while the upper part, due to inertia of motion, continues in the previous state of motion and so the person falls forward. (*iii*) A ball thrown vertically upwards in a running train comes back to hand also vertically, if the motion of the train is not changed in the meantime, because the ball retains the same horizontal motion which it acquired from the train. (*iv*) A pendulum bob once set in motion goes on oscillating for some time, and (*v*) also a cyclist paddling a free-wheel bicyle enjoys rest for some time due to inertia of motion. (*vi*) Before taking a long-jump an athlete runs from a little distance in order that the inertia of motion might help him in his exertion to jump.

(2) The second aspect of the law provides us with the definition of **force.** The idea of force has really been derived from this aspect of the law. As an inert body must continue in its state of rest or in its state of uniform rectilinear motion as the case may be, unless impressed forces act on it to change its state, we find from this that a force is that which tends to set a body in motion or to alter the state of motion of a body on which it acts.

Force.—It is not possible for any inert body to change its state by itself, whether the state be of rest or of motion. The change wherever it is, can only be effected by some external cause, which is termed *force.* Hence *a force is that, which acting on a body, changes or tends to change the state of rest or of uniform motion of the body in a straight line.* The definition of force is evidently derived from Newton's first law of motion.

80. The Second Law of Motion :—

Momentum*.—It is a property, a moving body possesses, by

* Newton used the expression 'change of motion' instead of 'change of momentum'. 'Motion of a body', he states, 'is the quantity arising out of the

virtue of its mass and velocity conjointly, and is *measured by the product of mass and velocity.*

For instance the momentum possessed by a 400-ton train moving with a velocity of $\frac{1}{2}$ mile per minute is equal to the momentum possessed by a 200-ton train moving with a velocity of 1 mile per minute. For, $400 \times \frac{1}{2} = 200 \times 1$.

The great havoc sometimes done by a cyclone is due to the great momentum of the moving mass of air. The mass of air may be small, but its velocity is very great, and so the momentum (*i.e.* mass $\times$ velocity) is large.

By taking the hammer at a distance before striking a nail in order to drive it into a piece of wood, a greater velocity of the hammer is acquired and consequently a greater momentum is obtained.

[N.B. It must be noticed that **momentum at any instant = mass $\times$ velocity** at that instant (and not mass $\times$ speed), *i.e.* momentum is a vector quantity and it should also be noted that there is no connection between the *momentum of a moving body* and the moment of a force (Art. 65).]

81. The Units of Momentum :— Unit momentum is the momentum possessed by unit mass moving with unit velocity.

The **C G S** unit of momentum *is the momentum possessed by a mass of 1 gm moving with a velocity of 1 cm per sec*	The **F P S** unit of momentum *is the momentum possessed by a mass of 1 lb moving with a velocity of 1 ft per sec*

82. Measurement of Force :— The second law of motion gives us a method of *measuring* force.

Let a *constant force P* continuously act on a particle of mass m and let u be the velocity and f the acceleration at any instant of time during the action of the force. Then by Newton's second law of motion,

$P \propto$ rate of change of momentum (mu) of the particle

$\propto$ ($m \times$ rate of change of u), for the mass m is constant

$\propto mf$

$= k \times mf$, where k is a constant.

Now, if we choose our unit of force as that which acting continuously produces unit acceleration in unit mass, we have $m=1$, $f=1$ when $P=1$. Hence k must be equal to 1 and we get,

$$\mathbf{P = mf.} \qquad (1)$$

Hence, we may write, force = mass $\times$ acceleration

---

mass and *velocity* con[jo]intly'. The idea contained in this expression is this. A marble swiftly mov[in]g over the floor has more '*quantity of motion*' in it than when moving slowly. But a heavy roller though moving slowly, has [m]ore 'quantity of motion' in it than the swiftly moving marble. That is, the [prod]uct of the *mass* and the *velocity* is a measure of the 'quantity of motion'. [Ev]idently, Newton meant by 'motion' what we call *momentum*.

83. Verification of Newton's Second Law of Motion :— According to the second law, a given force P acting upon a given mass m always produces a constant acceleration f, as given by $P=mf$. To verify it, the motion of a body falling freely under gravity may be observed. The driving force here is due to gravity (Art. 96) and may be, for all practical purposes, taken as constant. The acceleration with which the body falls, *the acceleration due to gravity*, (Art. 97) is found to be constant [*vide* Determination of g by Atwood's machine or by the *falling plate* method, Guinea and Feather experiment (Arts. 106 & 110)]. The same truth is also established by an *inclined plane* method [Art. 111(*a*)], where the driving force, for a given inclination of the plane, is a constant fraction of the force of gravity and the ball rolls down with an acceleration which is found constant.

An easy and convenient method of experimentally proving the second law is by means of a *Fletcher's Trolley*.

Description of the Fletcher's Trolley Apparatus.—A schematic diagram of the apparatus is given in Fig. 61, while the actual apparatus is shown in Fig. 62. It consists of a stout metal bed B_1, B_2 about one and a half metres long, with two parallel rails (R, R)

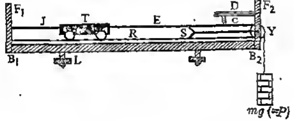


Fig. 61

fixed longitudinally on it. The bed is provided with levelling screws L. A trolley T provided with wheels can run on the rails almost without friction. To prevent a head-on crash, the front end of the bed has two projecting springs (S, S) called friction brakes, which arrest the moving trolley at this end. A specially made paper tape E has its one end attached to the trolley, and passing over a smooth pulley Y fixed at the end B_2 of the bed

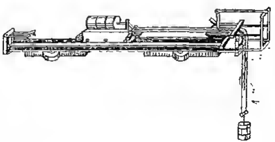


Fig. 62

has at its other end a hanger on which a suitable load m may be

placed, whose weight mg acts as the driving force. F_1 and F_2 are two end frames fixed at the two ends B_1 and B_2 of the bed. By means of a thread the trolley may be tied to F_1, if required.

The frame F_2 has a metal reed D fixed to it and placed lengthwise with the bed holding an inked brush C vertically down whose free end is in touch with the tape. The reed can be swung a bit so as to make it vibrate at right angles to its length, when the brush traces out a wavy curve on the tape below as the latter is made to move across it.

Experiment.—The bed is made horizontal by means of the levelling screws in order that the trolley may actually run on a horizontal surface. The metal reed is drawn a little at right angles to itself and then let go, when it vibrates to and fro. Its time period T is determined with the help of a stop-watch by counting a definite number of vibrations. By means of a thread the trolley is tied to the end-post F_1 and a suitable load is put on the hanger at the hanging end of the tape. The thread is then cut, when the trolley begins to move forward under the pulling weight (mg). The inked brush traces out of the wavy curve (Fig. 63) on the tape. The time period of the tracing point being constant, such a curve can be conveniently used to measure short intervals of time accurately. The tape is taken out and placed flat on a table and a straight line, which serves as the reference line, is drawn centrally from one end to the other of the wavy curve. The points of intersection of this line with the wavy curve being marked the distances ab, bc, cd, etc which are three consecutive points of intersection are accurately measured. The average velocity of the moving system during the interval ab is ab/T, and the same for the successive intervals are bc/T, cd/T, etc. The increase of velocity in the interval bc over that of the preceding inteval ab is $\frac{bc-ab}{T}$, whence acceleration is $\frac{bc-ab}{T^2}$. This acceleration, whatever is the interval from which it is determined, is found to be the same. Thus the acceleration is constant, when the driving force and the mass moved are constant. This verifies the second law of motion.

Fig 63

84. The Impulse of a Force:—*The impulse of a force acting on a body for any time is the* **product of the force and the time for which the force acts.**

Suppose a particle of mass m moving at any instant with velocity u is acted on with a constant force P for time t. Now $P=mf$, if f be the acceleration produced. If the velocity of

the particle at the end of an interval t measured from that instant be v,

$$v = u + ft = u + \frac{P}{m} . t.$$

Hence, by transposition, impulse $= P \times t = m(v - u) = mv - mu$ ... (2)

That is, *impulse = change of momentum.*

85. Impulsive Force :—

An impulsive force is a large force acting on a body for a short time, the impulse of the force being finite but the displacement of the body during the short interval negligible. Its whole effect is given by its impulse only.

Suppose the initial position and motion of a body are known when a force begins to act on it. The effect of the force on the body will be generally wholly known, if the final position and motion of the body can be known *i.e.* if the displacement it causes and the change of momentum it produces, be known. In the case of an impulsive force the displacement being negligible, its whole effect will, therefore, be given by the change of momentum it produces on the body, *i.e.* its whole effect is given by its impulse.

For a body initially at rest, $u = 0$ and therefore equation (2) becomes,

$$\mathbf{Pt = mv.} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

86. The Unit of Force :—From what has been shown above, the unit of Force may be defined as, (*i*) *That force which acting on a unit mass produces unit acceleration* [see equation (1), Art. 82].

(*ii*) *That force which acting for unit time on unit mass initially at rest creates in it unit velocity* [see equation (2), Art. 84].

(*iii*) *That force which acting on any mass at rest for unit time produces in it unit momentum in the direction of the force* [see equation (3), Art. 85].

Two systems.— There are *two systems* of force-units, (*a*) the absolute, and (*b*) the gravitational. The absolute units do not vary throughout the universe, but the *disadvantage* of the gravitational units of force is that they are not constant, because they depend upon the value of the acceleration due to gravity g, which varies, though slightly, at different places (see Art. 98).

(a) Absolute or Dynamical Units of Force.—

Dyne

The C.G.S. absolute unit of force is called a dyne, which is the force that can produce an acceleration of one centimetre per second per second when acting on a mass of one gram.

Poundal

The F.P.S. absolute unit of force is called a poundal, which is the force that can produce an acceleration of one foot per second per second when acting on a mass of one pound.

(b) Gravitational Unit.—The weight of a body is the force with which it is attracted by the earth. The acceleration with which a body falls freely is denoted by 'g' the value of which in the F.P.S. system is 32·2 ft. per sec. per sec., and in the C.G.S. system the value is 981 cm. per sec. per sec. So—

(i) The weight of 1 lb., *i.e.* a force of 1 lb.-wt. acting on a mass of 1 lb., produces an acceleration of 32·2 ft. per sec. per sec.

But the force of 1 poundal acting on a mass of 1 lb. produces an acceleration of 1 ft. per sec. per sec.

∴ **Weight of a pound** (also called a pound-weight written as **lb.-wt.**) = 32·2 (i.e. g) **poundals.**

∴ **m pounds-weight** (m **lbs.-wt.**) = mg **poundals.**

Hence, a force of 1 poundal = 1/32·2 of weight of one pound
= wt. of 16/32·2 oz.
= wt. of half an ounce nearly.

(ii) Again, the weight of one gram, which is expressed as a force, of 1 gm-wt., acting on a mass of 1 gram produces an acceleration of 981 cm. per sec. per sec.

But the force of 1 dyne acting on a mass of 1 gram produces an acceleration of 1 cm. per sec. per sec.

∴ **Weight of a gram** (called **a gram-weight**) = **981** (i.e. g) **dynes.**

∴ **m grams-weight** (m **gms.-wt.**) = mg **dynes.**

Hence, a force of 1 dyne = 1/981 of a gram-weight.

Generally, if m lbs. be the mass of a body, the only force acting on it is its weight, W. So, by substituting W for P, and g for f in the formula, $P = mf$, we get, $W = mg$,

i.e. **weight of a body (in dynes) = mass (in grams) × g ;**
(where **g** = **981**)

and **weight of a body (in poundals) = mass (in lbs.) × g ;**
(where **g** = **32·2**).

Note.—A force of 1 dyne can be practically realised by the weight W of one milligram, $W = mg = 1/1000 \times 981 = 1$ dyne (nearly).

The **gravitational unit of force** is the weight of unit mass. Hence—

The C.G.S. gravitational unit of force is a force equal to the weight of a gram.	*The F.P.S. gravitational unit of force is a force equal to the weight of a pound.*

∴ **The gravitational unit of force = g × absolute unit of force.**

[Note—(1) The weight of a pound has different values at different places of the earth due to the difference in the value of g.

(ii) The formula, $P=mf$, is true only when all the forces are expressed in absolute units, *i.e.* in poundals or dynes, and not in pounds-weight or grams-weight.

(iii) In solving problems using the above formula, (*a*) reduce all the forces into absolute units (if they are given in gravitational units, *i.e.* in lbs.-wt. or gms.-wt.) by multiplying with the corresponding value of *g*.

(iv) Finally, if necessary, reduce the forces to gravitational units by dividing by '*g*'.]

87. Relation between a Dyne and a Poundal :—

1 Poundal $=1/32{\cdot}2$ of wt. of a pound

$=1/32{\cdot}2\times 453{\cdot}6$ wt. of a gram ($\because$ 1 pound $=453{\cdot}6$ grams.)

$=981/32{\cdot}2\times 453{\cdot}6$ dynes ($\because$ 1 gm.-wt. $=981$ dynes)

$=13{,}800$ dynes (in round numbers).

Examples. (1) *Express in dynes the force due to 1 ton weight* ($g=981{\cdot}4$ cms. per sec.^{2}).

1 ton-weight $=2240$ lbs.-wt. $=2240\times 453{\cdot}6$ gms.-wt.

$=2240\times 453{\cdot}6\times 981{\cdot}4$ dynes $=9{\cdot}97\times 10^8$ dynes.

(2) *A force equal to the weight of 10 lbs. acting on a body generates an acceleration of 4 ft. per sec. per sec. Find out the mass of the body.*

Here $P=$ wt. of 10 lbs. $=10\times 32$ poundals; $f=4$ ft. per sec. per sec.

$\therefore$ By the formula $P=mf$, we have $10\times 32=m\times 4$, or, $m=80$ lbs.

(3) *A train weighing 400 tons is travelling at the rate of 60 miles an hour. The speed of the train is reduced to 15 miles per hour in 30 seconds. Find the average retarding force on the train.*

400 tons $=400\times 2240$ lbs.; 60 miles an hour $=88$ ft. per sec.

15 miles an hour $=22$ ft. per sec.

We have, by equation (2), Art. 84, $Pt=mv-mu$.

or, $P\times 30=(400\times 2240\times 22)-(400\times 2240\times 88)$.

$$\therefore\ P=-\frac{400\times 2240\times 66}{30}=-1{,}971{,}200 \text{ poundals.}$$

(4) *On turning a corner a motorist rushing at 45 miles an hour finds a child on the road 100 ft. ahead. He instantly stops the engine and applies brakes so as to stop within 1 ft. of the child (supposed stationary). Calculate the time required to stop the car, and the retarding force. (Car and the passenger weigh 2000 lbs.)* (*Pat. 1939*)

Here $u=45$ miles per hour $=66$ ft. per sec.

The final velocity $v=0$, and the distance travelled before the car stops $=100-1=99$ ft.

If f be the acceleration generated by the force we have, $v^2=u^2+2fs$;

or, $0=66^2+2f\times 99$; whence $f=\dfrac{-66^2}{2\times 99}=-22$ ft. per sec.^{2}.

Again, $v=u+ft$; or, $0=66-22t$; whence $t=66\div 22=3$ sec.;

or, the time required to stop the car $=3$ secs.

$\therefore$ The retarding force, $P=mf=2000\times 22=44{,}000$ poundals.

(5) A constant force acts for 3 secs on a mass of 16 lbs., and then ceases to act. During the next 3 secs the body describes 81 ft. Find out the magnitude of the force in lbs.-wt. and poundals (Acceleration due to gravity=32 ft. per sec per sec.) (Pat. 1947)

If the force P acts for t secs., the impulse $P\times t=m(v-u)$

Here $u=0$; we have $P\times 3=16v$ (1)

After the force ceases to act, the body describes 81 ft. in 3 secs. So the uniform velocity during this period $v=81/3=27$ ft.

∴ From (1), $P=\dfrac{16\times 27}{3}=144$ poundals (or, $\dfrac{144}{32}=4.5$ lbs.-wt.)

Otherwise thus—

The uniform velocity during the last 3 secs $=81/3=27$ ft.

So, 27 ft. is the final velocity of the first 3 secs.

Hence, considering the first period of 3 secs., we have,

$u=0,\ v=27,\ f=?$

$v=u+ft$, or, $27=0+f\times 3$,

∴ $f=27/3=9$ ft. per sec.^{2}

Hence $P=mf=16\times 9=144$ poundals (or, 4.5 lbs. wt.)

88. Physical Independence of Forces:—The latter part of Newton's second law of motion states that the change of motion produced by a force takes place in the direction of the force.

If two or more forces act simultaneously on a body, each force will produce the same effect independently of others. Hence their combined effect is found by considering the effect of each force on the body independently of others and then compounding their effects. This principle is known as the Principle of *Physical Independence of Forces*.

Illustrations:—(i) A stone dropped from the top of the mast of a ship, which is travelling without rolling, falls at the foot of the mast, whether the ship be in motion or not, and the time taken by the stone to fall is the same in the two cases.

This is because the two forces, the vertical force of gravity and the horizontal force due to which the ship moves forward, act independently of each other, i.e. one is unaffected by the other and acts in its own direction in full. The stone at the point of being dropped has the same horizontal motion as the ship and thus continues unabated during all the time the stone moves downwards on being dropped. So with respect to the mast the stone is at relative rest so far as the motion in the horizontal direction is concerned. Evidently, it must strike the foot of the mast, when dropped down, though the ship is in motion as it does on being dropped when the ship is at rest. The time taken by the fall in both the cases will be equal, because the distance covered in both the cases being the same, it is governed only by the force in the vertical direction, i.e. the force of gravity which is unaffected, according to the above principle, by the motion of the ship in the horizontal direction (which is without any component in the former direction).

(*ii*) A circus *rider* is another good illustration. When in the course of running, the rider jumps in a vertical direction from the horse's back, his horizontal velocity, which is the same as that of the horse, remains unchanged and independent of the vertical velocity. For this reason he is able to slight again on the horse's back and does not fall behind.

89. Pull, Push, Tension, and Thrust :— There are several *ways* in which a force may be exerted, the most familiar of these being by *pulls* or *pushes*.

A **Pull** is usually applied along some length of a substance, as for example, along a string, or a chain; and it is said that the string is in a *state of tension*. The *pull* is also spoken of as a *tension*. A pull may, as well, be exerted along a rigid substance say, a rod etc.

A **Push** cannot be exerted along flexible substances like strings or threads. Pushes can only be applied by rigid substances.

A push distributed over an area is often spoken of as a thrust. If any one presses a stone with his finger, the finger exerts a thrust on the stone tending to push it away.

90. The Third Law of Motion :—

If one body exerts a force on a second body, the second body exerts an equal and opposite force, called the **reaction** of it, on the first. The mutual force (per unit area) between two bodies is known as **stress.** So the third law is also sometimes called the **law of Reaction** or the **law of Stress.**

This law is a result of experience. It states that the action between two bodies is mutual. *The law is true whether the two bodies concerned are at rest or in motion and whether they are in contact or act from a distance.* Since every force must necessarily be accompanied by an equal and oppositely directed reaction, all forces in nature are in the nature of a stress between portions of matter.

(i) Illustrations of action force and reaction force—

Imagine a body of W lbs.-wt. resting on a table. This weight is exerting pressure downwards on the table. But if W were the only force, the weight would have gone through the table. As it does not do so, its motion must have been resisted by an equal and opposite force R, called the *reaction* exerted by the table *upwards* along the same line of action when W acts on the table *downwards*.

(*ii*) When there is a load on a hand, the hand is subjected to a downward force by the weight of the load, and the hand also applies an equal force on the load. If now, the hand is moved with the load, an additional force must be applied to the hand.

(iii) If a man raises a weight by a string tied to it, the string exerts on the man's hand exactly the same force as it exerts on the weight but in the opposite direction.

(iv) A ladder leaning against a wall presses on it and tends to push the wall back. This action is equal and opposite to the counter-thrust called the reaction, exerted on the ladder by the wall, which keeps the ladder in position.

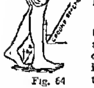


Fig. 64

(v) When a man, at the time of walking (Fig. 64), presses his feet against the ground slantingly in a backward direction, the reaction force of the ground has a substantial component in the horizontal direction forward. This enables the man to advance.

It is to be noted that it is difficult to walk over a slippery ground because sufficient pressure of the feet cannot be exerted slantingly on the ground on account of friction being small and so the reaction force not sufficient.

(vi) A boatman presses one end of a bamboo pole against the ground [Fig. 64(a)] and the boat on the water moves forward. Here the bamboo pole presses the earth and the earth sets up the reaction force along the pole in the opposite direction. The component of the reaction force in the horizontal forward direction communicated to the boat through the boatman makes the boat move forward.

Fig. 64(a)

(ii) When a magnet attracts a piece of iron, the iron also attracts the magnet with an equal and opposite force (*vide* Magnetism Part II, Vol. II). This may be verified by holding the magnet in hand and suspending the iron in front of it when the iron moves towards the magnet and repeating the experiment by holding the iron in hand and suspending the magnet in front of it at the same distance when the magnet moves in the same way towards the iron. If the action of the magnet on the iron is the action-force, that of the iron on the magnet is the reaction-force.

N.B. Newton's third law of motion really gives us an insight into the forces set in nature. The assertion of the law is that forces ... exist singly; whenever they appear, they appear in pairs. If one of them is an action-force, the other is a co-existent equivalent opposite

force to be called its reaction-force. The question then arises "If a force acting on a body has an equal and opposite reaction, why should the body move at all? *The body moves because the action and the reaction do not act on the same body or the same part of the body.* Take, for example, the case of a body falling to the earth from above. The earth exerts a gravitational attractive force (*vide* Chapter V) on the body which being of small mass moves toward the earth. At the same time, the small body attracts the earth towards itself with an equal force which is here the reaction-force. But this reaction-force acts on the earth and not on the body. The earth, being very massive, does not appreciably move towards the body under such a small force. Thus, in considering a mechanical problem what is needed at the beginning is to ascertain the particular body whose motion we want to consider, and then look out to ascertain which force, the action-force or the reaction-force, acts on the body.

(*viii*) **Horse pulling a Cart.**—This is another example which shows the equality of the action and reaction forces contemplated in the third law of motion.

Let a cart C be pulled by a horse H (Fig. 65), the two being connected by a string. The tension T in the string exerted by the horse pulling the cart C forward is the action-force and the tension T exerted by the cart on the horse pulling the horse backward is the reaction-force. How is the motion of the system possible in spite of the fact that the tension T in the string is always equal and opposite?

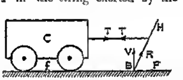


Fig. 65

The horse's foot exerts a force on the ground downwards in an oblique direction, and, in consequence, the ground produces an equal reaction R on the horse's foot in the opposite direction. The vertical component V of this reaction supports the weight of the horse, and the horizontal component F tends to drive the horse forward. The whole system moves forward provided F is large enough to overcome the frictional force f *between the wheels of the cart and the ground.*

The relations between T, F and f are given by,

$F-T=mx$, and $T-f=Mx$, when x=common acceleration of horse or cart, whose masses are respectively m and M. By addition, $F-f=(m+M)x$.

N.B.—It should be noted in the above illustrations that (*a*) the reaction lasts only so long as the action is present; (*b*) the action and reaction act on different bodies and never on the same body and so they can never produce equilibrium, because for equilibrium two equal and opposite forces must act on the same body.

91. The Principle of Conservation of Linear Momentum :—

When two or more bodies move under their mutual actions and reactions only, and no external forces act on the system, the sum-total of their momenta along any direction is constant. The law holds both for finite and impulsive forces.

Let two bodies, A and B move under their mutual action and reaction only, there being no other external forces acting on them. Then, by Newton's third law of motion, the action of A on B at any instant is equal and opposite to the reaction of B on A. Again, so long as there is action, there is also reaction. That is, the time for which the *two forces* (*action and reaction*) act is the same for both. So, the impulses of the two forces are equal and opposite. That is, the change of momentum produced in A is equal and opposite to the change of momentum produced in B. In other words, the total change of momentum of A and B, taken together, is zero, which means that the sum-total of momenta of A and B along any direction, is unchanged.

The result can be extended to the case of any number of bodies moving under conditions as stated above.

Illustrations.—(i) When a man jumps from a boat to the shore it is well known that the boat experiences a backward thrust which displaces it away from the shore. It is due to the impulsive force exerted by the man. The change of momentum of the boat caused by the force is equal and opposite to that of the man.

(ii) **Motion of a Shot and Gun.**—When a gun is fired, the powder is almost instantaneously converted into a gas at high pressure which by expansive action forces the shot out of the muzzle. The force on the shot at any instant, before it leaves the muzzle, is equal and opposite to that exerted on the gun backwards. The time for which both these forces (action and reaction) act being the same, their impulses are equal but opposite. So the change of momentum of the shot is equal and opposite to that of the gun. But both the shot and the gun being initially at rest the momentum produced in the shot is equal and opposite to that in the gun.

Suppose m and M are the masses of the shot and the gun respectively, v the velocity with which the shot emerges from the muzzle and V, the recoil velocity of the gun, supposing it to be free to move.

Thus, $$m(v-0)=M(V-0)\,;\ \text{or,}\ mv=MV.$$

Example. *A 14 lbs. shot is fired from a gun, the mass of which is 2 tons, with a velocity of 1000 ft. per sec. Find the initial velocity of recoil of the gun.*

Let u be the initial velocity of recoil of the gun. The backward momentum of the gun is equal and opposite to the forward momentum of the shot. Now, momentum of the shot $=14\times1000$ ft lb sec. units

Momentum of the gun $=(2\times2240)\times u$ ft lb sec. units

$\therefore\ (2\times2240)\times u=14\times1000$;

or, $$u=\frac{14\times1000}{2\times2240}=3.125 \text{ ft. per sec}$$

92. Circular Motion :—*If a particle is constrained to move in a circular path of radius* **r** *with a uniform speed* **v**, *it must have at any instant an acceleration of magnitude* $\mathbf{v}^2/\mathbf{r}$ *directed towards the centre of the path.*

Let a particle be constrained to move along a circular path of centre O and of radius r with a uniform speed v (Fig. 66). At any point P of its path, the velocity is v along the tangent PT. Afer an infinitely small time t, the position of the particle being P', its velocity should be v along the new tangent $P'T'$. Join P and P' to the centre and produce OP' to R intersecting PT at A. Now if the particle P were left to its inertial motion along PT without being subjected to any other external force, it would have reached the point A in time t, where $PA = vt$. The velocity v (along AT) at A can be resolved into two components, $v \cos \theta$ acting along AB parallel to $P'T'$ and $v \sin \theta$ acting radially outwards along OR (i.e. AR). Since t is very small, θ is also very small and as such A, P and P' are very close points. So $v \cos \theta = v$, and $v \sin \theta = v\theta$. Now as the particle is not allowed to move along PT and is rather constrained to follow the circular path, the particle which should have been at A due to its inertial motion, taken up the position P' attended with velocity v along $P'T'$ parallel to AB. So here only the cosine component of the velocity exists. The sine component, $v\theta$, is annulled by applying a radially inward force (hence an acceleration) of suitable magnitude on the particle.

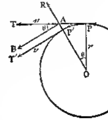


Fig. 66

At P there was no radial component of the tangential velocity, v. But at A, after t seconds, the radial component of the velocity$=v\theta$.

$\therefore$ The rate of change of outward radial velocity $= \dfrac{v\theta}{t}$

$$= \frac{v}{t} \cdot \frac{\text{arc } PP'}{r} = \frac{v}{r} \cdot \frac{\text{arc } PP'}{t} = \frac{v^2}{r} = w^2r = \text{outward radial acceleration}$$

($\because$ $v = wr$, cf. Art. 37).

This acceleration is the *centrifugal acceleration* of the particle arising out of its inertial motion and is directed radially away from centre. So in order to annul the effect of this acceleration and to make the particle move uniformly in a circular orbit, an equal and oppositely directed acceleration must act on the particle (due to some external force). This radially inwards acceleration is known as the centripetal acceleration and its magnitude is $\mathbf{v}^2/\mathbf{r}$ or $\omega^2\mathbf{r}$.

93. Centripetal and Centrifugal Forces :— When a body of mass m moves in a circle of radius r with a constant speed v, it is always subject to an acceleration v^2/r directed to the centre of the path.

Obviously then, there must be a force mv^2/r constantly acting *on the body* towards the centre of the path to constrain it to move in a circle. This force is known as the **centripetal** (Latin, *peto* to seek) **force.**

By Newton's third law of motion, an equal and *opposite* force, its reaction, is called into play. This force of reaction acts on the body at the centre and is directed away from the centre. It is known as the **centrifugal** (Latin, *fugio,* to fly) **reaction.**

The **centripetal** (*i.e.* centre-seeking) force is exerted on the revolving body by another body at the centre **towards** itself, along the radius while the **centrifugal** reaction is exerted by the revolving body on the body at the centre and is directed away from the centre, the magnitude being equal but the direction just opposite. The centripetal force, in nature, may arise in different cases due to different reasons namely, mechanical tension, gravitional force of attraction, magnetic or electric forces, etc.

The centrifugal reaction is sometimes loosely called the *centrifugal force.* But as has been indicated in Art. 92 the latter is the force due to the centrifugal acceleration arising out of the inertial motion of the body moving uniformly in a circle. Its magnitude is mv^2/r and it acts on the moving body in a radially outward direction.

(i) Take for example, the case of a man whirling in a circle at a constant speed v, a stone of mass m tied to one end of a string, the other end, at a distance r, being held by him (Fig. 67). A centripetal force mv^2/r continuously acts on the stone towards the centre of the circular path. The centrifugal reaction acts on the body at the centre, *i.e.* the hand, it being equal in magnitude but just opposite in direction. This is experienced by the hand and the man thinks as if the stone will fly outwards if he releases his grip. The tension T of the string is equal to either of these two forces and is given by $T=\dfrac{mv^2}{r}$.

It should, however, be noted that if the string be released or cut all on a sudden, then the rotating body flies off tangentially to the circular path and not away from the centre along the radius. This is because, as soon as the string is cut, the centripetal force, ceases to act and the motion of the stone continues due to inertia and takes place in the direction in which the stone was moving at the instant, *i.e.* in a tangential line. As soon as the centripetal force goes, the velocity component responsible for the centrifugal acceleration together with the other component, maintains the constancy of the tangential velocity in magnitude and direction.

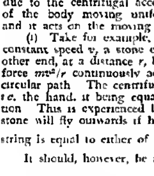


Fig. 67

N.B.—*Every cyclist must have noticed that the mud from a bicycle tyre flies off tangentially when there is not sufficient adhesive force (=centripetal force) between it and the tyre to keep it moving in a circle.*

(*ii*) A bucket containing water may be swung round in a vertical plane without the water falling down, if the motion is rapid enough. When the bucket is upside down, the weight of the bucket and water acting downwards is balanced by the centrifugal force acting vertically upwards.

Example. *A stone whose weight is 1 lb. swings round in a circle at the end of a string 4 ft. long and takes ½ second for every complete revolution. Calculate the stretching force in the string.*

The magnitude of the stretching force $= \frac{mv^2}{r}$.

Velocity of the stone, $v = \frac{\text{distance}}{\text{time}} = \frac{2\pi r}{\text{time}}$

$\therefore \quad \frac{mv^2}{r} = \frac{m \times 4\pi^2 r^2}{r \times t^2} = \frac{1 \times 4 \times 9{\cdot}87 \times 4}{1/4}$

$= 631{\cdot}68$ poundals $= \frac{631{\cdot}68}{32{\cdot}2}$ lbs.-wt. $= 19{\cdot}61$ lbs.-wt.

SOME MORE ILLUSTRATIONS OF CENTRIPETAL AND CENTRIFUGAL FORCES

(1) Motion of a Bicycle in a Curved Path.—Motion of a bicycle rider in a circular path is also an example of the *centripetal* and *centrifugal force*. A cyclist turning a corner has to incline his body inwards, *i.e.* towards the centre of the curved path (Fig. 68) for a safe journey.

At that time the forces acting are (*a*) mg, the total weight of the machine and the rider, acting vertically downwards through O, the C.G. of the system; (*b*) the centrifugal force mv^2/r, acting horizontally through O, where r is the radius of the curved path and v the speed; (*c*) the reaction Q of the ground acting at G directed along GO. This force provides the two component forces, a force F, being the horizontal component which acts along the ground and the component R which acts vertically to the ground. The couple formed by R and mg is balanced by the couple formed by F and mv^2/r.

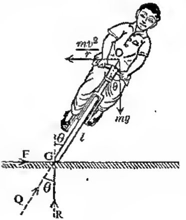


Fig. 68

The higher the speed, the greater will be the centrifugal force and so the couple due to it, and the rider will have to bend his body more *inwards* to increase the opposite couple.

Inclination of the Cycle.— If θ=*inclination of the cycle with the vertical*, l=distance *along the cycle* from O, *the C.G.* of the system to the ground G,

the couple formed by the centrifugal force being equal to that formed by the weight of the system, for a steady motion,

$$\frac{mv^2}{r} \times l \cos\theta = mg \times l \sin\theta; \text{ or, } \tan\theta = \frac{v^2}{r.g.}.$$

Thus for a given value of r, when v increases, θ must increase. So, if a cyclist rides with great speed along a curve of small radius he must incline inwards to the required extent to avoid a fall. Side slip shall occur, if θ is either too large or too small for the speed v, for a given value of r.

Example. *A cyclist is describing a curve of 50 ft. radius at a speed of 10 miles per hour. Find the inclination to the vertical of the plane of the cycle, assuming the rider and the cycle to be in one plane.*

Use the relation, $\tan\theta = v^2/rg$, of the above article. Here $v = \frac{44}{3}$ ft./sec.,

$r = 50$ ft., and $g = 32$ ft./sec.^{2}

Hence, $\tan\theta = \dfrac{44 \times 44}{3 \times 3 \times 50 \times 32}$ or $\theta = 7°40'$.

(2) The Banking of Tracks.—(*a*) A racing track for motor cars is constructed in such a way that it is banked inwards, such that a stationary car would have a tendency of sliping towards the centre of the track.

In this case the resolved part of the weight of the car along the inclined ground supplies the centripetal force necessary to keep the car moving and the other resolved part normal to the ground balances the upward reaction of the ground.

(*b*) While a railway line takes a bend, the outer rail is placed a little higher than the inner one, so that a train moving on it may have its floor inclined to the horizontal.

The wheels of the carriage are provided with flanges on the inner side for both the wheels in a pair, so as not to allow the wheels to move sidewise and cause derailment. If the rails are on the same level, while taking a bend, the tendency of the train to move in a straight line produces a pressure on the curved rails, the reaction of which at the flanges supplies the necessary centripetal force for the motion on the curved path. Such huge friction between the flanges and the rails may wear out the flanges quickly. To avoid this, the outer rail is so raised as to reduce the friction between the rails ' the flanges to nil, the inclination depending on the radius of the bend as also on the average speed of the train at the bend.

Let $ABCD$ (Fig. 69) be a vertical section of the carriage (mass$=m$) through the line shown by GO, joining the centre of gravity, G, of the carriage, and the centre of the circular track (radius$=r$). Suppose the outer rail is raised over the inner so that the floor AB of the carriage is inclined at the angle θ to the horizontal AE, when there is no lateral pressure exerted by the flanges of the wheels on the rails, when the carriage is moving with speed v.

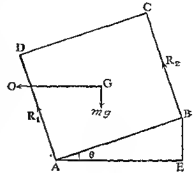


Fig. 69

In such a case, the reactions of the inner and outer rails will be normal to AB and are shown by R_1 and R_2.

Resolving vertically,

$$(R_1+R_2)\cos\theta=mg, \text{ (wt. of the carriage)} \quad \ldots \quad \ldots \quad (1)$$

Resolving horizontally,

$$(R_1+R_2)\sin\theta=\frac{mv^2}{r} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

Dividing (2) by (1), $\tan\theta=v^2/rg$ ... ... (3)

If trains of different speeds pass round the curve, pressures exerted by the flanges on the rails cannot be eliminated completely.

Example. *A railway carriage of mass 15 tons is moving with a speed of 45 m.p.h. on a circular track of 2420 ft. radius. If the rails are $4\frac{1}{2}$ ft. apart, find by how much the outer rail should be raised over the inner rail so that there is no side thrust on the rails.*

Use the relation, $\tan\theta=v^2/rg$, of the above article, when $v=45$ *m.p.h.* $=66$ ft./sec., $r=2420$ ft., and $g=32$ ft./sec.²

$$\therefore \tan\theta=\frac{66\times66}{2420\times32}=\frac{9}{160}.$$

Since θ is small, $\tan\theta=\theta=\sin\theta=\frac{9}{160}$.

The height of the outer rail above the inner$=4\frac{1}{2}\times\frac{9}{160}=\frac{81}{320}$ ft. $=3$ inches (nearly).

(3) The Centrifugal Drier.—This affords another example of the application of centrifugal force. This is used in laundries. The wet clothes (which are to be dried) are placed in a cylindrical wire cage which is caused to rotate at high speed. The water becomes separated from the clothing and moves off, as the adhesive force between it and the material of the clothings is not sufficient to keep it moving uniformly in a circle.

(4) Cream Separator.—A given volume of cream has smaller mass than the same volume of skimmed milk, and so a smaller force

is required to hold it in a circle of given radius. Hence, if cream particles and milk particles are set in rapid rotation, the milk particles will have greater tendency to move to the outerside of the vessel, the cream particles remaining nearer the centre.

(5) Flattening of the Earth.—Initially the earth consisted of a mass of fused matter. Because due to the rotation of the earth about its axis, a centrifugal force is generated which is greatest at the equator and decreases gradually towards and finally vanishes at a pole (*vide* Art 98), the earth bulged at the equator and got flattened at the poles. The model shown in Fig 70 is commonly used in the laboratory to exhibit a similar effect. It consists of a spindle having some circular rings made of thin metallic strips mounted around it so as to form the surface of a sphere. This body is fixed at the bottom to the spindle, while the top of it ends in a collar which fits on the spindle, and can slide up and down. When the spindle is rapidly rotated (by fitting it vertically on the axle of a horizontal whirling table), each particle of the strips forming the rings tends to move outwards due to the centrifugal force and causes the collar to slide down the rod. The body takes on a form flattened at the top and bottom, *i.e.* along the axis and bulged in the middle as shown by the dotted contour.

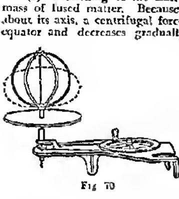


Fig 70

(6) Watt's Speed Governor.—The governor of an engine, first designed by James Watt, is a self-acting device by which the supply of power to an engine is automatically regulated such that the mean speed of the engine may remain constant at the rated value or within certain limits, when the load on the engine varies.

The *conical governor* (Fig 71), consists of two heavy metal balls A and B at the end of two arms hinged at the top of a vertical spindle V which is driven by the main shaft of the engine and rotates with it. Often the two arms are connected by two links (as shown in the figure) to a sleeve which, while it rotates with the spindle V, can slide up and down on it with the rise and fall of the balls. This to-and-fro motion of the sleeve is used to regulate the supply of steam to the engine.

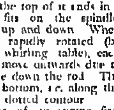


Fig 71—A Conical Governor

Theory of the Conical Governor.—Suppose A is a ball revolving about the vertical axis OY being suspended from the point O_1 by a slender rod, the joint at O_1 being such as to permit of free angular movement of AO_1 about O_1 in the plane AOY [Fig. 71(*a*)]. As A

revolves at a steady speed, AO_1 describes the surface of a cone, whose vertex is at O where AO_1 intersects OY. Let h be the height of O above the level of A and let r be the radius of the base of the cone. The forces acting on A in the plane AOY are its weight W, acting downwards, the centrifugal force F acting radially outwards and the tension T along AO_1, neglecting the weight of the rod. For steady motion these must balance one another. Taking moments about O, $F.h = W.r$

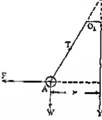


Fig. 71(a)

where $F = \frac{mv^2}{r} = \frac{W}{g} \times \frac{(wr)^2}{r} = \frac{W.w^2r}{g}$,

taking m=mass of A, v=linear velocity and w=angular velocity of it.

$$\therefore \quad \frac{W.w^2.r}{g} \times h = W.r \text{ ; or, } h = \frac{g}{w^2}.$$

That is, *the height of a conical governor is inversely proportional to the square of the angular velocity.*

[When the engine speeds up due to any decrease of load, the increased centrifugal force acting on the balls causes the balls to rise up (cf. *the height h* of the conical governor varies inversely as the square of the speed) and so the sleeve also rises up. If the speed of the engine falls due to any increase of load, the balls fall in level due to the decreased centrifugal force and so that sleeve also moves down. The sleeve has a groove (see Fig. 71) turned on it to receive the forked end of a lever through which, and other levers and links, if necessary, the sliding motion of the sleeve is transmitted and converted into the movement of a *valve*, which regulates the supply of steam, gas, or charge, as the case may be, and thereby keeps the speed of an engine constant.]

(7) A Body loses Weight due to the Earth's Rotation.—In its dirunal rotation, as the earth rotates about its polar axis (NS), a body on its surface also rotates in a circle with the same angular speed as that of the earth [Fig. 71(b)]. As a result of it, the body is subjected to a centrifugal force tending to make the body fly outwards along the radius of its own circle. A part of the weight of the body (which is a force directed towards the centre of the earth arising out of gravitational forces) is used up in counter-balancing the centrifu-

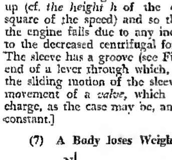


Fig. 71(b)

gal force and the remainder appears as the weight of the body. Thus, by the centrifugal forces generated by the rotations of the earth a body on the earth's surface loses a part of its weight. The loss of weight due to this cause is greatest at the equator and gradually diminishes towards and finally vanishes at each pole [*vide* Art. 98 *c*(*ii*)]

Example. *Calculate the apparent weight of a body of one ton mass at the equator, the radius of the earth being 4000 miles.*

The apparent weight $= m(g - v^2/r)$; 4000 miles $= 4000 \times 5280$ ft ; and 1 ton $= 2240$ lbs

$$\therefore \frac{mv^2}{r} = \frac{m \times 4\pi^2 r^2}{r \times t^2} = \frac{m \times 4\pi^2 \times r}{t^2} = \frac{2240 \times 4 \times 9.87 \times 4000 \times 5280}{(24 \times 60 \times 60)^2} \text{ (here } t = 24 \text{ hrs).}$$

$$= 250.2 \text{ poundals} \times \frac{250.2}{32.2} \text{ lbs -wt} = 7.77 \text{ lbs.-wt.}$$

Hence the apparent wt of the body $= 2240 - 7.77 = 2232.23$ lbs -wt

Questions

1 Explain clearly how the idea of 'inertia' of a body is deduced from Newton's First Law of Motion (Pat 1921)

2 State Newton's Second Law of Motion and explain how it enables you to measure forces (Pat 1918, '25, '30, 47, C U 1950)

3 State the laws of motion which are associated with the name of Newton in their final form, and add explanatory notes leading to the definition of a force and of the units for its measurement (Pat 1940)

4 State Newton's Second Law of Motion and show how starting from the law of parallelogram of velocities, we can arrive at the law of parallelogram of forces What is the relation between a poundal and a pound-weight? (Pat 1926)

5. State Newton's Laws of Motion and show how from the first we obtain a definition of force and from the second a measure of force. (Utkal, 1950)

6. The speed of a train of mass 200 tons is reduced from 45 m p h to 30 m.p h. in 2 min Find (*a*) the change in momentum, (*b*) the average value of the retarding force

[*Ans.* 9,856,000 F P S units; 1.145 tons wt]

7 A train of mass 175 tons has its velocity reduced from 40 miles per hour to zero in 5 minutes Calculate the value of the retarding force, assuming that it is uniform What has been the change in momentum?

Ans {Retarding force = 1.07 tons wt ; Change in momentum = 10266.6 tons ft /sec }

8 What are the units in common use for expressing a force? (Pat. 1947)

9 Explain the distinction between the absolute measure and the gravitational measure of force and show how one may be expressed in terms of the other

Express the weight of 10 kgm. in dynes and the value of a dyne in gm wt (Dac. 1942)

[*Ans* 10000*g* dynes, 1/*g* gm.-wt]

10 Find the pressure exerted on a vertical wall by the water from a fire-hose which delivers water with a horizontal velocity of 18 metres per sec from a circular nozzle of 5 cm diameter. The water is assumed not to rebound

[*Ans* 6.364×10^6 dynes]

11. Find the uniform force required to stop in a distance of 10 yards a lorry running on a level road at the speed of 15 m p h. Find also the time during which the force acts.

[*Ans.* 1.694 lbs.-wt.; 30/11 sec.]

12. An automobile of mass 3500 lbs. moving with a uniform velocity of 60 miles per hour is to be stopped in 1 sec. by the application of a uniform retarding force. What should be the magnitude of this force? (C. U. 1956)
[*Ans.* 9625 lbs.-wt.]

13. A man who weighs 100 lbs. slides down a rope hanging freely, with a uniform speed of 3 ft./sec. What pull does he exert upon the rope, and what would happen if at a given instant he would reduce his pull by one-third?

[*Ans.* (i) $100g$ F.P.S. units; (ii) $g/3$ F.P.S. units of acceleration downwards.]

14. When a man drags a heavy body along the ground by means of a rope, the rope drags the man back with a force equal to that with which the man drags the body forward. Why then does motion ensure? (Pat. 1928)

15. State Newton's Third Law of Motion and explain it carefully. Show how this leads to the principle of conservation of momentum. (Pat. 1953, '55)

16. Explain with the aid of a diagram the flight of a bird. (Pat. 1931)

[Hints.—At the time of flying the bird strikes against the air with its wings, but as every action has its opposite reaction (Newton's Third Law of Motion), the forces OA, OB, due to reaction, act in opposite directions (Fig. 72). If OA, OB represent these reactions in magnitude and direction, then by the law of parallelogram of forces, the resultant force with which the bird advances is represented in magnitude and direction by OC.]

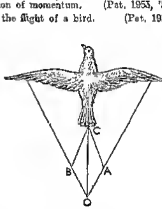


Fig. 72

17. A 10 gm. bullet is shot from a 5 kgm. gun with a speed of 400 metres/sec. What is the backward speed of the gun? (Dac. 1934)

[*Ans.* 80 cms./sec.]

18. Define momentum and impulse. State and illustrate by examples the Principle of Conservation of Momentum.

A 10 gm. bullet is fired from a kilogram gun suspended to move freely. This bullet now enters a block of wood of mass 990 gms. If the speed of the bullet is 500 metres/sec.; find the speed of recoil of the gun and the velocity imparted to the block. (G. U. 1957)
[*Ans.* 5 metres/sec.; 5 metres/sec.]

19. Explain why a force is needed to keep a body moving uniformly in a circle. Calculate this force in terms of the mass of the body, its uniform speed, and the radius of the circle. (Pat. 1944)

20. What would be the length of the year, if the earth were half its present distance from the sun?
[*Ans.* 129 days.]

21. Explain the following statement bringing out the scientific principles involved :—'If a small can filled with water is rapidly swung in a vertical circle, the water does not fall down.' (U. P. B. 1941)

22. Write a note on centrifugal and centripetal forces. (G. U. 1955)

23. What are 'Centripetal' and 'Centrifugal' forces and what are their relations with a body moving in a circular orbit? Discuss in detail their importance to man. (Pat. 1932; C. U. 1939)

24. What are 'Centripetal' and 'Centrifugal' forces, and how are they directed? Derive the magnitude of centrifugal force, and give three examples of its application.

A motor cyclist goes round a circular race course at 120 m.p.h. How far from the vertical must he lean inwards to keep his balance, (a) if the track is 1 mile long, (b) if it is 880 yds. long?

$$\left[Ans. \quad (a) \tan^{-1}\frac{121}{105}; \ (b) \tan^{-1}\frac{242}{105}\right]$$ (G. U. 1949)

25. Explain why a force is required to keep a body moving in a circle at constant speed. What is the name of this force? What happens to the moving body when the force is withdrawn?

A ball at the end of a string is whirled at constant speed in a horizontal plane. If the radius of the circle is 4 ft. and the speed of the ball is 10 ft./sec., calculate the magnitude of the radial acceleration. (G. U. 1956)

[Ans. 25 ft./sec.^{2}]

26. Why must a cyclist lean inwards to keep his balance when he is going round a circular course at high speed? Deduce a relation between the speed, inclination and the radius of the course. (Utkal, 1951)

27. What is the proper angle for banking a road around a curve of 200 ft. radius to allow for speeds of 40 m.p.h.?

[Ans. 28.1°]

CHAPTER V

GRAVITATION AND GRAVITY: FALLING BODIES: PENDULUM

94. Historical Notes.— One day while Newton was sitting under an apple tree in the garden of his village home at Woolsthorpe, a ripe apple, it is said, fell on his head. This simple event started him thinking why the apple should fall towards the earth. There must then exist some attractive force between the earth and any material body. Reasonings on this problem ultimately led him to found the doctrine of universal gravitation.

95. Newton's Laws of Gravitation:—(1) *In nature every material body attracts every other material body towards itself.*

(2) *The force of attraction between any two bodies varies directly as the product of their masses and inversely as the square of the distance between them.*

If m_1 and m_2 be the masses of two bodies and d the distance between them (Fig. 73) and if F be the force of attraction, which each exerts on the other, $F \propto m_1 m_2$ and also $F \propto 1/d^2$,

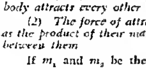


Fig. 73

$$\therefore \quad F \propto \frac{m_1 m_2}{d^2}, \text{ or, } F = G\frac{m_1 m_2}{d^2}$$

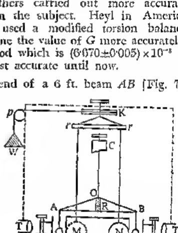
where G is a constant known as the *Universal Gravitation Constant*. Its value as determined by Boys in 1895 is 6.6576×10^{-8} in C.G.S.

units. The most accurate value of G claimed so far is by Heyl (1930) and is $(6{\cdot}670 \pm 0{\cdot}005) \times 10^{-8}$ C.G.S. units.

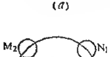
Let m_1 and m_2 in the above equation be each equal to 1 gram and d *equal* to 1 centimetre, then $G=F$, which means that *G is numerically equal to the force of attraction between two masses, each of one gram, when separated by a distance of one centimetre.*

95(a). Determination of the gravitational constant (G) :— Cavendish performed a torsion balance experiment in 1797 for the determination of the gravitational constant G. On account of its far-reaching importance it has since then become very famous. After him C. V. Boys, Poynting, and others carried out more accurate measurements on improved lines on the subject. Heyl in America comparatively recently (1927-1930) used a modified torsion balance experiment in an attempt to determine the value of G more accurately. The mean value of G by his method which is $(6{\cdot}670 \pm 0{\cdot}005) \times 10^{-8}$ is now-a-days considered to be the most accurate until now.

Cavendish's method.— At the end of a 6 ft. beam AB [Fig. 74 (a)] were hung two small lead balls P and Q each of two inches diameter. The beam was strengthened by two braces OA and OB connected to O in a short stout upright RO, attached to the middle point of AB. The beam was suspended at O from a torsion head C by means of silver-copper torsion fibre CO, 3 ft. long. Two large lead spheres, M and N, each of 1 ft. diameter, were suspended at equal distances near the balls P and Q, from the two ends of a rod, rr. rr could be rotated through any angle about a vertical axis with the help of a wheel K, a pulley p and the weight W. Thus MN could be made to take up positions either M_1N_1 or M_2N_2 on either side of AB [Fig. 74(b)]. The centres of the four balls P, Q, M, N, all lay on the circumference of a horizontal circle of radius 3 ft. The whole arrangement was entirely enclosed in a glass case, and rr could be rotated from outside and so also the torsion head C by an arrangement not shown in the figure. The observations were made with two telescopes T fitted in the walls of the room.



(a)



(b)

Fig. 74

In the position M_1N_1 of the large balls, attraction took place between P and M_1 as also between Q and N_1. These forces constitute a couple which turned the beam AB through an angle. The equilibrium position, which was difficult to be kept steady, was determined by the method of oscillation from the observation of the swings. The beam carried at each end a vernier that could move freely on a fixed scale. The vernier readings on the fixed scales were taken from outside the closed room by telescopes. The deflecting balls M and N were rotated on to the positions M_2 and N_2 such that PM_1 and PM_2 were equal and the new equilibrium positions were again estimated as before. In this way, the mean angle of twist θ of the beam due to interaction between the two balls at each end was known whence the value of G was calculated.

Calculation.—The deflecting force brought into existence due to gravitational attraction at each end of the rod $AB = G\frac{m_1m_2}{d^2}$,

where m_1, m_2 respectively are the masses of a big ball and a small ball and d is the distance between the centres of these balls when the mean deflection of the beam is θ. If l is the length of the beam, the moment of the deflecting couple $=\left(G\frac{m_1m_2}{d^2}\right)\times l$. In the position of equilibrium this couple is balanced by the torsion couple of the suspension wire, *i.e.* $c\theta$ where c = moment per unit twist $\left(=\eta\frac{\pi r^4}{2l_1}\right)$ where η = coefficient of rigidity of the wire, l_1 being its length and r its radius.

$$\therefore\quad G\frac{m_1m_2}{d^2}\times l=c\theta,\ \text{or,}\ G=\frac{c\theta d^2}{m_1m_2l}$$

Precautions.—1. To avoid draughts due to temperature fluctuation and air-currents, Cavendish housed the apparatus in a glass case and observations and adjustments were made from outside the case.

2. To avoid possible electrostatic attractions from outside, the glass case was gilt-covered. This precaution also served partly to equalise the temperature within the case.

Defects in Cavendish's experiment.—(1) The torsion fibre was thick and hence θ was small; (2) The vernier method of measuring deflection has only very limited accuracy; (3) The apparatus being large, the temperature fluctuations and hence draughts were not capable of close control; (4) Each sphere also attracted the more distant small sphere and thus a counter gravitational couple tending to decrease θ was present but not accounted for; (5) The rods supporting

the large masses M and N tended to increase θ; (6) The attraction on the beam AB also tended to increase θ.

C. V. Boys modified Cavendish's arrangements considerably whereby most of these defects were minimised or rectified.

96. Gravitation and gravity :— The force of attraction between any two material bodies is called *gravitation*. The term is more specially applied to the attraction between two heavenly bodies.

Now **gravity** is the force with which the earth attracts every body on or near its surface towards its centre. If the mass of the earth is M and the mass of any object on its surface is m, the force of attraction due to gravity $= G\,\dfrac{Mm}{R^2}$ where R is the radius of the earth. So gravity is a particular case of gravitation and may be called earth's gravitation. The force of gravity on body is called its *weight*.

97. The Acceleration due to gravity (g) :— A body, if dropped from a height, falls vertically towards the earth, *i.e.* as if it would pass through the centre of the earth; its velocity continuously increases as it falls. That is, it falls with an acceleration. Such motion is due to the attraction between the body and the earth, *i.e. due to gravity*. When a body falls freely from rest, as in the case when it is simply dropped from a height, it is experimentally found that the distance s through which it falls is proportional to the square of the time taken (*vide* Laws of Falling Bodies, Art. 110). That is, $s = kt^2$ where k is a constant. This is possible only if the acceleration of falling body is constant (cf. $s = ut + \frac{1}{2}ft^2$ when $u = 0$). *Thus a body dropped from a height falls vertically towards the earth with a constant acceleration.*

If then the acceleration is constant a heavy body as well as a light body dropped from a height should reach the ground simultaneously. That is also exactly what is found experimentally. Newton's famous *Guinea and Feather experiment* (Art. 110) conclusively proved that all bodies starting from the rest fall in vacuum with equal rapidity. That is, *the acceleration due to gravity at the same place is the same for all bodies.*

Consider, again, a body projected vertically upwards; its velocity gradually diminishes and is finally reduced to zero after which it begins to fall downwards again. This also clearly shows the existence of an acceleration directed towards the earth due to which the upward motion of the body is gradually reduced. Thus, all experimental observations lead us to the belief that any body moving in the field of the earth's attraction is subject to a constant acceleration acting vertically downwards and its value is the same for all bodies at the

same place. This acceleration is **called the acceleration** due to gravity and is conventionally **denoted by 'g'**.

We have seen in Art 96 that the force of attraction due to gravity varies inversely as the square of the distance of a body from the centre of the earth So the acceleration due to gravity (g) also varies in the same way, according to Newton's Second Law of Motion, mass of the body being constant It is affected due to rotation of the earth on the axis (*vide* Art. 98)

The value of the acceleration due to gravity at sea-level and in latitude 45° is generally taken as the **standard for reference** for values of acceleration The value of g at any place varies with its height above the sea-level, being less at the top of a high mountain than at its bottom. *The value of g is constant at the same place*, but varies with the latitude It is minimum at the equator and increases gradually to attain the maximum value at either of the poles At the equator, the value of g is about 978 cms. per sec per sec, and at the poles, it is about 983 cms per sec per sec, and the accepted mean value is taken to be 981 cms per sec per sec, or 32·2 ft. per sec per sec It also changes from one place to another due to various local conditions, the discussion of which is beyond the scope of this book

VARIATION OF 'g' WITH LATITUDE

Place	Latitude	Value in ft./sec.²	Value in cm./sec.²
Equator	0° 0′	32·09	978·10
Madras	13° 4′	32·10	978·36
Bombay	18° 53′	32·12	978·63
Calcutta	22° 22′	32·13	978·76
New York	40° 43′	32·16	980·19
Paris	48° 50′	32·18	980·94
London	51° 29′	32·19	981·17
Poles	90°	32·25	983·11

98. Variation of 'g' (or the Weight of a Body) from Place to :—Let R be the radius of the earth and D its mean density; the mass M of the earth will be given by,

$M=\frac{4}{3}\pi R^3 D$, assuming the earth to be a sphere.

(a) Above the Surface of the Earth.—At a height h above the surface of the earth, the force of attraction on a body of mass m, according to the law of gravitation $= G\dfrac{Mm}{(R+h)^2}$. For such positions external to the earth, the mass of the earth may be supposed to be concentrated at its centre.

So, the force of attraction, and hence the acceleration $\left[\dfrac{G.M}{(R+h)^2}\right]$ due to gravity, on a body above the surface of the earth, is *inversely proportional* to the square of the distance of the body from the centre of the earth. So 'g' will be less, as the distance of the body above the surface of the earth increases.

(b) Below the Surface of the Earth.—Again, consider a body of mass m at a depth 'h' below the surface of the earth [Fig. 74(a)]. Imagine a sphere concentric with the earth having a radius $(R-h)$, i.e. with its surface passing through points at a distance 'h' below the surface of the earth. It is known that the gravitational force of attraction within a hollow spherical shell is zero. Here the given body is on the surface of the inner sphere, but it is *inside* with respect to the portion of the earth outside the smaller sphere of radius $(R-h)$; so the outer portion has no attractive force on the body. The force of attraction on the body will be due to the inner solid sphere of radius $(R-h)$ towards the centre of the earth and is equal to,

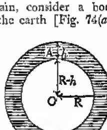


Fig. 74(a)

$$G.\frac{\frac{4}{3}\pi(R-h)^3.D.m}{(R-h)^2} = \tfrac{4}{3}\pi G(R-h).D.m,$$

where $\frac{4}{3}\pi(R-h)^3 \times D$ is the mass of the inner sphere.

The force of attraction, and hence g inside the earth, is therefore, *directly proportional* to $(R-h)$, that is, to the distance of the body from the centre of the earth.

So, g will be less inside the earth's surface, the greater the depth the lesser is the value of g. Hence the *maximum value* of 'g' is *on the surface of the earth*, and *the value of 'g' is minimum (i.e. zero) at the centre of the earth.*

(c) On the Earth's Surface.—In this case the value of g varies due to two reasons—

(i) The Peculiarity in the Shape ... **Earth.**—As the force of gravity on a body on the earth's su[rface] ... proportional to the square of the radius of the e[arth] ... greatest at the poles and least at the equa[tor] ... spheroid of which the polar radius is the le[ss] ...

Hence, the centre of gravity of a body or a system of particles rigidly connected together may be defined as the point through which the line of action of the weight of the whole body always passes, in whatever manner the body may be placed.

(a) Important Notes on the Centre of Gravity.—(i) Though the C.G. of a body is a point through which the line of action of the whole weight of the body always passes, it does not necessarily mean that the C.G. lies within the body itself in all cases. For example, a circular ring has its C.G. at the geometrical centre of the ring, which is in empty space.

(ii) If the size or shape of a body is changed, the C.G. of the body gets changed, though the weight is unaltered. A straight uniform wire has its C.G. at the middle point of its axis, but when the same wire is bent in the form of a ring, the centre of the ring, which is not within the body at all, becomes its new C.G.

(iii) As the weight of a body acts vertically downwards through its C.G., an equal force applied there in the opposite direction will make the body remain in equilibrium. Thus, when a rigid body is supported at its C.G., it remains in equilibrium, for the reaction at the support supplies an equal upward force. If the body is freely suspended by a string, the C.G. of the body will lie vertically below the point of attachment of the string.

102. C.G. of Geometrically Symmetrical Bodies:—By applying the principles of Statics, the position of C.G. of different bodies may be ascertained, as will be found in any standard book on Statics. But when the bodies are geometrically symmetrical and are of uniform density, the C.G. in their cases can be inferred as well. Thus the C.G. of,

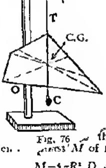


Fig. 76

(i) a straight wire, rod or beam, is at the middle point of the axis,

(ii) a parallelogram is at the intersection of its diagonals,

(iii) a triangular lamina is found by bisecting any two sides and joining the middle points so obtained to the opposite vertices when the point of intersection of these medians will give the C.G. Three equal particles placed at the vertices of the triangle have also the same C.G.,

(iv) a circular lamina, a circular ring, a solid or hollow sphere is at the geometrical

the ra (v) a cylinder (hollow or solid) is at the

then. . mass M of the middle point of its axis

$M = \frac{4}{3}\pi R^3 D$, ...tion of the **C.G. of an Irregular Lamina:**—

...of an irregular lamina, say, an irregular sheet ...termined by suspending it, with the help of

a stand O from the different corners of the lamina (Fig. 76). When it is suspended from one corner T by a string, the centre of gravity lies on the vertical line given by the plumb line TC through the point of suspension. This line is marked in chalk on one face of the lamina. The operation is repeated by suspending the lamina from another corner. The intersection of the two chalk lines gives the centre of gravity. On suspending the body from the other corners, the other vertical lines so obtained will also pass through the common point of intersection, called the Centre of Gravity.

104. Stable, Unstable and Neutral Equilibrium:— A body is in equilibrium, if the *resultant of the forces acting on it is zero*, and also if there is *no moment tending to turn the body about any axis.*

Suppose that a body is displaced slightly from its position of equilibrium. It may happen that the forces acting on the body tend to restore the body to its original equilibrium condition, or the force may tend to increase the displacement. In the former case the equilibrium is called **stable,** and in the latter case **unstable.** If, however, the forces have no tendency to increase or diminish the displacement, the equilibrium is called **neutral.**

A body is, hence, said to be in **stable equilibrium,** if it returns to the original position, when slightly displaced from the position of equilibrium. A cube resting on one of its faces, a glass funnel resting on its mouth (A, Fig. 77), are examples of stable equilibrium.

A body is said to be in **unstable equilibrium,** if after slight displacement, it moves still further from the position of equilibrium. A cone standing on its apex, a glass funnel standing on the end of its stem (B, Fig. 77), an egg standing on its end, are examples of unstable equilibrium.

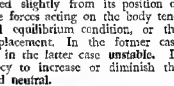


Fig. 77

A body is said to be in **neutral equilibrium,** when, after slight displacement, it neither returns to the original position nor moves further from it. A spherical ball resting on a horizontal plane, a cone or funnel lying on its side (C, Fig. 77), are examples of neutral equilibrium.

In stable equilibrium the centre of gravity of a body is at its lowest position, and a slight displacement tends to raise it. When the glass funnel A (Fig. 77) is slightly tilted, its C.G. is elevated and so the body returns to the original position as soon as it is allowed to do so.

In unstable equilibrium the C.G. is at its highest point, and a slight displacement tends to lower it. When the glass funnel B (Fig. 77) is slightly tilted, the C.G. at once occupies a lower point, and comes outside the base, and so can easily be overturned. Remem-

In performing an experiment the distances moved through by the right-hand weight P during the two stages of motion, (*a*) first, from its start from the upper platform till the rider is arrested, (*b*) second, from the instant when the rider is arrested till it is stopped on reaching the lower platform C, are noted and also the times taken by the two stages of motion are recorded by a stop-watch. Let the two intervals of distances be h_1 and h_2 and the times taken t_1 and t_2. Here h_1 is to be taken as the distance from the top of the upper weight P at a start to the ring B and h_2 as the distance from the ring B to the top of the same weight now on reaching the lower platform C.

(i) *Determination of g*.—According to the previous article, the acceleration of the moving body will be given by,

$$f = \frac{(P+Q)-P}{(P+Q)+P}\, g = \frac{Q}{2P+Q}\, g \qquad \ldots \quad (1)$$

assuming the formula, $P = mf$, which embodies Newton's Second Law of Motion. Starting from rest, the body moved through a distance h_1 in time t_1 with the acceleration given by equation (1)

$$\text{So, } h_1 = \tfrac{1}{2}\, \frac{Q}{2P+Q}\, g t_1^2 \qquad \ldots \quad (2)$$

P and Q being known, and h_1 and t_1 being noted, g is determined. The experiment may be repeated by altering h_1 at pleasure by shifting the position of the ring and noting t_1 in each case, when the value of g will be found the same.

(ii) *Verification of Newton's Laws of Motion*.—Here assume g to be known. The observed values of h_1 and t_1 will be seen to satisfy the relation (2) for all values of h_1 proving the correctness of the formula for the acceleration f given by relation (1). This indirectly verifies the truth of Newton's Second Law of Motion by which the relation (*a*) in Art. 103 is deduced.

The velocity acquired by the moving body at the end of the first stage of motion is given by,

$$v^2 = 2\, \frac{Q}{2P+Q}\, g h_1 \qquad \ldots \quad (3)$$

From the experiment it will be found that the value of v so determined exactly equals h_2/t_2, i.e. $h_2 = t_2 v$. The same result will be obtained on changing h_2 by altering the position of the lower platform C. This verifies Newton's First Law of Motion, for during the second stage of motion, i.e. from the ring B to the lower platform C, when the weights on the two sides of the string are equal, the velocity once acquired remains uniform in the absence of any resultant force on the system.

In deducing the above relation (1) which gives the acceleration f, the tension T is assumed constant throughout the string and this assumption is based on the truth of the Third Law of Motion. The

experimental verification of the value of f given by the relation (1) supplies an evidence in support of the Third Law as well.

(b) By the Falling Plate Method.—The method is very appropriate for measurement of the frequency of a tuning fork and has, therefore, been described in Chapter VI under Sound. It will be evident from the last equation of that article that the same experiment may be used to determine 'g' at a place, if a standard fork of known frequency is supplied.

107. Apparent Weight of a Man in a Moving Lift:—When a man is ascending or descending in a lift with uniform velocity, his weight exerted on the floor of the lift is equal and opposite to the reaction of the floor. When, however, the lift is rising upwards, the reaction is greater than the man's weight; and, when it is going downwards, the reaction is less than the man's weight.

Let m be the mass of the man, R the thrust on the floor of the lift, which is equal and opposite to the reaction of the floor on the man, and which may be called the man's 'apparent weight'.

The forces acting on the man are (a) his weight mg acting downwards, and (b) R acting upwards. Suppose the lift is *descending* with an acceleration f. Remembering that, force = mass × acceleration, we have, $mg-R=mf$; $\therefore R=m(g-f)=mg\ (1-f/g)$ ... ... (1)

Hence, the *man's apparent weight is less than his actual weight* 'mg' by 'f/g' *of the latter*, i.e. the man will appear to be lighter.

Similarly, when the lift is *ascending* with an upward acceleration f, we have, $R=mg(1+f/g)$ ... ... (2)

Hence *the man will appear to be heavier by 'f/g' of his actual weight.*

Example. *If a man weighs 16 stones on a lift which has an acceleration of 8 ft. per sec.², find the thrust on the floor due to its weight (i) when it is ascending, (ii) when it is descending.*

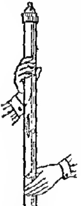
(i) We have, $R=mg\times(1+f/g)=16$ stones wt. $\times(1+\frac{8}{32})=20$ stones-wt.

(ii) In this case, we have, $R=mg\times(1-f/g)$ . . . . (*vide* eq. 1, Art. 107)
$=16$ stones-wt. $\times(1-\frac{8}{32})=12$ stones-wt.

108. Falling Bodies:—A very common experience is that if a heavy body like a stone and a light body like a feather or a piece of paper are dropped in air at the same time from a height, the heavy body reaches the ground much earlier. Such observations led Aristotle (384—322 B.C.), the great Greek Philosopher, to teach the world that a heavier body falls faster than a lighter body and according to him a body of 5 lbs.-wt. would fall five times faster than a body of 1 lb.-wt. Such idea prevailed until about two thousand years later when Galileo disproved it in 1589 A.D. He simultaneously dropped two heavy bodies, one large and one small, from the top of the *leaning tower of Pisa* before a large crowd of observers when it was found that the two bodies struck the ground together. Thus, he made the world know for the first time

that *all bodies fall with equal rapidity.* Why then a piece of paper falls more slowly than a stone? This is so, for when they fall through air, the air offers resistance to their motion. This air-resistance is too great for the weight of the paper and seriously affects its rate of fall; whereas, this resistance is very small compared to the weight of the stone and so the rate of fall of the stone is but little affected. If air could be removed and both the stone and the paper could be arranged to fall *freely*, i.e. unacted by any opposing force, both of them would fall with the same rapidity. In Galileo's time the air-pump was not invented and it was not possible to show that all bodies, heavy or light, should fall with equal rapidity, in the absence of air. After the invention of the air-pump in 1650 by Otto Von Guerike, Newton conclusively proved the truth of it experimentally by his well-known **Guinea and Feather experiment.**

109. Why should a Material Body fall to the Earth?—According to the law of gravitation, just as the earth pulls any body towards it, the body also pulls the earth towards itself with the same force. Why should then the body alone move to the earth and not the earth towards the body? Strictly speaking, the earth also must move but its motion is so small compared to that of the body that it cannot be taken notice of. Its motion, relative to that of the body, is extremely small due to its comparatively huge mass, for acceleration f is inversely proportional to the mass m for the same force P according to Newton's Second Law of Motion.



Guinea Feather Experiment. Fig. 80

110. The Laws of Falling Bodies:—

(1) *In vacuum all bodies starting from rest fall with equal rapidity.*

The acceleration due to gravity is the same for all bodies at the same place but the resistance of air influences the rate of fall differently in different cases. This will be evident by comparing the descent of a parachute with that of a lump of stone. The stone will fall very quickly and the observed difference in the rate of fall is due to the resistance offered by the air, the resistance increasing with the extent of the surface of the falling body. *Different objects will, however, fall at the same rate in a vacuum where the resistance to motion is nil.*

Guinea-Feather Experiment.—A wide glass tube (Fig. 80) about a metre long, having a cup screwed at one end and a stop-cock at the other is taken. A piece of paper and a small coin is introduced into the tube. On suddenly inverting the tube, it is found that the coin reaches the other end earlier than the piece of

paper. Next, by opening the stop-cock at which an air-pump may be connected, the air within the tube is exhausted. On now suddenly inverting the tube, it is found that the coin and the paper fall together and reach the other *end simultaneously*.

The following simple experiment also proves the same thing. A piece of paper is laid on metal disc (say, a rupee coin) of larger diameter and the combination is dropped down together. They are found to reach the ground simultaneously. Here the disc overcomes the resistance due to air and so the paper easily accompanies it.

(2) *The space traversed by a body falling freely from rest is proportional to the square of the time*, e.g. if the space traversed in one second is x feet, in two seconds it will be $x \times 2^2$ feet, in three seconds $x \times 3^2$ feet, and so on.

So, if s and t denote the space and time respectively, $s \propto t^2$.

This can be mathematically represented by the equation, $s = \frac{1}{2}gt^2$ [*vide* Art. 41], where g *is the acceleration due to gravity.*

(3) *The velocity acquired by a body falling freely from rest is proportional to the time of its fall*, e.g. if the velocity at the end of one second is x feet per second, at the end of two seconds it is $2x$ feet per second, and at the end of three seconds it is $3x$ feet per second, and so on. For this reason, a stone falling from a balloon at a great height will acquire such a large velocity that it will strike the surface of the earth almost like a rifle bullet. So, if v denotes the velocity and t the time, $v \propto t$. This can be mathematically represented by the equation, $v = gt$ [*vide* Art. 40].

111. Notes on the verification of Galileo's Laws of Falling Bodies:—The Atwood machine (Art. 106) may be used to verify Galileo's laws of falling bodies too. But the method, though direct, is only a rough one and the interest of the method lies in its antiquity only. The chief defects are,—the mass of the pulley which cannot be neglected, the friction of the pivot on which the wheel turns and the air-resistance.

A body falling freely from rest acquires a very large velocity after a short time, the acceleration due to gravity being large. To measure this velocity in a laboratory is a problem. Moreover, in Galileo's time clocks were not accurate enough to measure the short time involved in such a measurement. So Galileo used an inclined plane down which the motion of a rolling ball is much slower. A component only of the vertical acceleration depending on the inclination of the plane to the horizontal operates in this case to make the ball roll down. Thus, to test the *nature* of the acceleration due to gravity, he first, as it were, *'diluted'* it to make measurements easy.

(a) Verification by the Inclined Plane Method.—A fairly long wooden plank BA, say, about 4 to 5 metres long, is held in an inclined fashion (Fig. 81), there being a hinge at A about which it can be turned and thus its inclination θ to the horizontal can be altered. A straight groove is cut on the plank from B to A and a marble ball

when released from the top rolls down the plant along the groove. Commencing from the top of the groove, Galileo marked off positions a_0, a_1, a_2, a_3, a_4, etc. along the groove, making the intervals of successive distances proportional to 1, 4, 9, 16, . . . , *i.e.* proportional to the squares, 1^2, 2^2, 3^2, 4^2,...... Suppose these intervals are 10, 40, 90, 160 cms, etc. The ball moves when released from the top, with an acceleration which increases as the inclination of the plank is increased. The inclination of the plank is at first so adjusted that the ball rolls down a distance of, say, 10 cms in the first second. Galileo started the ball from the top and verified that the times of its describing the marked intervals were proportional to 1, 2, 3, 4, ...secs. Hence the distances described from rest were proportional to the squares of the times. **This verifies the second law** of Galileo for an inclined plane. Now this could be so, only if the acceleration is constant (cf. $s=ut+\frac{1}{2}ft^2$). The same result was obtained when the experiment was repeated with balls of different masses. This proved that the acceleration is independent of the masses and *is constant at the same place for a given inclination of the plane*. As the inclination was increased, the acceleration increased but remained constant for

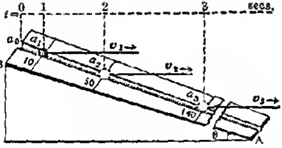


Fig 61—Inclined Plane Method.

any given inclination. From this he argued that when the inclination is vertical, this should also be constant, or, in other words, the acceleration due to gravity at a place is a constant quantity. **This verifies the first law.**

To measure time Galileo used the following device which may be called a water-clock. He took a vessel of large transverse section having a hole at the bottom. At first the hole was closed by the finger and the vessel filled with water. He removed his finger when the ball was started and the escaping water was collected. When the ball reached a mark, he again closed the hole and the water collected in the meantime was weighed. This weight gave a fair measure of the time elapsed, for the former is proportional to the latter approximately.

To measure the velocity acquired by the ball at the end of the 1st., or 2nd., or 3rd., etc.-secs., a smooth platform is held horizontally

at the position a_1, or a_2, or a_3, etc. respectively as the case might be. When the ball falls on the platform at a position, a_1, or a_2, etc. it moves forward over the platform with uniform velocity, the velocity being equal to that which the ball acquired in rolling down upto the position concerned. The velocity is measured by finding the distance travelled by the ball on the horizontal platform in a given time. The velocities, when so determined, at a_1, a_2, a_3, a_4, etc. will be proportional to 1, 2, 3, 4, etc. secs., *i.e.* proportional to time. **This verifies the third law.**

112. *The Falling of Rain-drops*:— A small rain-drop does not fall so quickly as a larger one, as the rate of fall of a smaller one is retarded more by the air.

The resistance of air is proportional to the area of cross-section through the centre of the drop, *i.e.* $\propto \pi \times (\text{radius})^2$.

But the weight of a drop $\propto$ its volume $\propto \frac{4}{3} \times (\text{radius})^3$.

Hence when the radius increases, *i.e.* for a drop of large size—the weight increases more rapidly than the resistance of the air. So *a larger drop falls more rapidly than a smaller one.*

It has been found that the maximum velocity of a very small rain-drop of diameter equal to $\frac{1}{1000}$ mm. is about 1·3 cms. per sec., and that of a larger drop of diameter equal to 0·46 cm. may be about 800 cms. per second.

113. Bodies projected Vertically Downwards:— If a body be projected vertically downwards with an initial velocity u, the equations of Art. 89 become,

$v = u+gt$,	where v is the velocity after a time t;
$s = ut+\frac{1}{2}gt^2$,	„ s „ distance fallen through;
$v^2=u^2+2gs$,	g being the acceleration due to gravity.

114. Bodies projected Vertically Upwards:— If a body is projected vertically upwards with an initial velocity u, we must substitute $-g$ for f, and the equations of Art. 113 now become,

$$v=u-gt\ ;\ s=ut-\tfrac{1}{2}gt^2\ ;\ v^2=u^2-2gs.$$

Greatest Height attained.— At the highest point the velocity of the body is zero; so if x be the greatest height attained by the body, we have $\quad 0=u^2-2gx.$

Hence the greatest height attained $=u^2/2g$.

Again, the time t to reach the highest point is given by,

$0=u-gt$, when $t=u/g$.

Similarly, u/g will be the time to reach the ground from the highest point. So the whole time of flight $=2u/g$.

Examples. (1) *A body is thrown vertically upwards with a velocity of 100 ft. per sec. Find (a) how high it will rise, (b) the time taken to reach the highest point, (c) the time of its returning to the ground.*

At the highest point the velocity of the body will be momentarily zero, and the body will then fall.

(a) Here, $u=100$ ft., $v=0$ at the highest point; $g=32$ ft./sec^2; $s=?$

We have $v^2=u^2-2gs$.

$\therefore 0=100^2-2\times32\times s$; $\qquad \therefore\ s=\dfrac{100\times100}{64}=156.25$ ft.

(b) Here $\qquad u=100$ ft./sec., $v=0$, $g=32$ ft./sec^2; $t=?$

We have $\qquad 0=100-32t$; $\qquad \therefore\ t=\dfrac{100}{32}=3\frac{1}{8}$ sec.

(c) Here $\qquad u=100$ ft./sec.; $g=32$ ft./sec.^{2}; $s=0$; $t=?$

We have $\qquad s=ut-\frac{1}{2}gt^2$; or, $0=ut-\frac{1}{2}gt^2$;

or, $t(u-\frac{1}{2}gt)=0$, whence either $t=0$, which is rejected;

or, $u=\frac{1}{2}gt$, i.e. $t=\dfrac{2u}{g}$. $\therefore\ t=\dfrac{2\times100}{32}=6\frac{1}{4}$ secs.

(2) *Two stones are projected vertically upwards at the same instant. One ascends 112 ft. higher than the other and returns to earth 2 seconds later. Find the velocities of projection of the stones ($g=32$ ft. per sec. per sec.)* (C. U. 1935)

At the highest point v will be momentarily zero, so we have, $0=u^2-2gs$,

$$\text{or,}\quad s=\frac{u^2}{2g}\ \text{for one stone. For the other stone,}\ s+112=\frac{u_1^2}{2g},$$

$$\therefore\quad 112=\frac{u_1^2-u^2}{2g} \qquad (1)$$

At the highest point $u-gt=0$; or, $t=u/g$. $\therefore$ Total time of flight $t_1=2u/g$, and for the other, the total time of flight $(t_1+2)=2u_1/g$.

$$\therefore\quad 2=\frac{2(u_1-u)}{g}=\frac{u_1-u}{16} \qquad (2)$$

From (1), $\qquad 112=\dfrac{u_1^2-u^2}{64}=\dfrac{(u_1-u)}{16}\times\dfrac{(u_1+u)}{4}=2\times\dfrac{u_1+u}{4}=\dfrac{u_1+u}{2}$,

$\therefore\ u_1+u=224$, and $u_1-u=32$, from (2)

or, $u_1=128$ ft./sec., and $u=96$ ft./sec.

(3) *A stone is dropped from a balloon at a height of 200 feet above the ground and it reaches the ground in 6 seconds. What was the velocity of the balloon just at the moment when the stone was dropped?* (C. U. 1932)

At the moment when the stone was dropped, it was moving upwards with the same velocity as the balloon. Let this velocity be u ft. per sec. upwards. So, here u is negative, and g is positive and the stone is falling downwards.

Here, $\qquad u=?$, $s=200$ ft., $t=6$ secs.; $g=32$ ft./sec^2.

We have, $s=(-u)t+\frac{1}{2}gt^2$, or, $200=-u\times6+\frac{1}{2}\times32\times6^2=-u\,6+576$;

$\therefore\ 6u=576-200=376$, $\quad$ or, $u=62\frac{2}{3}$ ft. per sec. $=62.6$ ft./sec.

(4) *It is required to pierce ... of a rifle bullet fired immediate... must have a velocity of 40 ft. pe... must it leave the muzzle of the ...*

$$\tfrac{3}{4}\ \text{mile}=\frac{1760\times3}{4}=1320\ \text{ft.}$$

Here $v=40$ ft./sec.; $u=?$; $g=32$ ft./sec.^{2}, $s=1320$ ft.

We have, $40^2=u^2+2gs$, or, $40^2=u^2+2(-32)\times1320$;

$\therefore\quad u^2=40^2+64\times1320=86080$; $u=293.4$ ft. per sec.

PENDULUM

115. Historical Note:— Galileo appears to have been the first to make use of the pendulum. One day when in the Cathedral at Pisa, 1593, he was watching a swing lamp and noticed that while the oscillations of the lamp gradually died away, the time taken by it to make one oscillation still remained the same. He timed the oscillations by beats of his pulse. This discovery, he pointed out, could be utilised to regulate clocks. In 1658 Huygens actually used the pendulum to regulate the motion of clocks.

116. *Some Definitions:—*

The Simple Pendulum.— A simple pendulum is defined as a *heavy* particle suspended by a *weightless, inextensible* but *perfectly flexible* thread, from a *rigid support* about which it oscillates without friction. In practice, however, a small metal bob suspended from a fixed support by a very fine long thread is taken to be a simple pendulum.

The Compound Pendulum.— Any body capable of oscillating freely about a horizontal axis is known as a compound pendulum. The metallic rod carrying at its lower end a heavy lens-shaped mass of metal, known as the bob, acting as oscillator, in a clock is an example of a compound pendulum.

The Seconds Pendulum.— It is a simple pendulum which takes one second in making *half* a complete oscillation (*i.e.* one vibration or swing). So it has a *period* of two seconds. When it is said that a pendulum *beats* one second, it means that it takes one second to make one swing.

117. Some Terms:—

Length of a Simple Pendulum.— It is the distance L from the point of suspension up to the centre of gravity of the bob, *i.e.* the distance between A and B [Fig. 82(*a*)]. That is, it is the length of the suspension thread plus the vertical radius r of the bob. It is also called the *effective length* of the pendulum.

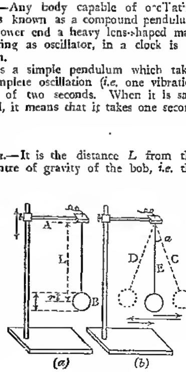


Fig. 82

Amplitude.— The maximum angular displacement a [Fig. 82(*b*)] of the bob, measured, on either side, from its undisturbed position (given by the vertical position E) up to the extreme position as shown at C or D, is called its amplitude. It should not exceed 4° for the motion to be *simple harmonic* [*vide* Art. 119]. The

amplitude of a simple pendulum gradually decreases as the bob swings, on account of air-resistance mainly.

Time period (or simply, Period).— It is the time taken by a pendulum to make *one complete oscillation*. One complete oscillation comprises *two* swings—one forward, another backward. An oscillation is usually reckoned from the extreme position D [Fig. 82(*b*)] to the other extreme position C and back to D next time; or, from the undisturbed position E (pendulum vertical) when, say, it is moving to the right, until when it passes through the undisturbed position E again moving in the same direction as shown by the arrows.

One *vibration* means the motion from one extreme position, say, D, to the other extreme position C, i.e. it is half of an oscillation.

Frequency.—It is the number of complete oscillations made by a pendulum per second at a place. Thus, if n=frequency, and t=time period, $nt=1$, or, $n=1/t$.

Phase.—The phase of a pendulum gives its state of *displacement* and *motion* at any particular instant, *i.e.* it determines the position of the pendulum in the path of motion and also the direction of motion at that instant.

118. The Laws of Pendulum :—The laws of oscillation of a simple pendulum are given by the following relation—

$$t=2\pi\sqrt{\frac{l}{g}},$$

where t=period of the pendulum, l=effective length; g=acceleration due to gravity at the place of oscillation.

Law 1. *The oscillations of a pendulum are isochronous.*—That is, a pendulum takes equal time to complete each oscillation whatever is the amplitude, provided the latter is small (within 4°). So time-period is independent of the amplitude of vibration. *This is also known as* **the law of isochronism.**

Law 2. *The period of oscillation of a pendulum varies directly as the square root of the length.* Mathematically, $t\propto\sqrt{l}$, or l/t^2=a constant for the place of observation. Thus, if the length be increased four times, the period becomes double. *This is known as* **the law of length.**

[**Note.**—The length of a pendulum changes with temperature; so the period t of a pendulum changes with temperature.]

Law 3. *The period of oscillation varies inversely as the square root of the acceleration due to gravity at the place of observation. This is known as* **the law of acceleration.** Mathematically, $t\propto 1/\sqrt{g}$, or $t^2\times g$=a constant, for the same pendulum.

Thus, if g be greater at a place, t will be less, i.e. the pendulum will vibrate more rapidly.

Law 4. *The period does not depend on the mass or material of bob of the pendulum, provided the length remains constant.* ... *is known as* **the law of mass.**

119. Simple Harmonic Motion*:— A body is said to execute simple harmonic motion (abbreviated as **S.H.M.**), if it does a to-and-fro periodic motion in a straight line such that its acceleration is always directed to a fixed mean position in that path, called the position of equilibrium, and is proportional to the displacement from that mean position.

120. Motion of a Simple Pendulum is Simple Harmonic:— Let the bob of mass m of a pendulum of length l [Fig. 83] be displaced through an angle θ from its undisturbed position B to the position C. If g be the acceleration due to gravity at the place, the weight mg of the bob can be resolved into two components $mg \cos\theta$ acting along CF, the direction of the string which is kept taut thereby, and $mg \sin\theta$ acting at C along CE at right angles to CF. The former is balanced by the tension of the string, while the latter tends to bring the bob back to its original position B with an acceleration $g \sin\theta$. If θ does not exceed 4°, $\sin\theta$ may be taken to *be equal to* θ *and so the acceleration* of the bob, $g \sin\theta = g\theta$. After crossing the mean position B, when the bob moves towards BD by virtue of its inertia and acquired velocity, the acceleration acts in the opposite direction, *i.e.* towards B and so the motion decreases and vanishes at the other extreme position D, when the direction of motion is reversed. ***This explains why a pendulum should oscillate at all.*** The acceleration, it is to be noted, is always directed towards the mean position B.

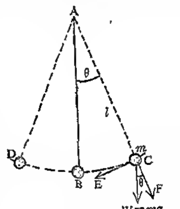


Fig. 83

$$\text{Again, } \theta = \frac{\text{arc } BC}{\text{length } AB} = \frac{\text{displacement}}{\text{length of pendulum } (l)}$$

$$\therefore \quad \text{Acceleration} = g.\theta = \frac{g}{l} \times \text{displacement} \qquad \ldots \quad (1)$$

That is, the acceleration is proportional to the displacement, because g and l are constants for the pendulum at a given place.

Thus, acceleration being proportional to displacement and always being directed to a fixed position B in the path of motion, the motion is simple harmonic, according to the definition of simple harmonic motion.

* For a detailed treatment of S.H.M., see Chapter II on S.H.M., in Sound, Part III of this volume.

Though a pendulum continues to oscillate for a long time, it, however, gradually stops due to the resistance of air and the friction at the point of suspension; otherwise a pendulum would have oscillated for ever, had there been no such resistance to stop it.

121. Period of a Simple Pendulum :— Mathematically, the motion of a pendulum, which is simple harmonic, is given by [*vide* Art 10, Part III].

$$\frac{\text{Acceleration}}{\text{Displacement}} = \omega^2, \text{ where } \omega = \text{angular velocity}$$

$$= \left(\frac{2\pi}{t}\right)^2, \text{ where } t = \text{time-period.}$$

That is, $t^2 = 4\pi^2 \times \dfrac{\text{displacement}}{\text{acceleration}} = 4\pi^2 \times \dfrac{l}{g}$, from (1) above.

$$\therefore \quad t = 2\pi\sqrt{\frac{l}{g}}.$$

122. Verification of the Laws of Pendulum :—

Law 1 (*The Law of Isochronism*)—To verify the first law, note with a stop-watch the total time of, say, 20 oscillations with different amplitudes, keeping the length constant. It will be found that the period t in each case remains constant.

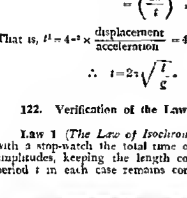


Fig. 84

It should be noted that the law is true only for small angles of amplitude (about 4°), so when noting the times of oscillation with different amplitudes, care should be taken not to exceed the maximum limit of 4°.

Law 2. (*The Law of Length*)—Find the *vertical* radius of the bob by means of a slide callipers, and hence determine the length from the point of suspension up to the centre of gravity of the bob. Observe the time taken for 20 complete oscillations, and thus find t, the period.

Change the length of the pendulum and again find the period. In this way get several values of the period for the corresponding lengths. It will be found that $t \propto \sqrt{l}$, *i.e.* the value l/t^2 will be a constant.

Law 3 *(The Law of Acceleration).*—This law can be verified by taking a pendulum to different places having different values of g. It will be seen that at a place where g is greater, the vibrations will be quicker. $t^2 \times g$ will, however, be found constant at different places for the same length of the pendulum.

Law 4 *(The Law of Mass).*—Keeping the effective length of the pendulum the same in every case, if the bob be replaced by another one of different *size* or of a different material, it will be found that the period t remains unaltered.

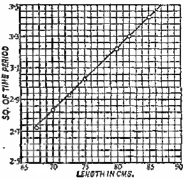


Fig. 85

By performing this experiment with bobs of different substances (such as wood, iron, brass, etc.), it can be shown that *the acceleration due to gravity at the same place is the same for all bodies.*

Graph.—Draw a graph with the length l (along the X-axis) and time period t (along the Y-axis). The relation between l and t will be an arm of a parabola (Fig. 84). The graph (in Fig. 85), which is a *straight line*, represents the relation between l and t^2. From any of these graphs, the length of the pendulum corresponding to a given time period of oscillation can be determined, but it is better to take the help of l and t^2 graph (straight line) for this purpose.

123. The length of a Seconds Pendulum:—The period of a *seconds* pendulum is 2 *seconds.* Hence from the formula for the period of oscillation, we have,

$$1=\pi\sqrt{\frac{l}{g}}\,;\ \text{or, } l=\frac{g}{\pi^2} \qquad \ldots \qquad \ldots \qquad \ldots \quad (1)$$

So the length of the seconds pendulum changes at different places depending on the value of g.

Taking the value of g to be 981 cms. per sec. per sec., the length of the seconds pendulum becomes [from eq. (1) above],

$$l=\frac{981}{\pi^2}=\frac{981}{9{\cdot}87}=99{\cdot}39 \text{ cms.}$$

Taking the value of g to be 32·2 ft. per sec. per sec.,

$$l=\frac{32{\cdot}2}{\pi^2}=\frac{32{\cdot}2}{9{\cdot}87}=3{\cdot}26 \text{ ft.}=39{\cdot}12 \text{ inches.}$$

Graph.—To determine the length of the seconds pendulum from the graph, draw the l and t^2 graph (Fig 85) and find the length corresponding to $t^2=4$.

124. The Value of 'g' by a Pendulum :—By carefully measuring the length and the corresponding period of a simple pendulum, the value of g at any place can be determined from the formula,

$$t=2\pi\sqrt{\frac{l}{g}}\text{; whence } g=\frac{4\pi^2 l}{t^2}=4\pi^2\times\frac{l}{t^2}.$$

Thus, when the value of l/t^2 at a place is (say) 21·84, g is given by $g=4\pi^2\times l/t^2=4\times9{\cdot}87\times21{\cdot}84=980{\cdot}68$ cms /sec.^{2}.

125. Loss or Gain of Time by a Clock on Change of Place :—The loss or gain of time depends on (*a*) *the latitude of the place* as the value of g varies with the latitude of a place (Art. 98). g is minimum at the equator and increases gradually towards a pole. But as the time-period t of a simple pendulum varies inversely as the square root of g, the period t of a pendulum will decrease as it is taken from the equator to a pole So, a pendulum clock will gradually gain time, *i e.* will go *fast*, when taken from the equator to a pole.

(*b*) The loss or gain of time also depends on *the height of a place above the sea level* As the value of g diminishes with the distance above and also below the surface of the earth, the time-period t of a pendulum clock will increase, and so the clock will lose time, *i e.* will go *slower* when taken to the top of a mountain or to the bottom of a mine.

126. Measurement of Height of a Hill :—

(i) *By a pendulum experiment.*—

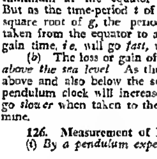


Fig. 86

Suppose g and g' are the respective values of the acceleration due to gravity at the bottom and at the top of a hill as measured by a pendulum experiment Then, as shown in Art. 100, at the bottom

$$g=\frac{GM}{R^2},$$ with usual notations,

where R=radius of the earth (Fig. 86). Acceleration g' at the top of a hill of height h will be given by $g'=\frac{GM}{(R+h)^2}$. From the above equations,

$$\frac{g}{g'}=\frac{(R+h)^2}{R^2}\text{; or, }\frac{R+h}{R}=\sqrt{\frac{g}{g'}} \quad ..(1)$$

Thus, if R is given, h will be known when g and g' are experimentally determined.

(ii) By a Clock.—

Suppose, a clock gives correct time at the foot of a hill and loses n secs. a day at the top of it.

Then, at the bottom, $1=\pi\sqrt{\dfrac{l}{g}}$ ... ... (2)

where l=length of its pendulum, which is really a seconds pendulum and at the top, where it makes $(86400-n)$ swings in 86400 secs.

$$\frac{86400}{86400-n}=\pi\sqrt{\frac{l}{g'}} \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

From (2) and (3), $\sqrt{\dfrac{g}{g'}}=\dfrac{86400}{86400-n}$ ... ... (4)

From (1) and (4), $\dfrac{R+h}{R}=\dfrac{86400}{86400-n}$ ... ... (5)

Thus, h will be found, if R is given and n determined.

127. The Disadvantages of a Simple Pendulum :—

(*i*) In obtaining the formula for the simple pendulum, the thread was assumed to be weightless and all the mass of the bob was assumed to be concentrated at its centre, but, in practice, neither of these conditions is strictly true.

(*ii*) The formula for the simple pendulum is true only for very small amplitudes, and corrections should be made for large amplitudes.

(*iii*) Corrections should also be applied for the effect of resistance on motion, and the buoyancy of the air, which raises the centre of gravity of the pendulum.

(*iv*) Errors are also introduced due to the slackening of the thread when approaching the limit of swing, and *due to the friction at the point of suspension* which may interfere with the free movement of the pendulum.

Examples. (1) *Find the length of a seconds pendulum at a place where $g=981$.* (*C. U. 1912, 1919*)

For a simple pendulum, we have. $t=2\pi\sqrt{l/g}$.

For a seconds pendulum, $t=2$ secs.; $\therefore\ 2=2\times\dfrac{22}{7}\sqrt{\dfrac{l}{981}}$

Hence $l=\dfrac{49}{22\times22}\times981=99{\cdot}31$ cms. (nearly).

(2) *Two pendulums of lengths 1 metre and 1·1 metre respectively start swinging together with the same amplitude. Find the number of swings that will be executed by the longer pendulum before they will again swing together ($g=978$ cms. per sec.^{2}).* (*C. U. 1909*)

Let t_1 and t_2 be the periods of oscillation of the pendulums of lengths 1 metre and 1·1 metre respectively; 1 metre=100 cms. and 1·1 metre=110 cms.

Then we have, $t_1=2\pi\sqrt{\frac{100}{978}}$; and $t_2=2\pi\sqrt{\frac{110}{978}}$.

Suppose the pendulum of 1·1 metre length makes n_2 swings, and the other makes (n_1+n_2) swings before they again swing together :

$$\text{then,} \quad n_2t_2=(n_1+n_2)t_1; \qquad \text{or,} \quad n_2(t_2-t_1)=n_1t_1 \quad \ldots \quad (1)$$

$$\text{But} \quad (t_2-t_1)=\frac{2\pi}{\sqrt{978}}(\sqrt{110}-\sqrt{100})$$

$$\therefore \text{ From (1),} \quad \frac{2\pi n_2}{\sqrt{978}}(\sqrt{110}-\sqrt{100})=\frac{2\pi n_1\sqrt{100}}{\sqrt{978}};$$

$$\text{or,} \quad n_1=\frac{10}{\sqrt{110}-10}n_2=\frac{10(\sqrt{110}+10)}{10}n_2$$

$$=(\sqrt{110}+10)n_2=20{\cdot}5n_2 \text{ (nearly)}=\frac{41}{2}n_2 \text{ (nearly)}$$

To get a whole number, the least value for n_2 is 2, and therefore, $n_1=41$ (nearly)

(3) Supposing a pendulum to be so constructed that it beats seconds at a place where $g=980$, how would its length have to be changed so that it may beat seconds at a place where $g=340$?

The period of a seconds pendulum is 2 seconds

$$\text{We have,} \quad t=2\pi\sqrt{\frac{l}{g}} \qquad \text{Hence } 2=2\pi\sqrt{\frac{l}{980}}$$

$$\text{Again,} \quad 2=2\pi\sqrt{\frac{l_1}{340}} \qquad \therefore \sqrt{\frac{l}{980}}=\sqrt{\frac{l_1}{340}}$$

$$\text{or,} \quad \frac{l_1}{l}=\frac{340}{980} \qquad l_1=\frac{17}{49}l$$

Hence the length has to be shortened to $\frac{17}{49}$ of its original length

(4) A pendulum which beats seconds at a place where $g=32{\cdot}2$ is taken to a place where $g=32{\cdot}197$. How many seconds does it lose or gain in a day?

Let t_1 be the original period and t_2 the new period of the pendulum. In this case t_1 is equal to 2 secs, but this fact is not required

$$\text{We have,} \quad t_1=2\pi\sqrt{\frac{l}{32{\cdot}2}},\ t_2=2\pi\sqrt{\frac{l}{32{\cdot}197}}$$

$$\textit{Hence,} \quad \frac{t_1}{t_2}=\sqrt{\frac{32{\cdot}197}{32{\cdot}2}}=\sqrt{\frac{32{\cdot}2-0{\cdot}003}{32{\cdot}2}}=\sqrt{\left(1-\frac{0{\cdot}003}{32{\cdot}2}\right)}$$

Because period $\propto\frac{1}{\sqrt{g}}$, we have $t_2>t_1$, and so the pendulum will lose time

Let n=no. of secs. lost per day. The number of secs. in a day is $24\times60\times60$, or 86400. $\therefore$ $(86400-n)t_2=86400\times t_1$;

$$(86400-n)=86400\times\frac{t_1}{t_2}=86400\times\sqrt{\left(1-\frac{0\cdot003}{32\cdot2}\right)}=86400\left(1-\frac{0\cdot003}{32\cdot2}\right)^{\frac{1}{2}}$$

$$=86400\left(1-\tfrac{1}{2}\times\frac{0\cdot003}{32\cdot2}\right)\text{ approx.}=86400-4;\quad\therefore n=4\text{ secs.}$$

Hence the pendulum loses 4 secs. per day.

(5) *A pendulum which beats seconds at the Equator gains five minutes per day at the Poles. Compare the values of g at the two places.*

Let g_1 and t_1 denote the value of g and *period* respectively at the equator, and g_2 and t_2 those at the poles.

Because the pendulum beats seconds at the equator, $t_1=2$ seconds.

We have, $t_1^2=4\pi^2\dfrac{l}{g_1}$: or, $4=4\pi^2\dfrac{l}{g_1}$; or, $g_1=\pi^2l$ ... ... (1)

Now, at the poles, the pendulum gains 5 minutes per day, that is (5×60) seconds in $(24\times60\times60)$ secs. $\therefore$ It gains $\dfrac{5\times60}{24\times60\times60}$ sec. per sec.,

i.e. it gains $\dfrac{1}{288}$ sec. in one vibration; or, $\dfrac{2}{288}$ sec. in one complete oscillation.

Because it gains $\dfrac{2}{288}$ sec. in one oscillation, its period, $t_2=\left(2-\dfrac{2}{288}\right)=\dfrac{574}{283}$ secs.

$$\therefore\quad\left(\frac{574}{288}\right)^2=4\pi^2\frac{l}{g_2}\text{ ; or, }g_2=4\pi^2l\times\frac{288^2}{574^2}\qquad\ldots\qquad\ldots\qquad\ldots\quad(2)$$

From (1) and (2), $\dfrac{g_1}{g_2}=\dfrac{\pi^2l}{4\pi^2l\times\dfrac{288^2}{574^2}}=\dfrac{\pi^2}{4\pi^2\times\dfrac{288^2}{4\times287^2}}=\dfrac{287^2}{288^2}=\dfrac{289}{291}$ approx.

(6) *A pendulum of length l loses 5 secs. in a day. By how much must it be shortened to keep correct time?* (C. U. 1952)

There are 86400 seconds in a day. As the pendulum loses 5 secs. a day, it beats $(86400-5)$, or, 86395 times in one day, *i.e.*, in 86400 seconds.

$\therefore$ Time of one vibration (time of one swing), $t=\dfrac{86400}{86395}$ (and.. not 1 sec.),

But the time of one swing, i.e. half oscillation is $\pi\sqrt{l/g}$.

$$\therefore\text{ We have, }\pi\sqrt{\frac{l}{g}}=\frac{86400}{86395}.\quad\therefore\ \pi^2\frac{l}{g}=\left(\frac{86400}{86395}\right)^2\qquad\ldots\qquad\ldots\quad(1)$$

In order to keep correct time, let the length of the pendulum be shortened by x. In this case, it becomes a true seconds pendulum and its time of one vibration becomes 1 second.

$$\text{Then we have, }\pi\sqrt{\frac{l-x}{g}}=1;\qquad\therefore\ \pi^2\frac{l-x}{g}=1\qquad\ldots\qquad\ldots\quad(2)$$

From (1) and (2), $\pi^2\dfrac{x}{g}=\left(\dfrac{86400}{86395}\right)^2-1=\left(1+\dfrac{5}{86395}\right)^2-1$

$$=\left(1+\frac{2\times5}{86395}+\text{etc}\ldots\right)-1,\text{ from Binomial theorem.}$$

$$=\frac{10}{86395}\text{(neglecting other terms).}$$

130. Christian Huygens (1629—1695): A Dutch Physicist and contemporary of Newton He ranks with Galileo as an investigator of Nature. His chief claim to immortality relates to the development of the wave theory, though his contributions to Mathematics and Astronomy are no less He discovered the Orion Nebula and was the inventor and perfector of pendulum clocks. Elected to the Royal Society of London he delivered in 1663 his famous lecture giving the laws for the collision of elastic bodies He thoroughly studied the properties of curves, particularly the Cycloid. He died a bachelor.

131. *Sir Isaac Newton* (1642—1727).—An English Physicist and Mathematician and a genius with few equals The foundations of most of our physical sciences rest on his different works His treatise *Principia* is an immortal gift to posterity. In it are contained, amongst others, the foundations of Mechanics—the laws of motion, given in Latin, and their applications to motion of heavenly bodies under gravitation

Sir Isaac Newton

He was born at Woolsthorpe, Lincolnshire, England on the Christmas day of 1642, a posthumous son From his boyhood he was philosophic in temperament Educated at the Trinity College, he received the M.A degree from Cambridge in 1665 That year *the black plague* broke out and he removed to Woolsthorpe where during the next few years he made the greatest discoveries. Once while sitting under an apple tree in his home garden, it is said, a ripe apple fell on his head. Why should the apple fall towards the earth? He thought and concluded that there must be some attractive force between the earth and any material body He knew the three laws of planetary motion which Kepler

had discovered before, as also the Galilean laws of falling bodies. In nearly a circular path the moon moves round the earth once in a month. A force is necessary to keep the moon in its orbit. The question arose in Newton's mind—was this force of the same nature as the force which makes an apple fall? He founded the doctrine of universal gravitation from reasonings on this question. During his short period of stay at Woolsthorpe he also worked out the principles of Differential Calculus, for he found that the existing mathematical knowledge of his times was not adequate to deal with the problems relating to continuously varying quantities. He next devoted his attention to the studies of optics and revealed the composition of white light by the use of a prism. He also advanced a theory on the propagation of light, namely the *corpuscular* theory. He was a great practical optician too and constructed a reflecting type of telescope. He also worked on the viscosity of fluids.

In 1669 he was appointed Professor of Natural Philosophy at the Cambridge University. He was elected to Parliament and acted for twenty-five years as the President of the Royal Society and was knighted by Queen Anne in 1705. A remark of his made at the death-bed show how modest he was though so great. He said, "If I have seen farther than others, it is by standing on the shoulders of giants." At the age of fifty he developed a nervous breakdown, after which he did not do much scientific work and turned to theology. He died at the ripe old age of eighty-five, a bachelor and was buried at Westminster Abbey.


Henry Cavendish

132. Henry Cavendish (1731—1810):— He ranks with Scheele, Priestley and Black in founding the science of chemistry. He belonged to a noble and rich family of England and lived a life devoted to science. He discovered in 1771 that Hydrogen and Oxygen when burning together form water. His researches on the chemistry of the air practically led to our present knowledge of the composition of the air. In 1773 his electrical investigation led him to establish the law of inverse squares for electric forces. He successfully measured by means of a Coulomb Torsion balance the force of gravitational attraction between two lead spheres which he set up. Perhaps this was the first time that such a small mechanical force was experimentally measured by any worker. He calculated the value of G, from the masses concerned and their distance apart. This enabled him to calculate the density of the earth and so "Cavendish is often said to have weighed the earth."

Questions

1. What is meant by the phrase "Constant of gravitation is 6.5×10^{-8} c g s unit"? (R. U. 1954)

2. Calculate the mass of the sun given that the distance between the sun and the earth is 1.49×10^{13} cm. and $G = 6.66 \times 10^{-8}$ c.g.s. unit. Take the year to consist of 365 days (P. U. 1942)
[*Ans.* 4.247×10^{30} lb]

3. A body is weighed at the surface of the earth, at the sea level and at the top of a mountain. State, in general terms, how the position will affect the weight and mass of a body. Give reasons for your answer as far as possible (C. U. 1920; *cf*. Pat. 1932)

4. State where a body weighs more—at the poles or at the equator. Give reasons. How do you prove this difference in weight experimentally? (C. U. 1931, '40)

5. Distinguish between mass and weight. How are the mass and weight of a body affected by variations of latitude? Is weight an essential property of matter? (C. U 1941, *cf* Nag U. 1950, Pat 1932)

6. Describe an Atwood's machine and explain how you would use it to determine the value of g in the laboratory. (Pat. 1955)

7. State Newton's law of gravitation. Obtain an expression for the acceleration due to gravity in terms of the mass of the earth, the radius of the earth, and the gravitational constant (R U 1955)

8. What is meant by "acceleration of gravity"? How do you prove that it varies from place to place on the earth's surface? How does it vary? (C U 1933; *cf* All 1939; U P, B 1943)

9. How does the rotation of the earth affect the acceleration due to gravity? (R U 1955)

10. A light string passes over a smooth pulley and has masses of 240 gm and 250 gm attached to its ends. Calculate the value of g, if the system starting from rest moves a distance of 160 cm in 4 seconds (Anna U. 1950)
[*Ans.* 980 cm per sec.^{2}]

11. Two masses of 80 and 100 gm are connected by a string passing over a smooth pulley. Find the tension of the string when they are in motion. Find also the space described in 4 secs ($g = 981$ c g s units). (M U 1951)
[*Ans.* 87200 dynes, 872 m]

12. A man weighing 10 stones is sitting in a lift which is moving vertically with an acceleration of 8 ft per sec.^{2} Prove that the pressure on the base of the lift is greater when it is ascending than when it is descending and compare the pressures (Pat 1931)
[*Ans.* $R/R_1 = 5/3$]

13. A mass of 10 lb is hung from a spring balance attached to a lift. The lift is (*a*) ascending with an acceleration of 4 ft /sec.^{2}, (*b*) ascending with a uniform velocity of 4 ft /sec. Calculate how the reading of the spring balance will be affected in each case ($g = 32$ ft /sec.^{2}) (Pat 1933)
[*Ans.* (*a*) The balance will read more by 1.25 lb
(*b*) The balance will record the same all along]

14. Show that for a falling body the distance through which it falls down during a given number of secs is equal to the distance travelled during the first sec. multiplied by the sq. of the number of secs (C. U. 1946)

15. A body of mass 50 gms. is allowed to fall freely under the action of ...ity. What is the force acting upon it? Calculate the momentum and the ...inetic energy it possesses after 5 seconds ($g = 980$ cms per sec.^{2}) (C. U. 1937)

[*Ans* 49×10^3 dynes, 245×10^3 c g s units; 60025×10^3 ergs]

16. How would you experimentally show that the acceleration of a freely falling body is uniform? (Utkal, 1948, '50, '54)

17. State the laws of falling bodies and illustrate them by suitable examples. (C. U. 1941)

18. How is the period of swing of a pendulum related to the wt. of the bob, its length, and the amplitude of the swing? Hence state the laws of oscillation of a simple pendulum and state how you would verify them experimentally. What is meant by *effective length*?

(C. U. 1913, '15, '17, '19, '21, '24, '32, '36, '40, '47, '49, '53; Pat. 1946)

19. Explain why a pendulum should oscillate if the bob is drawn aside and let go. (Pat. 1946)

20. Obtain an expression for the period of a simple pendulum. What is the practical use of this formula? (R. U. 1951)

21. What is a simple pendulum? State the laws of vibration of a simple pendulum. Explain in general terms how a clock will gain or lose as it is taken from the surface of the earth to the top of a hill and to the bottom of a mine. (C. U. 1957)

22. State the laws of vibration of a simple pendulum and find the length of a seconds pendulum at a place where g is 980 cms./sec.^{2}. (C. U. 1951)
[*Ans.* 99·29 cm.]

23. A faulty seconds pendulum loses 20 secs. per day. Find the required alteration in length so that it may keep correct time; given $g=32$ ft./sec.^{2}. (Pat. 1953)

[*Ans.* 0·0015 ft.]

24. (*a*) How will you proceed to determine the 'g' of a place with a pendulum? Give the practical directions necessary and state reasons.

(U. P. B. 1947, '48; G. U. 1949)

(*b*) What is the effect of the height above, or the depth below, the surface of the earth, on the periodic time of a pendulum? (G. U. 1949)

[*As 'g', decreases, the periodic time of a pendulum increases and hence a clock will go slower.*]

25. A pendulum which keeps correct time at the foot of a mountain loses 16 seconds a day when taken to the top. Find the height of the mountain. Neglect the attraction due to the mountain and take the radius of the earth as 21×10^6 ft.

[*Ans.* 3890 ft. approx.]

26. A pendulum which beats seconds at a certain place where 'g' is 981 cm./sec.^{2} is taken elsewhere where 'g' is 978·5 cms./sec.^{2}. Calculate the number of seconds it loses or gains in a day? (Pat. 1939)

[*Ans.* It loses 1 minute 58·89 seconds.]

27. Will a pendulum clock gain or lose, when taken to the top of a mountain? (C. U. 1917, '19; *cf.* U. P. B. 1941)

28. When a ball suspended by a string is made into a 'seconds pendulum', does the actual length of its string equal the length of the equivalent simple pendulum? If not, why? (C. U. 1912)

[**Hints.**— *As the ball has a certain dimension, the actual length of the string will not be equal to the length of the equivalent simple pendulum. The distance between the point of suspension and the centre of gravity of the ball will be the length of the equivalent simple pendulum.*]

29. What precautions or corrections are necessary in an experiment with a simple pendulum? (C. U. 1953)

30. A clock which keeps correct time when its pendulum beats seconds was found to be losing 4 minutes a day. On altering the length of the pen-

dulum it gained $2\frac{1}{2}$ minutes a day. By how much was the length altered, if the length of the seconds pendulum is 99 177 cms ?

[*Ans.* 8 97 mm.]

31 A hollow pendulum has a hollow spherical bob attached to its thread. Will the period alter if the hollow bob is half filled with mercury ? (C. U. 1950)

CHAPTER VI

WORK : ENERGY : POWER

133. Work :—Work is said to be done *by* or *against* a force, when its point of application moves in or opposite to the direction of the force and is measured by the product of the force and the displacement of the point of application in the direction of the force. The work may also be determined by the product of the displacement and the component of the force in the direction of the displacement

When a man raises a weight, the force which he exerts does work against the force of gravity which acts downwards. Work is done by a horse when it draws a carriage against the force of friction, called into play between the carriage and the ground, which opposes the motion

Suppose a force P acts on a body at A in the direction AX and it moves to B in a given time (Fig 87)

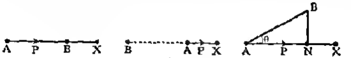


Fig 87

(i) If the displacement AB is in the direction AX [Fig. 87(*a*)], the work done by P, is $W=P\times AB$, and is called positive work.

(ii) If the displacement AB is in a direction opposite to the direction of P [Fig 87(*b*)], the displacement measured in the direction of $P = -AB$, and the work done by the force P, is $W=-P\times AB$, and is called negative work. This work is done against P

(iii) If the displacement AB is in a direction different from the line of action of P, say, making an angle θ with AX [Fig 87(*c*)], then the displacement measured in the direction of P is $AN= AB\cos\theta$, where BN is the normal from B on AX. Therefore, work done by P is $W=P\times AN=P\times AB\cos\theta=AB\times P\cos\theta$. That is, work $=$force$\times$component of the displacement of the point of applica-

tion of the force in the direction of the force=displacement × component of the force along the direction of the displacement.

N.B. It should be noted from above that *no work* is done by or against a force at *right angles* to its own direction, because θ in this case is 90°.

134. The Units of Work :—*Unit work is done when unit force moves its point of application, in its own direction, through unit distance.* As the unit of force is measured in the two systems, the *absolute* and *gravitational*, so the unit of work may also be measured in the above two systems:

(*a*) *The absolute unit of work* in the C.G.S. system is one **Erg**; *it is the work done when a force of one dyne moves its point of application through a distance of one centimetre in its own direction.*

The absolute unit of work in the F.P.S. system is one **Foot-Poundal**: *it is the work done when a force of one poundal moves its point of application in its own direction through a distance of one foot.*

(*b*) *The gravitational unit of work* in the C.G.S. system is the **Gram-Centimetre**; *it is the work done in lifting a mass of one gram through a vertical distance of one centimetre.*

[For practical purposes the unit chosen by the engineers is the *Kilogram-metre.*]

The gravitational unit of work in the F.P.S. system is the **Foot-Pound** (ft.-lb.); *it is the work done in raising a mass of one pound through a vertical distance of one foot.*

Since the weight of a gram is nearly 981 dynes, 1 gram-centimetre = 981 ergs.

1 erg = 1 dyne-cm.; 1 foot-poundal = 421,390 ergs.

Note. The erg being very small, **three additional units** of work (or energy) are used by electrical engineers for practical purposes, *viz.*—

(*i*) **The Joule** = 10^7 ergs.

(*ii*) The Watt-hour = 3,600 Joules = $(3{,}600 \times 10^7)$ ergs, *i.e.* one Joule per second for one hour.

(*iii*) The Kilowatt-hour (kWh) = $3{,}600 \times 1000$ Joules = $(1000 \times 3600 \times 10^7)$ ergs, *i.e.* 1000 Joules per second for one hour = 36×10^{12} ergs.

The Kilowatt-hour (kWh) is the legal supply unit fixed by the Board of Trade and is called the **Board of Trade Unit.**

135. Conversion of Foot-Poundals into Ergs :—

$$1 \text{ poundal} = \frac{1}{32 \cdot 2} \text{ of the wt. of 1 lb.} = \left(\frac{1}{32 \cdot 2} \times 453 \cdot 6\right) \text{grams-weight}$$

$$= \left(\frac{1}{32 \cdot 2} \times 453 \cdot 6 \times 981\right) \text{dynes; and 1 foot} = 30 \cdot 48 \text{ cms.}$$

Hence, 1 foot-poundal $= \frac{30.48 \times 453.6 \times 981}{32.2}$ ergs $=(4.2139 \times 10^5)$ erg approximately.

136. Relation between the two Units of Work:—Since the gravitational unit of force is g times the absolute unit of force,

gravitational unit of work = $g \times$ absolute unit of work.

Since the weight of a pound is 32.2 poundals,

1 foot-pound $= 32.2$ foot-poundals $= 32.2 \times 421,390$ ergs.

$= 1.356 \times 10^7$ ergs $=$ **1.356 Joules**

(since, 1 foot-poundal $= 421,390$ ergs).

137. Power:—The power or activity of an agent, say a dynamo or an engine, is the rate at which it does work, *i.e.* the work done by it in unit time, when the work is done continuously.

When we consider the time taken by an agent to perform any work, we consider what is called the **power** of the agent. The average power used in any operation is the ratio of $\frac{\text{total work done}}{\text{time taken}}$.

c **138. The Units of Power:**—(*a*) The **C.G.S. absolute unit of**
cal. **power is one erg per second.**
the m This being too small for practical purposes, two additional units
Sup employed in electrical engineering, *viz.*—
and it move The watt=1 Joule per sec $=10^7$ ergs per sec.

The Kilowatt = 1000 watts

° **F.P.S. absolute unit of power** is one foot-poundal per

'tional unit of power is one foot-pound per

(*a*)

the British practical unit of power and is 'eering very largely.

(*i*) If the displacement AB ˋ)=33,000 ft-lbs. per min.=550 ft-lbs the work done by P, is $W=P \times AB$,

(*ii*) If the displacement AB is iing capacity of a horse, James Watt, direction of P [Fig. 87(*b*)], the displaceirried out an experiment in which tion of $P=-AB$, and the work done by thecoal pit by a horse through a and is called negative work. This work is don. Thus, the work done was

(*iii*) If the displacement AB is in a directi550 ft-lbs. in one second line of action of P, say, making an angle θ witpower, which he termed then the displacement measured in the direction $AB \cos\theta$, where BN is the normal from B on AX. T. done by P is $W=P \times AN=P \times AB \cos\theta=AB \times P \cos\theta$ T. = force × component of the displacement of the point olbs $=(746 \times 10^7)$

Hence **1 H.P.** $= 550$ **ft.-lbs.** per sec. $= 746 \times 10^7$ ergs per sec.
$=$ **746 watts**; ($\because$ 1 watt$=10^7$ ergs per sec.),

and 1 Kilowatt $= \frac{1000}{746} =$ **1·34 H.P.**

140. Conversion of Kilowatt-hour into Foot-pounds :—

Since 1 Kilowatt$=$1·34 H.P.$=(1·34 \times 500)$ ft.-lbs. per sec.,

and Work$=$Power$\times$Time in seconds,

we have, 1 **Kilowatt-hour** $=(1·34 \times 550)\times(60 \times 60)$ ft.-lbs.
$=$ **2,653,200 foot-pounds.**

[Remember.—The amount of work done by an average horse is only $\frac{3}{4}$ H.P. The average amount of work done by an active man is $\frac{1}{7}$ H.P. The power of motor car engines varies from 6 to 30 H.P.; that of a jeep from 20 to 80 H.P.; those of gas engines from $\frac{1}{2}$ to 270, while the power of a large battle cruiser may reach up to 120,000 H.P.].

141. Distinction between Work and Power :—As power is the rate of doing work, it involves a time-unit and its average value is measured by the ratio of the work done to the time taken in doing the work, if the work is done continuously.

That is, power$=\frac{\text{work}}{\text{time}}$. Some examples of power are, 1 H.P. $=550$ ft.-lbs. per sec.; 1 watt$=10^7$ ergs per sec., etc. Thus from the above, Work$=$Power$\times$Time.

So 'watt-hour' or 'kilowatt-hour', which are products of 'power' and 'time intervals', are units of work.

Examples. *(1) A man whose weight is 10 stones runs up a flight of stairs carrying a load of 10 lbs. to a height of 20 ft. in 10 seconds. Find the mean power during this interval.*

10 stones$=14\times10=140$ lbs.

Total work done in 10 secs.$=(140+10)\times20=3000$ ft. lbs.

$\therefore$ The work done per sec.$=\frac{3000}{10}=300$ ft.-lbs. So, power$=\frac{300}{550}=0·545$ H.P.

(2) A man weighing 140 lbs. takes his seat in a lift which weighs 2 tons. He is taken to the 3rd floor, which is at a height of 75 ft. from the ground floor in 2 minutes. Calculate the work done and the power required in this process. [1 ton$=$2240 lbs.] *(Pat. 1929)*

The total weight of the man and the lift$=140+2240\times2=4620$ lbs.

The work done in raising 4620 lbs. through 75 ft.

$=$force$\times$distance$=4620\times75=346,500$ ft.-lbs.

The unit of power in the F.P.S. system is one hors[illegible] is 550 ft.-lbs. per second. $\therefore$ Power$=$rate of doing work.

$=\frac{346,500}{2\times60}$ ft.-lbs. per second $=\frac{346,500}{2\times60\times550}$ H.P. $=5·25$ H[illegible]

142. Mechanical Energy :—The capacity of [illegible] mechanical work is known as its mechanical ener[illegible]

by the total work the body can do under the circumstances (*position, configuration,* or *motion*) in which it is placed

Obviously, the unit of energy should be identical with that of work. Therefore *erg, foot-pound, Joule,* etc. which are units of work are also **units of energy.**

The falling water at Naigra does work in driving the dynamos which generate electricity. Hence the elevated water of the falls has got energy The wound spring moves the hands of a watch, and so it has energy. Wind has energy, for work is done by it when it drives a boat

143. Distinction between Energy and Power:—The *energy* of a body indicates the total amount of work the body, under the circumstances in which it is placed, can do and has no reference to the time in which that work is to be done, while *power* denotes the rate at which work is done and is irrespective of the total work done.

144. The two Forms of Mechanical Energy:—Mechanical energy may have either of the two forms, *potential* and *kinetic.*

(a) Potential Energy.—A body may possess energy by virtue of its position or configuration; such energy is called potential energy and is measured by the amount of work the body can do in passing from its present position or configuration to some standard position or configuration, usually called the *zero position* or *configuration*

(*i*) *Potential energy due to position*—

A lifted weight, like a **Pile-Driver,** can do work in falling down under the force of gravity, to the original position So it has potential energy Water stored up in elevated reservoirs in municipal water supply, formations of ice on a mountain top, are also similar instances of potential energy For bodies raised above the surface of the earth, the earth's surface is usually taken as the zero-position.

(*ii*) *Potential energy due to configuration*—

A coiled spring as in the case of a watch or a gramophone, a bent or compressed spring, compressed air, etc have potential energy for, in recovering the normal configuration (condition), each one of them can do work.

Potential Energy of a Raised Body.—Consider a body raised above the earth's surface In this raised *position* the body has potential energy

Let m=mass of the body, g=acceleration due to gravity; h= vertical height through which the body is raised from the ground level.

The potential energy= work done in raising the body$=mg\times h=mgh$. If m be taken in pounds and h in feet, then the potential energy,

P.E. $= mgh$ **ft.-poundals (where** $g=32.2$)$=mh$ **ft.-pounds.**

If m be taken in grams and h in centimetres,

P.E. $= mgh$ ergs. (where $g=981$)$=mh$ **gm.-cms.**

(b) Kinetic Energy.—A body in motion has energy due to its motion; such energy is known as *kinetic energy* and is measured by the amount of work the body can perform against external impressed forces before its motion is stopped. The bullet fired from a rifle, the rotating fly-wheel of an engine, a falling body, a swinging pendulum, a cannon ball in motion have all got kinetic energy.

Kinetic Energy of a Body moving with Velocity v.—Consider a body in *motion*. At the instant of consideration, let the velocity of the body be v.

Let the mass of the body be m and suppose it is brought to rest by a constant force P resisting its motion, which produces in the body an acceleration $(-f)$, given by $P=mf$.

Let s be the distance traversed by the body before it comes to rest.

We have, $0=v^2+2(-f)s$ [*vide* Art. 89, eq. (*iii*)];

$\therefore f.s=\frac{1}{2}v^2$. Therefore, the *K.E.* of the body=work done before coming to rest $=P\times s=mf\times s=m\times f.\ s=m.\ \frac{v^2}{2}=\frac{1}{2}\ mv^2$.

Hence, the kinetic energy of a body moving with a velocity v is equal to half the product of the mass and the square of the velocity.

Note.—If m be taken in pounds and v in feet per second, the kinetic energy, **K.E.** $=\frac{1}{2}mv^2$ *ft.*-poundals (lb. × ft.²/sec.² = ft. × lb. × ft./sec.² = **ft.-poundals**)

$=\frac{1}{2}mv^2/g$ ft.-pounds, (where $g=32{\cdot}2$).

If m be taken in grams and v in cms. per second,

K.E. $=\frac{1}{2}mv^2$ ergs (gm. × cms.²/sec.² = cms. × gm. × cms./sec².

= cms. × dynes = ergs) $=\frac{1}{2}mv^2/g$ gm.-cms. (where $g=981$).

145. Potential Energy and the State of Equilibrium:—*The state of stable equilibrium* of a body corresponds to **minimum** *of potential energy*, because the centre of gravity of a body, when in stable equilibrium, occupies the lowest possible position and any displacement tends to raise the position of the centre of gravity and thus increases the potential energy of the body. When *the potential energy of the body is* **maximum**, any displacement will give rise to a couple tending to move the body further, and thus, in this position *the equilibrium of the body is* **unstable.** Again, when the body is *in the state of* **neutral** *equilibrium its potential energy will remain* **constant** for any small displacement.

146. Transformation of Energy and the Principl[e] [of Conser]vation of Energy:—If a body is at some height abov[e] [the ground] it has got some gravitational potential energy. If it [is allowed] to fall freely through a distance, it loses an amount of [potential energy] equivalent to the work done by the weight of the bo[dy and gains an] equal amount of kinetic energy. Just before the

referred to, is really not a loss, for an equivalent energy reappears in each case mostly as heat and partly as sound. When a falling body touches upon the ground, the mechanical energy is reduced to zero but is transformed in equivalent quantity mostly into heat and partly into sound. Thus, we find that whatever be the system of forces acting on a body, conservative or non-conservative, the total energy of the system will be found to remain constant, if we take into account all the different forms of energy to which energy is admissible namely, mechanical, thermal, magnetic, electrical, acoustical and light energies. Sometimes it becomes really difficult to trace out the different forms into which energy transforms itself and makes us doubt the validity of the principle but when closely examined, it will be found that the situation arises not due to any defect in the universal character of the principle but due to our inadequate knowledge of the transformations. Consider the various transformations of energy in the case of an ordinary steam engine connected to a dynamo for the generation of electricity. When the coal burns, we get *heat energy*. The heat does work in changing water to steam, which then expands. The expanding steam exerts force and causes the piston to move, and thus runs the engine. Thus, the heat energy is transformed into *mechanical energy*, and when the engine drives a dynamo, which generates electricity, the mechanical energy is converted into *electrical energy*. This energy can be transmitted by wires and made to do useful work such as driving tram cars where electrical energy is reconverted into *mechanical energy*; lighting lamps in houses, where electrical energy is reconverted into *light energy*; and in this way various other transformations may also take place but whatever are the transformations, the guiding principle remains that the total energy of the whole system will be constant.

147. The Principle of Conservation of Energy is obeyed by a Swinging Pendulum:— In the undisturbed position the pendulum acts like a plumb line and hangs vertically. At this position, the centre of gravity of the pendulum, which is practically the same as the centre of the spherical bob, lies at the lowest level which may be called its zero or ground level as shown by the point *B* in Fig. 88. The vertical position of the pendulum is its mean position: for, when the pendulum is made to oscillate by drawing it aside and then let go, it swings about this position with almost equal amplitude on either side of it in each oscillation. When it moves to one side of the mean position, the centre of the bob rises and the bob

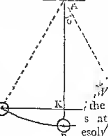


Fig. 88—A Pendul

gains potential energy. When it is at the extreme end position, as shown by C or D, its whole energy is potential, there being no kinetic energy, for the bob is at rest momentarily there. The vertical height BK, through which the bob rises when at the extreme position C or D, multiplied by the weight of the bob, gives the potential energy gained

From the extreme position C or D, when the bob moves towards the mean position B, the potential energy is gradually transformed into kinetic energy till finally the whole of the potential energy is transformed into kinetic energy when the bob reaches the mean position B, its ground or zero level. At this position the whole energy being kinetic, it attains its maximum velocity. At positions intermediate between B and C or D, the energy of the bob is partly potential and partly kinetic One crossing the mean position by virtue of inertia and acquired velocity, when the bob begins to move to the other side, the kinetic energy of the bob gradually reduces at a rate in which its potential energy increases till finally the whole of the kinetic energy is again transformed into potential energy at C or D If there had been no friction of the air or at the point of support no energy would have been lost by the pendulum and it would oscillate with the same amplitude for ever, once being set into motion Thus for an ideal pendulum *oscillating in vacuum* the sum of the potential and kinetic energies at any instant should be constant.

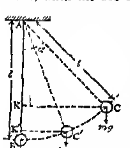


Fig 89—A Swinging Pendulum

Mathematical Proof.—Let the position C denote the extreme end position for a pendulum and C' any subsequent position while moving towards the mean position B (Fig 89) Draw CK and $C'K'$ perpendiculars on AB

At C, the total energy (which is wholly potential) $= mg \times BK$. At $P.E. = mg \times BK'$, and $K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m \times (2g \times KK') = mg \times KK'$
abo $mg \times (BK - BK')$
poten

Let
vertical he Total energy at $C' = P.E. + K.E.$

The po $= (mg \times BK') + mg(BK - BK')$
mgh If m b
$= mg \times BK =$ energy at start
P.E. $= m$
If m be ta $= mg \times l(1 - \cos \alpha)$

P.E. $= mg$th of the pendulum, and $\alpha =$ amplitude.

148. Total Energy of a Falling Body is Constant:—The potential energy of a body of mass m at a height h (Fig. 90) above the ground$=mgh$.

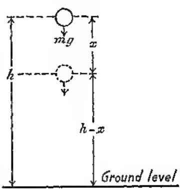


Fig. 90

When it falls through a distance x, its potential energy at the time$=mg(h-x)$.

Its kinetic energy at that instant$=\frac{1}{2}mv^2$ (where v is the velocity acquired during this interval$=\frac{1}{2}m\times 2gx$ ($\because$ $v^2=2gx$) $=mgx$.

$\therefore$ At the instant, potential energy+kinetic energy

$=mg(h-x)+mgx=mgh=$potential energy in the beginning.

Hence, neglecting the effects of air resistance, it is seen that the total amount of energy (kinetic+potential) of the body remains constant as it falls. When the body strikes the ground, it is brought to rest and loses its kinetic energy. Then the potential energy is also reduced to zero. The energy, however, is not destroyed. It is converted mainly into heat, the body and the ground being warmer as the result of the impact; a small part of the energy is also converted into sound energy.

149. A Particle sliding down a Smooth Inclined Plane obeys the Principle of Conservation of Energy throughout its Motion:—

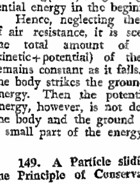


Fig. 91—A Body Sliding down an Incline.

Consider a particle of mass m, say, which is allowed to slide down a *smooth* inclined plane AB having an inclination α to the ground level BC (Fig. 91). Suppose the particle starts from rest at a point whose height from the ground level is h. The *P.E.* at this point is mgh and the *K.E.* is zero, so that th total energy at start$=(mgh+0)=m$

Let v be the velocity acquir in the particle at any instant w case, particle has slided down throu plane tance x along the inclined (W sin θ) acceleration down the plane the plane. $g\cos(90-\alpha)=g\sin\alpha$. s at right $v^2=2g\sin\alpha\times x$. esoly

$\therefore$ $K.E.=\frac{1}{2}mv^2=mgx\sin\alpha$ But $x\sin\alpha$ is ev and tical height through which the particle has descen

height above the ground at this position is $(h-x \sin \alpha)$ and therefore, its $P.E.=mg(h-x \sin \alpha)$.

$$\therefore \quad K.E.+P.E.=mgx \sin \alpha+mg(h-x \sin \alpha)$$
$$=mgh, \text{ which is}$$

independent of x and is equal to the initial total energy. So the total energy remains constant as the particle slides down the inclined plane and thus the principle of conservation of energy is obeyed by the sliding particle

150. A Projectile obeys the Principle of Conservation of Energy throughout its Motion :— Let a particle of mass m be thrown from the ground (Fig 92) with a velocity u at an angle α with the horizontal. At start its total energy $=K.E.+P.E.$

$$=\tfrac{1}{2}mu^2+0=\tfrac{1}{2}mu^2$$

Fig 92—A Projectile

Suppose, v is the velocity of the particle at an angle θ with the horizontal at the instant when it is at any vertical height h above the ground

Since the only acceleration acting on the particle is due to gravity, i e g vertically downwards, its horizontal velocity all along remains unchanged, and so,

$$v \cos \theta=u \cos \alpha \qquad (a)$$

and considering the motion of the body vertically upwards, we have

$$(v \sin \theta)^2=(u \sin \alpha)^2-2gh \qquad (b)$$

Squaring equation (a) and adding it to equation (b), we have,

$$v^2=u^2-2gh$$

$$K.E.=\tfrac{1}{2}mv^2=\tfrac{1}{2}m(u^2-2gh)=\tfrac{1}{2}mu^2-mgh.$$

At this position, the vertical height of the particle above the ground being h, its

$$P.E.=mgh$$

$\therefore$ Total energy $=P.E.+K.E.$ (neglecting air resistance to motion, etc)

$$=mgh+(\tfrac{1}{2}mu^2-mgh)$$

abo $\quad =\tfrac{1}{2}mu^2=$ initial energy.

poten, the same at all heights.

Let **Perpetual Motion :—** The principle of conservation of vertical h... icates the impossibility of the existence of a "perpetual

The p... hine, i.e a machine which, when once set in motion, will mgh. If m l... notion perpetually without the supply of an equivalent

$P.E.=$ ... ergy from outside. Even when no useful work is done

$=$. If m be ... ne, the energy, supplied in the beginning, will be ... up in overcoming frictional and other resistances and

$P.E.=mg$ ... ultimately come to a stop.

152. The Velocity of the Bob of a Pendulum at its Lowest Point:—When the bob of the pendulum, of length l cm., is set free from its extreme position C, it moves in an arc of a circle CBD, B being the lowest position (Fig. 93). From C draw a perpendicular CK on AB. At C the bob of the pendulum has potential energy $mg \times BK$, which represents the work done in raising the bob from B to C, *i.e.* vertically through BK. When the bob is released from the position C, it gradually loses its potential energy and gains kinetic energy. At the lowest point B, it loses all its potential energy $mg \times BK$, and the kinetic energy $\frac{1}{2}mv^2$, which it gains, is equal to this.

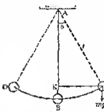


Fig. 93

$\therefore$ $m.g.\ BK = \frac{1}{2}mv^2$, where v is the velocity of the bob at B

or, $g(AB - AK) = \frac{1}{2}v^2$; or, $g(l - l\cos\theta) = \frac{1}{2}v^2$;

or, $v^2 = 2gl(1-\cos\theta)$; $\therefore$ $v = \sqrt{2gl(1-\cos\theta)}$

or, $v = \sqrt{2gl \times 2\sin^2\dfrac{\theta}{2}} = 2\sqrt{gl} \times \sin\dfrac{\theta}{2}$.

Example. *The heavy bob of a simple pendulum is drawn aside so that the string makes an angle of 60° with the horizontal and then let go. Find the velocity with which the bob passes through its position of rest.* (Pat. 1940)

(Draw the diagram and proceed as explained in the preceding article.)

$\theta = (90° - 60°) = 30°$. $\therefore$ $v = \sqrt{2gl\left(1 - \dfrac{\sqrt{3}}{2}\right)}$; or, $v = \sqrt{0{\cdot}268 \times gl}$.

153. Other Forms of Energy:—As already stated, the mechanical energy, which a body possesses may be due to either or both of the two forms, kinetic and potential. Besides mechanical energy there are also other forms of energy, *e.g.* heat, light, sound, electrical, magnetic, and chemical energy.

154. The Sun is the ultimate Source of all Energy:—The sun is generally considered to be the ultimate source of all forms of energy. We get considerable amount of energy from solar radiation in the form of heat, light, etc. For example, the energy of the steam engine is derived from coal. Coal again is nothing but wood decomposed and subjected to great pressure of the earth for thousands of thousands of years. The energy in the wood is due to the sun's [illegible] on trees and plants. When the coal burns, the stored-up chemical energy derived from solar radiation is given back [illegible] and light.

155. Further Examples of Transformation of Energy

(1) Mechanical.—

(*a*) Kinetic to potential.—The bob of a pendulum

the normal position (maximum kinetic energy position) to the extreme position of swing (*b*) **Potential to kinetic.**—A body falling from a raised position to the earth; a pendulum returning from an extreme position of swing towards the normal position (*c*) Kinetic to heat—Heat produced by rubbing two stones; a moving wheel stopped by applying brakes. (*d*) Kinetic to sound.—Sound produced when a reed vibrates. (*e*) Kinetic to electrical—A dynamo.

(2) Heat.—(*a*) Heat to mechanical—Heat engines. (*b*) Heat to light—White hot ball; filament in a bulb (*c*) Heat to sound.—Singing flame. (*d*) Heat to electrical—Thermo-electric phenomena. (*e*) Heat to chemical—Water formed by igniting a mixture of hydrogen and oxygen (*f*) Heat to mechanical.—Molecules in a gas produced by heating a liquid.

(3) Light.—(*a*) Light to electrical—Photo-electric cell (*b*) Light to chemical—Photography

(4) Sound.—(*a*) Sound to mechanical—Forced vibration and resonance (*b*) Sound to electrical—Telephone transmitter.

(5) Magnetic.—(*a*) Magnetic to heat—Rapid magnetisation and demagnetisation repeated in a specimen of iron (*b*) Magnetic to mechanical—Electromagnet.

(6) Electrical.—(*a*) Electrical to mechanical—Electric motors; tram cars (*b*) Electric to heat—Electric iron; electric furnace. (*c*) Electrical to light—Electric lamps (*d*) Electrical to sound—Calling bell; Telephone (*e*) Electrical to chemical—Charging of batteries, electro-plating (*f*) Electric to magnetic—Electromagnet.

(7) Chemical Energy.—(*a*) Chemical to heat—Burning of a fuel—petrol, kerosine, coal, etc (*b*) Chemical to light.—Burning magnesium wire, gas lighting (*c*) Chemical to electrical—Voltaic cells (*d*) Chemical to mechanical—Explosives

156. Different Examples of Work done:—Work is measured by the product of the force and the distance through which the point alt application of the force moves in the direction of the force

potei (*i*) *Work done in raising a load vertically upwards.*

Le
vertical fi *w* represents the work done, $w = mgh$, where m is the mass of and h the vertical height through which the load is raised.

The p
mgh. If m *Work done in taking a load up along an inclined plane.*

P.E. = ncase, $w = mg \sin \alpha \times l$, where l is the length of the inclined
=. If m be tan. 149] and α the inclination of the plane to the horizon,
P.E. = mg $= mgh$, where h is the height of the inclined plane.
k done in taking the load up the inclined plane is the

same as that required to raise the load m vertically through a height h. Hence, the work done in raising a body to a height h against gravity is independent of the path along which the body is taken and depends only on the vertical height.

(iii) *Work required to generate a velocity v in a body originally at rest.*

$W = P \times S$, where P is a force which generates an acceleration f in a body of mass m; and S is the distance traversed by the body in time t.

Here $P = mf$; and $S = \frac{1}{2}ft^2$.

$\therefore \quad W = P \times S = mf \times \frac{1}{2}ft^2 = \frac{1}{2}m(f^2t^2) = \frac{1}{2}mv^2$,

where v is the velocity acquired by the body after time t starting from rest.

157. Summary of Results:— ...

Quantity	Symbol	Quantity	Symbol
Displacement or distance	s	Relation between distance & speed	$v^2 = u^2 + 2fs$
Time	t	Mass	m
Velocity	$v = \frac{s}{t}$	Force	$P = mf$
Acceleration	$f = \frac{v_2 - v_1}{t}$	Momentum	$M = $ [illegible] in
Distance (uniform motion)	$s = vt$	Kinetic energy	case, K plane ($W \sin \theta$)
Relation between speed and time	$v = u + ft$	Potential energy	the plane. s at right
Relation between distance & time	$s = ut + \frac{1}{2}ft^2$	Work	esol and

Here $m=100\times 2240=224{,}000$ lbs.; $u=30$ miles per hour $=44$ ft. per sec.

$$\therefore \quad F=\frac{mu^2}{2s}=\frac{224{,}000\times 44^2}{2\times 120}=1{,}806{,}933\cdot 3 \text{ poundals}$$

$$=56{,}466\cdot 6 \text{ lbs.-wt. (taking } g=32).$$

(b) We have, $v=u+ft$; or, $0=u-\dfrac{Ft}{m}$; or, $Ft=mu$;

$$\therefore \quad F=\frac{224{,}000\times 44}{10}=985{,}600 \text{ poundals}=\frac{985{,}600}{32}=30{,}800 \text{ lbs.-wt. } (\because g=32).$$

(3) *Find the energy stored in a train weighing, 250 tons and travelling at the rate of 60 miles per hour. How much energy must be added to the train to increase its speed to 65 miles per hour.* (C. U. 1925)

Mass $=250$ tons $=250\times 2240$ lbs.

Velocity $=60$ miles per hour $=88$ ft. per sec.

$\therefore$ The kinetic energy of the train $=\frac{1}{2}\times(250\times 2240)\times 88^2$ foot-poundals

$=2{,}168{,}320{,}000$ ft.-poundals.

Again, 65 miles per hour $=\dfrac{65\times 1760\times 3}{60\times 60}=\dfrac{286}{3}$ ft. per sec.

$\therefore$ The $K.E.$ of the train, when the speed is 65 miles per hour

$=\frac{1}{2}\times(250\times 2{,}240)\times\left(\dfrac{286}{3}\right)^2=2{,}544{,}764{,}444\cdot 4$ ft.-poundals.

$\therefore$ The energy to be added $=2{,}544{,}764{,}444\cdot 4-2{,}168{,}320{,}000$

$=376{,}444{,}444\cdot 4$ ft.-poundals.

(4) *If clouds are 1 mile above the earth and rainfall is sufficient to cover 1 square mile at sea-level, $\frac{1}{2}$ inch deep, how much work was done in raising the water to the clouds.* (C. U. 1920; G. U. 1950)

If w lbs. be the mass of rain water, and h ft. the height of the clouds above the surface of the earth, the work done in raising w lbs. of water through h ft.

$=w\times h$ *foot-pounds*. Here $h=1760\times 3=5280$ ft.

The volume of rain water $=1$ square mile $\times\frac{1}{2}$ in. $=(5280)^2\times\dfrac{1}{2\times 12}$ cu. ft.

The mass of 1 cubic foot of water $=62\cdot 5$ lbs

$\therefore$ Mass of rain water $=(5280)^2\times 1/24\times 62\cdot 5$ lbs.

$\therefore$ The work done $=(5280)^2\times\dfrac{1}{24}\times\dfrac{125}{2}\times(5280)=(5280)^3\times\dfrac{1}{24}\times\dfrac{125}{3}$

$=383\cdot 328\times 10^8$ *foot-pounds*.

Questions

1. *Show that if a piston is moved along a cylinder against pressure, the work done in a stroke is equal to the product of into the volume swept out by the piston. Explain clearly the the work will be given by this calculation.* (Pat. 19[illegible])

[Pressure = force on unit area.

Work done = force $\times$ distance = (pressure $\times$ area) $\times$ distance th[illegible] piston moves = pressure $\times$ volume swept out. The work is e[illegible] pressure is measured in dynes per sq. cm. and volume in c.c.]

2. What is the work done when a weight of 500 kilograms falls through a height of 50 metres and is then stopped? Assume the normal value of gravity.

[*Ans.* $24{,}525 \times 10^{4}$ ergs] (Dec. 1933)

3. How much power is required to pump water at the rate of 90 litres per minute to a height of 20 metres?

[*Ans.* 294 watts]

4. Water is pumped up from a well through a height of 30 feet by means of a 5 horse-power motor. If the efficiency of the pump is 85%, find in gallons the quantity of water pumped up per minute (1 gallon of water weighs 10 lbs) (C. U 1952; C. U. 1954)

[*Ans.* 467.5 gallons approx.]

5. An engine is employed to pump 6,000 gallons of water per minute from a well through an average height of 21 feet. Find the horse-power of the engine, if 45% of the power is wasted

[*Ans.* 69.42]

6. What is the potential energy of the water which fills a cubical tank of each side 10 ft. and whose base is 20 ft above the ground?

[*Ans* 1.55×10^{6} ft.-lb]

7. A railway train is going up hill with a constant velocity. What is the source from which the energy of the train is supplied?

Describe the various transformations of energy that go in this case

(C. U. 1918)

[**Hints**—The energy of the train is derived primarily from the burning coal This is utilised in running the train against friction and air resistance, and also in raising the train up hill against the force of gravity and thus work is done The energy of the coal is derived from the sun So the sun is the ultimate source of supply of energy]

8. A solid mass of 100 gms is allowed to drop from a height of 10 metres Calculate the amount of kinetic energy gained by the body, g being 980 cms per sec^{2}. (Dec 1930)

[*Ans* 98×10^{6} ergs]

9. A shot travelling at the rate of 200 metres per second is just able to pierce a plank 2 inches thick What velocity is required to pierce a plank 6 inches thick? (Pat 1941)

[*Ans* $200\sqrt{3}$ metres per sec]

10. A mass of 10 lbs falls 10 ft from rest and is then brought to rest by penetrating 1 ft into sand, find the average thrust of the sand on it

[Ans 110 lbs -wt] (Utkal, 1950)

11. Distinguish between pound, poundal, and pound weight

...rove that in the case of a body falling freely under gravity the sum of ...tial and kinetic energies is constant.

(Pat. 1925, '36, '49; C U 1932, '41)

...lain the meaning of the 'Principle of Conservation of Energy'. Show ...ciple is applicable at every stage of the journey of a particle falling ...gravity from a height till it reaches the ground (C. U. 1933)

...weighing 1 oz is dropped from the top of a tower 60 ft high ...rest by penetrating 5 ft into mud Find the average thrust (Pat. 1919)

...ndals]

14. *A pendulum consisting of a ten-gram bob at the end of a string thirty centimetres long oscillates through a semi-circle; find its velocity and kinetic energy when it passes its lowest point. Specify the units in which your answer is given.* (Pat. 1935)

[**Hints.**— *At the starting point the bob has got only potential energy* $=mgh$.

At the lowest point the energy is all kinetic$=\left(\frac{1}{2}mv^2\right)$ *which is equal to* $mgh=(10\times981\times30)$ *ergs. Hence find* v.]

[*Ans.* $v=242.61$ cms. per sec.; $K.E.=294{,}300$ ergs.]

15. A body falls under gravity and strikes the ground. Explain how the phenomenon supplies an illustration of the transformation of energy. Does it also illustrate the principle of consevation of energy?

(C. U. 1917, '36, '54; Pat. 1931)

16. What are the practical units of power in the F.P.S. and C.G.S. systems? Write out the relations between these units. (C. U. 1956)

17. A steel ball of 100 gms. drops through a height of 10 metres. What is its velocity when it reaches the ground? ($g=980$ cms. per sec.^{2}). (C. U. 1950)

[*Ans.* 1,400 *cms. per sec.*]

CHAPTER VII

FRICTION

158. Friction :—No solid surface is perfectly smooth. In other words, all solid surfaces are rough more or less. So when two continuous solid surfaces (which are dry) are in contact and any attempt is made to move either over the surface of the other, it is always attended with a resistance which tends to oppose the motion. Such resistance to motion is called *friction.*

Friction arises on account of the adhesion, *i.e.* the mutual forces of attraction between the molecules of the two contacting surfaces, and the interlocking of the irregularities present on the contacting surfaces.

Friction can be thought of as equivalent to a *force acting along the plane of contact between two surfaces* opposite in direction to

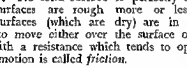


Fig. 94

in
case,
plane
$W \sin \theta$)
the plane.
s at right
esolv and

any force attempting to produce a relative motion bet
surfaces. This will explain why a force is necessa

book along a table, a rectangular box along the ground, and so on. Consider next a more general case when two plates are pressed together by normal forces N (Fig. 94) To overcome friction and to cause sliding between the two surfaces, a certain force P (its value depending on the value of N and the nature and condition of the two surfaces), acting along the common plane of contact, will be required.

Friction is perverse.—That is, it always opposes motion irrespective of the direction in which the motion may take place. In Fig 95(a) a very closely fitting piston working in a cylinder of an engine is shown moving outwards under a force P, while in (b) it is moving inwards at the return stroke In either case the motion of the piston will be opposed by frictional forces f operating along its surfaces of contact

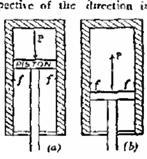


Fig 95

Friction may be divided into the following categories —

(a) Static and kinetic friction,
(b) Rolling friction,
(c) Fluid friction

158.(a). (i) Static friction and its limiting value:— Frictional force is self adjusting but it can exert itself only up to a limited maximum value As the force attempting to drag a surface over another is gradually increased from zero, the frictional force opposing it also increases equivalently The two contacting surfaces remain in static equilibrium up to a maximum value of the applied force. That is, up to this stage the frictional force which acts in opposition is equal to the applied force When the applied force just exceeds this maximum value, the body on which the force is applied begins to slide This maximum value of the applied force is a measure of the limiting value of static friction between the two surfaces, and is called the force of limiting friction—often also called the force of friction F

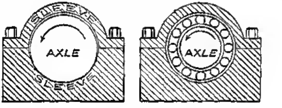
(ii) Kinetic or sliding friction.—It is found that the force, F, necessary to start sliding of one surface on another is greater than necessary to maintain sliding That is, the force of sliding or friction is less than that of limiting friction

It must be remembered that if two surfaces are separated of liquid, such as a lubricant, the nature of the friction changed.

...ling Friction.—This is also a kind of kinetic friction between two solid surfaces in contact but one of them

rolling or tending to roll on the other, as in the case of a marble rolling on the floor, a football rolling on the turf, a rope passing over a rolling pulley, etc. It is a common experience that the force required to drag a rectangular box along the ground is much greater than that required to move it on rollers. **This means that rolling friction is much smaller than sliding friction.** That is the reason why vehicles are mounted on wheels instead of on runners, and ball bearings are used instead of sleeve bearings. There are a number of different types of ball and roller bearings known collectively as **anti-friction bearings.** Basically, all these consist of the rolling elements (balls or rollers), the *race rings* on which are provided tracks for the rolling elements and in the majority of cases a separator for the rolling elements known as the cage.

Sleeve and Ball-bearings.—Fig. 96(*a*) illustrates a sleeve type of bearing where it will be seen that the rotating *axle slides* on the



(*a*) Sleeve-bearing. (*b*) Ball-bearing.
Fig. 96

bottom of the sleeve at low speeds. It, however, tries to climb up the side of the sleeve at increased speeds.

Fig. 96(*b*) illustrates a ball type of bearing where it will be seen that the axle rotates on the balls *without sliding*. The groove, in which the balls themselves roll on account of reaction, is called the '*race*'.

(c) Fluid Friction.— Friction occurs when a liquid or gas is made to pass around a stationary body or the body made to move in a liquid or gas, *i.e.* it manifests itself when there is relative motion between the two. It arises in the propulsion of a ship through water or automobiles, trains and aeroplanes through the air and so o
For more elaborate considerations of air-friction, read Chapter · in
Aeronautics (Appendix A). Take the case of a rain-drop f case,
through air. Its speed depends upon its size and not upon its plane
above the ground (*vide* Art. 112). Starting with zero velo $W \sin \theta)$
velocity increases as the drop descends until the retarding the plane.
friction equals the downward pull of gravity. When thi s at ight
of equilibrium is reached, the body falls with a steady vel esol
its **terminal velocity.** For small particles like *fog* , and
terminal velocity is low and the air-flow around them

when, however, the particles are large, the terminal velocity may exceed the *critical velocity* and *turbulent flow* sets in around a moving body, which then determines the frictional resistance mostly. The considerations are important for aeroplanes which move in the air and the ships in the water.

159. The Role of Friction :—Friction is useful in many ways, though it is also wasteful in other ways.

Usefulness of Friction.—Friction is important in our daily life. If there were no friction, walking would have been impossible, nails and screws would not remain in the wood, fibres of a rope would not hold together, a ladder would not rest on the ground, locomotive engines would not draw a train on the rails, and so on. In designing automobiles and their parts, steps are taken to increase friction where it is needed. Brake linings in automobiles require special materials and tires are given special thread designs for purposes of increasing the friction consistent with minimum wear and tear.

Wastefulness of Friction.—Friction is ordinarily looked upon as an evil. It is inevitably present, to an extent large or small whenever there is motion of one body relative to another in contact. The effect of friction is to reduce the relative motion to certain extents and, to that extent, there is loss of mechanical energy of the moving member. So, in designing engines and all other moving machineries, precautions are taken to reduce friction in the bearings to the minimum. Ball and roller-bearings entail much lesser friction than sleeve-bearings and that is why these bearings are rapidly replacing the latter type in modern machineries. Lubrication of the surfaces in contact further decreases the frictional wastage of energy, as also the wear and tear.

160. Limiting Friction :—Let a rectangular block of wood D rest on a horizontal table BC (Fig. 97). The forces acting on the block are its weight W, acting vertically downwards, and the reaction R of the table acting normally upwards at the surface of contact. In this case R is equal and opposite to W, there being no motion in the vertical direction. Now, suppose a small force P is applied to the block parallel to the surface BC. If the body is still at rest, *an equal force F opposite in direction to P*, must have been called into play to oppose motion on account of friction arising from contact between the two bodies. As the

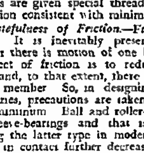


Fig. 97

P is gradually increased, the opposing force of friction

F which is a self-adjusting one, also increases at the same rate until a certain maximum value is reached. If the applied force be increased beyond this value, the block begins to move. The magnitude of this maximum force, when the block is just on the point of sliding is a measure of what is called **the force of limiting friction.**

When the block has once started to move, a *smaller* force would be sufficient to keep the block moving with a constant velocity; this smaller force is called **Kinetic friction or Dynamic friction.** The same considerations also apply to rolling friction. But it should be remembered that rolling friction is even less than kinetic friction.

161. The Laws of Limiting Friction :— The following generalisations, known as the laws of friction, are due to A. T. Morin, a Frenchman, though some of these facts were previously established by A. Coulomb, another Frenchman who published the results of a large number of experiments on the subject in 1781.

(*i*) Friction always opposes motion.

(*ii*) The force of friction is proportional to the normal reaction between the two surfaces in contact.

(*iii*) It is independent of the extent of the areas of the surfaces in contact, but depends on the material, nature and condition of the surfaces in contact.

162. The Co-efficient of Friction :— If the normal reaction acting across two solid surfaces in contact be equal to R, and F denotes the force of limiting friction, the ratio, F/R is found to be a constant and is called the co-efficient of *static friction* or *limiting friction* and more universally as **co-efficient of friction** and is generally denoted by μ, i.e. $\dfrac{F}{R}=\mu$. For any pair of surfaces in contact, the co-efficient μ is always less than unity.

163. The Angle of Friction :— In the case of limiting friction if the normal reaction and the frictional forces be compounded into a single force, which is sometimes referred to as *resultant* or *total reaction*, the angle, which this resultant makes with the normal reaction, is called the angle of friction.

Consider a small block D resting upon a horizontal plane and acted upon by a force P making an angle α with the (Fig. 98, *a*) [P may be considered as the resultant of som force and the force of gravity on the block D]. As long brium exists, the reaction of the supporting surface is ec a reactive force R which will be equal, opposite and collin force P. R may be replaced by its two components, F ar

tangentially and normally, respectively, to the surface of contact. The component F will represent the friction between the surfaces,

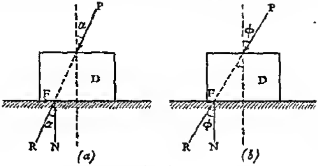


Fig 98

and the component N, the normal pressure so that $\frac{F}{N} = \tan \alpha$, for equilibrium. Suppose the sliding of the block impends when the force P makes an angle ϕ with the vertical (Fig. 98, b) then $\frac{F}{N} = \tan \phi$ ..(1).

Again, from the condition of sliding to begin, $\frac{F}{N} = \mu$ ... (2)

(where μ = co-eff. of friction, or limiting friction)

From (1) and (2), $\tan \phi = \mu$ .. ... (3)

The limiting angle ϕ whose tangent is equal to the co-eff. of friction is the angle of friction or angle of static friction.

[Note.—The above furnishes the idea of how friction affects the reaction exerted by a supporting surf ce acted on by a force When motion impends, the total reac n R exerted by the supporting surface is inclined to the normal by the angle of static friction ϕ and acts so as to oppose the motion.

When motion is not impending, the total reaction R inclines to the normal by whatever angle is necessary to maintain equilibrium. alk or an ideal surface ($\mu=0$), ϕ is also zero, *i.e.* the total reaction is poten mal to the supporting surface]

Le 64. **Cone of Static Friction**:—In the preceding considerations, vertical li P (Fig. 98) was supposed to be in the plane of the figure. We

The p ever, generalise it and say that if the force P remains con-

mgh If m n a cone generated by a line making the angle of static

P.E. = m with the normal to the supporting surface, the block D

If m be ta e to be in equilibrium whatever is the magnitude of the is cone is called the ***cone of static friction.***

P.E. = mg

165. Determination of Co-efficient of Friction :—

(i) Horizontal Plane Method.—Place on a horizontal wooden table a rectangular block of wood [Fig. 99] to act as a slider. The contacting surfaces of both these pieces of wood should be as smooth as possible. The slider is attached to a light string which is passed over a light pulley fixed at the end of the table. A scale pan is attached to the end of the string passing over the pulley. The pulley should be so fixed that the position of the string above the table should be horizontal.

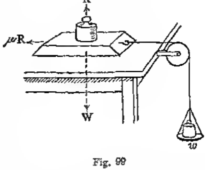


Fig. 99

Weigh the slider and put a known weight on it. Now put weights on the scale pan until the slider is just on the point of motion. Near about the slipping point, gently tap the table to ascertain the required weight to be placed on the scale pan. If W be the total weight of the slider and the weight placed on it, and W' the total weight of the scale pan including the weight w placed on it, the value of the limiting friction $= W'$, and that of the normal reaction $= W$. So we have, $\mu = W'/W$.

Repeat the experiment several times with different weights on the slider and again on reversing the block.

The ratio W'/W for each set of experiment will be approximately the same. The mean value of the ratio is the value of μ.

(ii) Inclined Plane Method.—Place a rectangular slab of wood D on an inclined plane AB [Fig. 99(a)] and gradually increase the inclination of the plane to θ, until D just begins to slide down the plane. Ascertain this by gentle tapping as in the last method. When this is the case, the friction $F(=\mu R)$ acts up the plane and balances the component $(=W \sin \theta)$ of the weight acting down the plane. The normal reaction R acts at right angles to the plane, AB. Resolving in directions perpendicular and parallel to the plane, we have, $W \cos \theta = R$, and $W \sin \theta = F$.

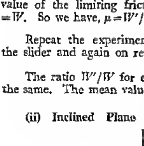


Fig. 99(a)

Hence $\frac{F}{R} + \frac{W \sin\theta}{W \cos\theta} = \tan\theta$. But $\frac{F}{R} = \mu$.

$\therefore$ $\mu = \tan\theta$; or, the co-efficient of friction is simply the tangent of the angle at which sliding begins. Again $\tan\theta = \frac{\text{height}}{\text{base}}$ of the plane $= \frac{BC}{AC}$. Hence, the co-efficient of friction is obtained by taking the height of the plane and dividing it by the base.

Repeat the experiment several times and calculate the mean value of μ.

166. The Angle of Repose:—In the case of an inclined plane the angle of incline θ, which the plane AB [Fig. 99(*a*)] makes with the horizontal AC when a body D on it just begins to slide down, is called *the angle of repose*. It is proved above that the tangent of this angle is equal to the co-efficient of limiting friction. It is also equal to *the angle of friction.*

If the inclination of the inclined plane AB is greater than the angle of repose, the force component down the inclined plane is greater than that required to overcome the friction F and the difference between them produces an acceleration.

167. Co-efficients of Friction (μ):—

Static Friction		Rolling Friction	
Wood on wood	0·3 to 0·5	Rubber tires on Concrete	0·03
Metal on metal	0·3 (average)	Ball-bearing on Steel	0·002
Metal on wood	0·2 to 0·6	Cast Iron on Rails	0·004
Leather on wood	0·3 to 0·5	Roller bearings	0·002 to 0·007
Leather on metal	0·3 to 0·6		
Greased surfaces	0·05		

168. Laws of Kinetic (or Sliding) Friction:—

(1) The frictional force is proportional to the normal reaction between the two rubbing surfaces. [The force necessary to maintain sliding is less than limiting friction, *i.e.* the frictional force here is less than limiting friction.]

(2) The frictional force is independent of the area of contact between the two surfaces, but depends on the material, nature and condition of the surfaces.

(3) The frictional force is independent of the velocity of sliding, provided the velocity is low.

169. Co-efficient of Kinetic (or Sliding) Friction :—

If the normal reaction between a sliding body and the supporting surface be R, and F denotes the force (less than limiting friction) necessary to maintain a low steady velocity of sliding, once it has been started, then the ratio, F_k/R, is a constant for the given two surfaces and is known as the co-efficient (μ_k) or kinetic (or sliding) friction. That is, $\mu_k = F_k/R$.

The effect of kinetic friction on a body is to oppose the motion of the body with a constant force, μR. If the sliding body be of mass m and moving under a constant applied force P, the acceleration of the body $=(P-\mu_k R)/m$. If surfaces are smooth ($\mu_k = 0$), acceleration $= P/m$.

THE MACHINES

170. The Machines :— A machine is a contrivance by which a force applied at some point of it is overcome by means of another force applied at some other point of it with alteration in direction or magnitude or both. It used to be the practice to call the former force the *weight* and the latter force the *power*. But as the force to be overcome is not necessarily that of gravity, it is better practice to name it the **resistance (or load)** and since the term *power* is used in connection with *rate of work*, it will be better to use the term **effort** in referring to the driving force in a machine. The points at which the effort and the resistance act are usually termed the **driving point** and the **working point** respectively.

171(a). Mechanical Advantage.— The ratio, $\dfrac{\text{load}}{\text{effort}}$, is called the mechanical advantage of a machine. The term *force-ratio* is sometimes used instead of mechanical advantage. Ordinarily, a machine is so constructed that the mechanical advantage is greater than one. If in a machine, this ratio is less than *one*, it would be more accurate to call it *mechanical disadvantage*.

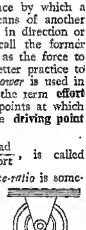


Fig. 100—A Simple Wheel and Axle.

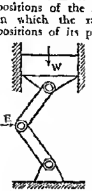
(b) Velocity Ratio.—

The ratio, $\dfrac{\text{displacement of driving point}}{\text{displacement of working point}}$, is called the velocity ratio of a machine. In some machines it is a constant while in some others it is not.

Thus, in a simple wheel and axle (Fig. 100) the displacement, say a, of the effort E will bear a constant ratio to the displacement, say b, of the load W.

That is, its velocity ratio $= \dfrac{a}{b}$.

In a toggle joint [Fig. 100(a)] the ratio of the displacement of the effort E to that of the load W will be different for different positions of the moving parts of the machine. In such a machine in which the ratio is variable, the velocity ratio for any given positions of its parts is the ratio of the displacement of the driving point to the displacement of the working point, when these displacements are indefinitely small.



A Toggle Joint
Fig. 100(a)

172. Efficiency of a Machine:— In all machines some work is always wasted in overcoming *friction*. The result of it is that the work done by the effort in a given time, called *total work* or *work input* ($= E \times a$), is always greater than the work done on the resistance or load ($= W \times b$), called—*useful work or work output*. The difference of the latter from the former=*lost work*$=(Ea - Wb)$.

The **Efficiency** is defined as the ratio, *useful work/total work*. Efficiency evidently will always be less than unity. Often it is expressed as a *percentage* by multiplying with 100.

173. Mechanical Advantage=Efficiency × Velocity Ratio:— Let E be the effort and W the load. The *mechanical advantage*$=\dfrac{W}{E}$.

Suppose the displacement of the *driving point* is a and that of the *working point* is b.

$$\text{Then, } \textit{Efficiency} = \frac{\text{useful work}}{\text{total work}} = \frac{W \times b}{E \times a} = \frac{W/E}{a/b} = \frac{\text{Mech. advantage}}{\text{Velocity ratio}}$$

or **Mechanical advantage=efficiency × velocity ratio.**

174. The Principle of Work:— In any actual machine, the useful work obtained in overcoming the resistance is always less than the total work done by the effort. This is because (i) work has to be done in lifting its parts which have weight, and (ii) because there is always some internal friction which has to be overcome. A *perfect* or *ideal* machine is one which has no weight and no internal friction. For it the useful work is equal to the total work and the *efficiency of the machine is unity*. So the **principle of work**, *viz whatever be the machine, provided there is no friction and that the weight of the machine is neglected, the work done by the effort is always equivalent to the work done against the load* (that is, $E \times a = W \times b$), is a universal principle relating to a machine. It is no new principle but is the same principle known as the **principle of conservation** of energy.

175. What is gained in Power is lost in Speed.— From the principle of work, $E \times a = W \times b$, assuming the machine to be an ideal one. If in a machine the effort E is less than the resistance W, the

distance a through which the driving point moves will be greater in the same proportion, than the distance b through which the working point moves in the same time. This is, in popular language, expressed as, "*What is gained in power* (effort) *is lost in speed.*" The meaning of the statement is that whenever mechanical advantage is gained it is gained at a proportionate decrease of speed.

There is never any gain of work in a machine, though mechanical advantage is generally arranged for.

176. The uses of a Machine :—

(1) This enables one to lift weights or overcome resistances much greater than one could do unaided, as in the case of a pulley-system, a wheel and axle, a crow-bar, a simple screw-jack, etc.

(2) This enables one to convert a slow motion at some point into a more rapid motion at some other desired point, *viz.* a bicycle, a sewing machine, etc. An opposite effect may also be arranged in practice when necessary. Such changes of speed are brought about by belting, gearing, etc.

(3) This enables one to use a force acting at a point to be applied at a more convenient point, as in the use of a poker for stirring up a fire, or to use a force acting at a point in a more convenient manner, *e.g.* lifting of a mortar-bucket to the top floor by means of a rope passing over a pulley fixed at the top of the building, the other end of the rope being pulled down by an agent remaining on the ground.

(4) This enables one to convert a rotatory motion into a linear motion or *vice versa*, as in the case of a *rack and pinion*, etc.

(5) This enables one to convert a reciprocating (to-and-fro) motion into a rotatory motion or *vice versa*, *e.g.* a crank used in the heat engine.

177. Types of simple Machines :—

The following six simple machines represent the types of principles used in making practical machines:—

(1) Pulley, (2) Inclined plane, (3) Lever, (4) Wheel and axle, (5) Screw, and (6) Wedge.

178. The Pulley :—A pulley is a simple machine which consists of a grooved wheel, called the **sheave**, over which a string can pass. The wheel is capable of turning freely about an axle passing through its centre. The axle is fixed to a framework, called the **block**. The pulley is termed **fixed** or **movable** according as its block is fixed or movable.

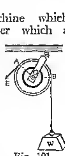


Fig. 101

(1) The Single Fixed Pulley.—In this (Fig. 101) the load W is attached to one end of the string and the effort E is at the other end. With a perfectly smooth pulley and a weightless string, the tension of the string will be the same throughout. Hence, the

of weight W can be supported by a force, $F=W/2$ acting up the plane.

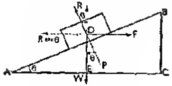


Fig. 104

Case II.—Let the force F act horizontally, *i.e.* parallel to the base AC (Fig. 104).

The vertical and horizontal components of R are $R\cos\theta$ along ED and $R\sin\theta$ along FD.

$R\cos\theta=W$, and $R\sin\theta=F$.

The mechanical advantage, $\frac{W}{F}=\frac{R\cos\theta}{R\sin\theta}=\cot\theta=\frac{\text{base}}{\text{height}}$ of the plaoe

180. The Lever :— The knowledge of the principle of the lever is as old as Archimedes. A lever is a simple machine and consists of a rigid bar (straight or bent) having one point fixed about which the rest of the lever can turn. This fixed point is called the **fulcrum.** The forces exerted on or by the lever may be parallel or inclined to one another. As in all machines, the driving force is called the **effort** (or power), and the working force, the **weight** (or resistence or load) and let them be denoted by E and W respectively. The perpendicular distances between the fulcrum and the lines of action of the effort and the weight are called the **arms** of the lever. The ratio of the arm 'a' of the effort to the arm 'b' of the weight, in the position of equilibrium, is often called the **leverage,** *i.e.* leverage$=a/b$. The mechanical advantage$=\frac{\text{weight}}{\text{effort}}=\frac{W}{E}$.

The principle of the lever is practically the principle of moments which may be stated as, "*If a lever is in equilibrium, the sum of the moments tending to turn it clockwise round any point is equal to the sum of the moments tending to turn it anti-clockwise round that point.*"

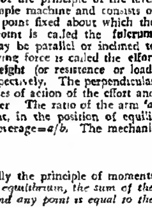


Fig. 105

So for a lever, if it be in equilibrium, *clockwise moment round the fulcrum=contra-clockwise moment round the same point.*

Experiment.—Let a metre-stick AB balance on the sharp edge of a wedge-shaped piece of wood (Fig 105) and let a load W say, 200 gms., be suspended by a string from a point 20 cms. from the fulcrum F. Now find, by experiment, a point on the other side of F such that an effort E, say, weight of 100 gms. applied at the point, will just support the load W. This

point will be found to be at 40 cms. from the fulcrum. It is seen at once that the product of (200×20) is equal to the product of (100×40). If instead of an effort of 100 gms., an effort of 400 gms. is taken, the point of balance will be found now at 10 cms. from the fulcrum. Again it is seen that the product of (200×20) is equal to the product of (400×10). The driving moment in this example, *i.e.* the moment of E about F is anti-clockwise and the working moment, *i.e.* the moment of W about F is clockwise

180(a). The Straight Levers :— When the lever is straight, and the effort and the weight act perpendicularly to the lever, the following three distinct classes of levers are found in practice.

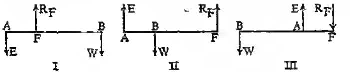


Fig. 106

(I) $E \times AF = W \times BF$, (II) $E \times AF = W \times BF$, (III) $E \times AF = W \times BF$.

or, $E = \dfrac{BF}{AF} \times W$; or, $E = \dfrac{BF}{AF} \times W$; or, $E = \dfrac{BF}{AF} \times W$;

(I) R_F (reaction at fulcrum $= E + W$). (II) $R_F = W - E$. (III) $R_F = E - W$.

Evidently, in class I and II type of levers, if F be taken very near to W, the ratio $\dfrac{BF}{AF}$ can be made very small, *i.e.* a small effort E can be used to overcome a large resistance W, i.e. there is mechanical advantage in these cases. In class III type of levers, a large effort E overcomes a small resistance W, which shows a mechanical disadvantage. This arrangement gives W a large movement for a small movement of the effort E, a fact which is just opposite to what happens in the other two types of levers. The practical use of Class III type of levers lies often in *convenience*; for, in practice, it may not always be possible to find a convenient point to apply the effort relatively at greater distance referred to the fulcrum than that of the load.

181. Common Application of the Principle of Levers :— The lever principles, as described above, are used in our daily life in various ways. Levers may be simple or double. Three common

appliances representative of the three classes of straight levers are shown below in Fig. 107.

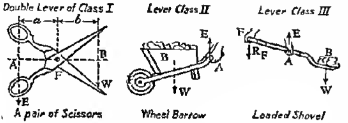


Fig. 107

182. Examples of the Three Classes of Levers:—

Class *I*.—A common balance, pump handle of a tube-well, a spade used in digging earth, a crow-bar used in moving a weight at one end, etc. A pair of scissors and a pair of pincers are examples of *double levers* of this class.

Class II.—A cork squeezer, a crow-bar with one end in contact with the ground, etc. A pair of ordinary nut-crackers is an example of *double levers* of this case

Class III—The human fore-arm (when a load is placed on the palm and the elbow is used as fulcrum, the tension exerted by the muscles in between acts as effort), the upper and lower jaws of the mouth, a pair of forceps used in a weight box, and a pair of coal-tongs are examples of *double levers* of this class

183. The Wheel and Axle:— It is a simple machine, and may also be looked upon as a modification of the lever. It consists of two cylinders of different diameters capable of turning about a common fixed axis, the larger of which is called the *wheel* and the smaller the *axle* [Fig 108]. The load W to be raised is attached to a rope coiled round the axle and the effort E is applied to a rope coiled round the wheel in the opposite direction, so that when the rope round the wheel is uncoiled, the rope round the axle is coiled up and thereby the weight is raised. Fig 108, shows a section where OB is the radius r of the axle and OA the radius R of the wheel Taking moments about O, the axis, $E \times OA = W \times OB$.

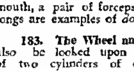


Fig. 108
The Wheel and Axle.

$\therefore$ The mechanical advantage $= \dfrac{W}{E} = \dfrac{OA}{OB} = \dfrac{\text{Radius of wheel } (R)}{\text{Radius of axle } (r)}$.

The **windlass** by which water is drawn from a well is of the same class as the wheel and axle, the crank-handle of which serving the purpose of the wheel. The **capstan** [Fig. 108 (*a*)] used on board a ship for raising an anchor is also of this class. In it the length of the lever arm takes the place of the radius of the wheel and the radius of the barrel corresponding to the radius of the axle.

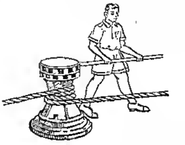


Fig. 108(*a*)—The Capstan.

184. Screw:—An accurately cut screw has many important applications in modern industrial machines. The screw gauge and the spherometer which are two very common laboratory instruments also work on the principle of the screw and the nut.

A screw can be considered as an **inclined plane** wrapped round a cylinder. The connection between the inclined plane and the srew is shown in Fig. 109.

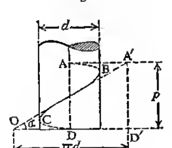


Fig. 109

It shows a solid cylinder having one turn *ABCD* of a helix marked on its surface. The right-angled, triangle *A′OD′* is the development of that part of the surface of the cylinder which is below the helix, p = pitch of the helix, α inclination of the helix, d = diameter of the helix.

Then, $\tan \alpha = p/\pi d$.

Actual screws are of metal or wood and differ from the above ideal screw in that they always have a protuberant thread (forming the helix) cut on the cylinder. This enables the screw to work in a nut which is a hollow coller on the inside surface of which a similar screw is cut, the threads of the screw fitting in the grooves of the nut. The screw is rotated in the nut or the nut on the screw by a force applied on a wheel or lever attched to the rotor. On account of the rubbing between the rotor and the stator some friction is inevitable and so the useful work obtained in an actual screw is less than the work

that should be got out from an ideal screw. So the mechanical advantage of an actual screw is less than the velocity ratio and the efficiency of the screw is always less than unity.

Threads of screws are generally triangular or square in section as shown in Figs. 110, (*a*) and (*b*), respectively. Screws are conventionally represented as in Fig. 110, (*c*). A screw and a nut form a

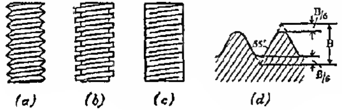


Fig 110

relative pair. The Whitworth V-thread in which the angle of the thread is 55°, shown in Fig 110, (*d*) is perhaps the most used thread in Engineering.

Pitch (*p*) *of screw-thread*.—The distance through which a screw moves when it is rotated once about its axis is called the pitch of the screw It is the same as the axial distance between two consecutive threads of it as shown in Fig 14

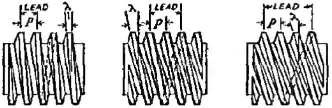


Fig 110(a)

Lead of screw-thread.—It is the actual distance a nut on the thread would travel in making one complete rotation. When the screw is single-threaded, the pitch and the lead are equal; when double-threaded, the lead is twice the pitch. In general, when the screw is *n*-threaded, the lead is *n* times the pitch. Fig. 110(*a*) shows in the three diagrams from left to right, the lead of a single-, double-, and treble-threaded screw.

Back-lash.—This error is present in almost all instruments with nut and screw. If due to wear, or any imperfection in manu-

facture, the screw is a loose fit in the nut, it may so happen that equal rotations of the screw-head in opposite directions produce unequal linear movement of the screw, or any rotatory motion can be given to a screw without causing any translatory movement of the nut when the latter should move; then an error called **back-lash** exists

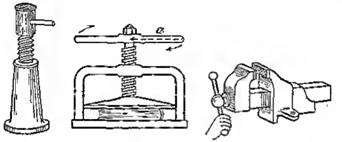


Fig. 110(b)

in the instrument. When a screw and nut principle is utilised for measuring a small distance, as in the case of a spherometer, the screw should always be turned in the same direction to avoid *back-lash*.

185. Some common Applications of the Screw :—

A *screw jack* [Fig. 110(b)] used for lifting heavy loads like an automobile, a *screw-press* or *letter-press* used for compressing bound

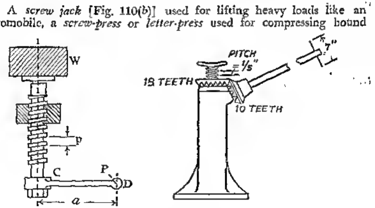


Fig. 110(c)

books etc., a *vice* used in workshops for holding jobs with a strong grip, are common examples of a screw.

186. Velocity Ratio and Efficiency of a Screw-jack :—

Let a single-threaded screw [Fig. 110(c), left] of pitch p working in a nut support a load of weight W and a force P be applied in the horizontal plane to the end D of a lever CD (length $= a$) fitted on the screw. In one complete turn of the lever arm, distance travelled by the effort $P = 2\pi a$, while the load is moved up through a distance p.

The velocity ratio $(V.R.) = 2\pi a/p$. Since the mechanical advantage $(M.A.)$ is W/P,

$$\text{the efficiency} = \frac{\text{work got out}}{\text{work put in}} = (W \times p)/(P \times 2\pi a) = M.A. \times \frac{1}{V.R.}$$

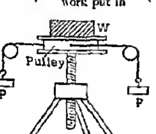


Fig 110(d)

Examples. *(1) In an experiment, a screw-jack is arranged to be driven by a pulley as shown in Fig 110(d). The lead of the screw (single threaded) is ½ inch, and the diameter of the pulley is 12 inches. Equal dise weights P of 4.5 lbs are seen to raise a load W of 280 lbs slowly and steadily. Find the efficiency of the jack.*

In this case (single threaded screw), the lead is equal to the pitch of the screw

$$V.R. = \frac{\text{distance moved by effort } P \text{ for one complete turn of screw}}{\text{distance moved up by load}}$$

$$= \frac{2\pi(12/2)}{1/2} = \frac{\text{circumference of pulley}}{\text{pitch of screw}} = 3.14 \times 12 \times 2 = 75.36.$$

$$M.A. = \frac{\text{load}}{\text{effort}} = W/P = \frac{280}{2 \times 4.5} = 31.11. \qquad \therefore \text{ Efficiency} = \frac{\text{work got out}}{\text{work put in}}$$

$$= \frac{W \times \text{pitch of screw}}{P \times \text{circumference of pulley}} = 31.11 \times \frac{1}{75.36} = 0.413, \text{ i.e. } 41.3\%$$

(2) In the screw-jack shown in Fig. 110(c), [right] the cross bar is 7 in long, the levelled wheel has 10 teeth engaging with a wheel of 18 teeth which raises a screw of pitch ⅕ in. Show that the velocity ratio is 198

(3) The length of each arm of a screw-press [Fig. 110(b)] is 6 in. and the pitch of the screw 1/4 in. Forces of 1¾ lb.-wt. are applied to each arm. Find the resistance overcome.

Work put in $= 14 \times 2\pi \times 6/12$ ft.-lbs.

Work got out $= W \times 1/(4 \times 12)$ ft-lbs, where $W =$ resistance in lbs.-wt.

Neglecting friction, $\frac{14 \times 2\pi \times 6}{12} = \frac{W}{4 \times 12}$, $\therefore$ $W = 4224$ lbs wt

187. Wedge :—A wedge is a simple machine consisting of a solid block of metal or wood shaped as an inclined plane. A small driving force applied to the wedge results in a much larger splitting or separating force. It is commonly used for raising heavy bodies, for propping up a sinking wall, for widening a gap, for breaking strong cover joints, etc. [Fig. 111(*a*)].

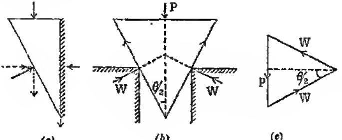


Fig. 111

A double wedge (angle of the wedge $=\theta$) is shown in Fig. 111(*b*) being used for widening a gap. The separating forces generated produce equal reactions W, W at the edges of the gap. The forces P, W, W can be represented by a triangle shown in Fig. 111(*c*), neglecting friction.

Here $P=2\ W\ \sin\ \theta/2$ ... (1), and $M.A.=W/P=\dfrac{1}{2\sin\theta/2}$ ... (2)

The action of an axe, or a knife, or a nail may be treated as that of a combination of two wedges (Fig. 112).

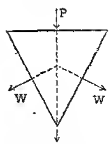


Fig. 112

Example. *The angle of a wedge is 10°. Find the splitting force exerted by it when driven by a force of 15 lbs.-wt. and the mechanical advantage. Neglect friction.*

From equation (1) of the preceding article, $P=15=2\times W\times\sin\ 10°/2$,

or, $W=\dfrac{15}{2\sin 5°}=\dfrac{15}{2\times 0{\cdot}0872}=86$ lbs.-wt.

$M.A.=W/P=86/15=5{\cdot}73$.

188. Magnification of Displacement by the Use of Levers :—

In machines and instruments it becomes very often necessary either to magnify or reduce the displacement of a moving element.

This is realised in practice usually by using a lever or levers. Fig. 113 represents an arrangement of double levers used to magnify a small displacement d_0 caused by an effort E at the end A_1 of a lever

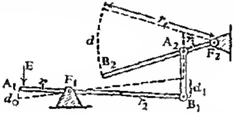


Fig 113—A Magnifying Device of Levers.

A_1B_1 to a large displacement d (in two stages) at the free end B_2 of a second lever. The fulcrum of the first lever is F_1 and that of the second F_2, the working point B_1 of the first lever being rigidly connected by a stout rod to the driving point A_2 of the second lever. What happens, when the driving end A_1 of the first lever is given a finite displacement d_0 by the action of the effort E, is shown by the dotted lines.

$$\text{Overall magnification} = \frac{d}{d_0} = \frac{d}{d_1} \times \frac{d_1}{d_0}$$
$$= (r_4/r_3) \times (r_2/r_1)$$

189. Rack and Pinion:—A rack is a toothed wheel of infinite diameter. A rack and pinion in gear are shown in Fig. 113(a). When the rack is fixed, the pinion (with its attachments) rolls on it on being rotated. When the pinion axle is fixed in position and the rack is movable, the latter with its attachments moves as shown when the pinion is rotated.

Fig 113(a)—Rack and Pinion

190. The Common Balance:—A common balance is an instrument of utmost usefulness. It provides us with a ready means of measuring the *mass* of a body. We do not measure the *weight* of a body directly with it, though we ordinarily say that we do. A balance of this type is used by the grocer and this shows its importance in our daily life. A *sensitive* balance of this type, usually referred to as a *physical* or *chemical* balance, is an indispensable necessity in the laboratory.

Description.—It consists of a **horizontal** rigid **beam** balanced at its centre on a **knife-edge** which rests on an agate plane fixed on the top of a vertical **pillar** (Fig. 114).

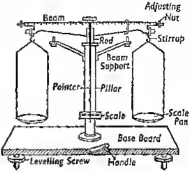


Fig. 114—A Physical Balance.

Two scale **pans** of equal weight are suspended from stirrups (or hangers) carried by knife-edges at the two extremities of the beam. The distances between the central knife-edge and those at the extremities are called the **arms** of the balance, which should be equal. A long **pointer** attached to the centre of the beam moves over a **graduated** arc (scale) fixed on the **pillar**. For accuracy, the pointer should swing evenly to equal distances on each side of the middle mark of the scale. There is a **lever arrangement** (handle) by which the pillar, which supports the beam, can be lowered and the beam arrested in order to preserve the sharpness of the knife-edges.

The method of use.— *To use the balance,* it is first of all levelled by levelling screws provided at the base **board** and then adjusted by means of two *screws* (adjusting nuts) at the two extremities of the beam until the pointer oscillates equally on both sides of the middle division. The *body* to be weighed is then placed on the *left-hand pan,* and *weights* from the weight box are added on the *right-hand pan,* until the pointer oscillates in the same way, as in the case of the unloaded beam; it is then that the weights on the two sides are balanced. As the arms are equal, the two weights on the two pans are also equal. So the weight on the right-hand pan is equal to the weight of the body.

Weight Box.—The weight box [Fig. 114(*a*)] which is supplied along with a balance contains the following weights: 100, 50, 20, 20, 10, 5, 2, 2, 1, grams. Besides these, the box contains a few fractional weights, from 500 mgms. (*i.e.* 0·5 gm.) down to 10 mgms. (*i.e.* 0·01 gm.).

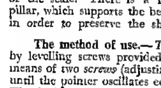


Fig. 114(*a*)—The Weight Box.

Rider.—For accurate weighings by means of a good **balance**, a bent piece of wire of mass 10 mgms. (*i.e.* a centigram) called a *rider,* is often used. Each **arm** of a good **balance** is divided into 10 equal parts (Fig. 114) and the rider can be placed on the right arm at any one of the points by means of a sliding rod from outside the case, in addition to the weight from

the weight box already placed on the pan, until the pointer swings equally on both sides. When the rider is placed on the 10th division, *i.e.* at the end of the arm, it is equivalent to adding 10 mgms on the corresponding pan of the balance. If the rider is placed on any other division, say the 1st, the equivalent weight on the pan becomes $(\frac{1}{10} \times 10)$, *i.e.* 1 mgm, and so the rider placed on the nth division is equivalent to adding n mgms on the corresponding pan

(a) Principle of Measurement.— A common balance is an example of class I type of a lever in which the two arms, AF and BF, of a beam AB, F being the fulcrum, are equal [Fig. 114(*b*)] Neglecting the weight of AB, and of the two scale-pans hung at A and B, if the beam remains in equilibrium at the horizontal position when a weight W_1 is placed on the pan at A and a weight W_2 on the pan at B, we have, by taking moments about F,

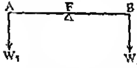


Fig 114(b)

$$W_1 \times AF = W_2 \times BF \quad \ldots \quad (1)$$

But $BF = AF$ $\therefore$ $W_1 = W_2$. That is, the weight on the pan is equal to the weight on the other at the position of balance of the beam in the horizontal position This is the principle of measurement by a common balance

In practice, the weight of a given body is balanced by the combined weight of a number of standard masses of known values Let m be the mass of the given body and m', the combined value of the standard masses required for balancing.

Then from (1), the two weights being equal, $mg = m'g$, where g is the acceleration due to gravity at the place, or $m = m'$. That is, an unknown mass is measured in terms of some standard masses supplied.

Note.—(i) *Weighing* by a balance means the determination of a known mass which has the same weight as that of the unknown mass, and mass being proportional to weight, a common balance is used only to compare the masses; for let W, W' be the weights of two bodies in poundals or dynes, as the case may be, and let their masses be m and m' respectively. Then we have, $W = mg$, and $W' = m'g$, where g is the acceleration due to gravity at that particular place,

$$\therefore \quad \frac{W}{W'} = \frac{mg}{m'g} = \frac{m}{m'}.$$

Thus, the weights of two bodies at a given place are proportional to their masses.

(*ii*) The position of equilibrium for any two masses is unaltered by taking the balance to another place where the value of g is different when weighing is done by a common balance.

191. Theory of the Common Balance:—Suppose the beam AB having *equal* arms AF and BF turns round the fulcrum F, which to diminish friction, is made of an agate or steel knife-edge resting on a smooth agate plane [Fig. 114(c)]. Let W_0 be the weight of the beam and pointer. Let us assume that the centre of gravity of the beam and pointer, through which W acts, lies at G on the line FG which is perpendicular to the beam through F and is below F. Let two scale pans of equal weight S be hung at A and B. If now nearly equal weights W and W' be placed on the pans at A and B respectively, and thereby the beam be tilted from the horizontal through θ, we have, by taking moments round F,

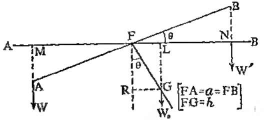


Fig. 114(c)

$$(W+S).FM=W_0.FL+(W'+S).FN$$
$$=W_0.GR+(W'+S).FN \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

Also, $FM=FA\cos\theta=a\cos\theta=FN$, and $FL=GR=h\sin\theta$.

$$\therefore \quad (W+S)\,a\cos\theta=W_0\,h\sin\theta+(W'+S)\,a\cos\theta \ \ldots \quad \ldots \quad (2)$$

$$\text{or,} \quad \frac{\sin\theta}{\cos\theta}=\tan\theta=\frac{(W-W')a}{W_0h} \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

192. Requisites of a good Balance:—A good balance must be (*a*) *true*, (*b*) *sensitive*, (*c*) *stable* and (*d*) *rigid*.

(*a*) **True.**—A balance is said to be true if the beam of the balance is horizontal when *equal weights* or *no weights*, are in the pans. Equation (3) shows that $\theta=0$, when either $W=W'$, or $W=0=W'$. Therefore, it follows from the assumptions made in arriving at equation (3) that a balance will be true provided the arms are (*i*) of exactly equal length, (*ii*) of exactly equal weight, *i.e.* the C.G. lies on the perpendicular to the beam at its middle, and (*iii*) the pans are equal in weight.

(*b*) **Sensitive.**—A balance is said to be sensitive, if for a small difference between W and W', the angle of tilt θ is large. Equation (3) shows that for a given difference between W and W', $\tan\theta$ or θ will be large if a is large, and W_0 and h small. Therefore the conditions of sensitivity are that the beam should be (*i*) long, (*ii*) light, and (*iii*) its centre of gravity as near the fulcrum as possible.

(*c*) **Stable.**—A balance is said to be stable, if it quickly returns to its position of rest after being deflected, with equal weights in the pans. Equation (2) shows that, when $(W+S)=(W'+S)$, the only

restoring couple (i.e. the couple which tends to restore the beam to its position of rest) arises from the weight of the beam. Hence for stability, $W_4 h$ should be large (of course the C.G. must be below the fulcrum F). That is, for a given value for W_4 consistent with the rigidity of the instrument, the condition for stability is that h should be large, *i.e.* the C.G. as much below the fulcrum as possible.

(*d*) **Rigid.**—A balance is said to be rigid, if it be sufficiently strong so as not to bend under the weights it is intended to carry.

Note.—For a balance to be sensitive, the C.G. of the beam should be as near the fulcrum as possible; while to be stable, the C.G. should be as much below the fulcrum as possible. Evidently, great sensitiveness and quick-weighing are incompatible in the same balance. In practice, these opposite conditions, however, do not present much difficulty; for, in balances requiring high sensitivity as in the laboratory balances, accuracy of weighing forms the main criterion and quickness of weighing can be sacrificed to some extent. On the other hand, in commercial balances as used by grocers, etc. when large masses are used, speed in weighing is more looked for than high accuracy. A compromise between the two opposite conditions is adopted when it is desired to combine the qualities of sensitiveness and quick weighing in the same balance to a moderate extent. This is done by making h neither too small, nor too large.

193. Test of Accuracy of a Balance:—Let a and b be the lengths of the arms of the balance, S and S' the weights of the scale pans. Now, if the beam is horizontal with empty pans, we have, by taking moments about the fulcrum, $S \times a = S' \times b$ .. . (1), provided the C.G. of the beam lies on the perpendicular to the beam through the fulcrum.

Again, the beam will be horizontal, if equal weights W, W are placed on the pans. We have, then, $(W+S)a = (W+S')b$ . . (2). From (1) and (2) we get, $Wa = Wb$, or $a = b$, *i.e.* the arms must be of equal length; and since $Sa = S'b$, we have $S = S'$, *i.e.* the scale pans must be of equal weight. So, to test the accuracy of the balance, first see if the beam is horizontal when the scale pans are empty. Then put a body on one of the scale pans, and put weights on the other pan to balance it. Next interchange the body and the weights on the two pans. If the beam of the balance is still horizontal, the balance is true, otherwise it is not.

194. Weighing by the Method of Oscillation:—The operation of weighing by a sensitive balance takes a very long time before the beam comes to rest. It is, however, unnecessary to wait till the pointer comes to rest, for we can calculate the position which the pointer would occupy if the balance comes to rest. This can be done by observing the readings of the scale corresponding to the turning points of the pointer while the balance is swinging. The position so determined is called the 'resting-point' (written, R.P.) for a [illegible]icular adjustment of weights and load, or for empty pans. It is

more accurate and much quicker to perform the weighing by this method, which is called the **Method of oscillation.** *This method is suitable when the weight to be taken is small.*

Procedure.—Imagine the scale divisions, over which the pointer moves, to be numbered from left to right, as shown in Fig. 114(d). Slowly raise and lower the beam two or three times so that the pointer swings over about 3/4 of the scale divisions. When after two or three oscillations the motion becomes regular, take a reading (say, 4) of the turning point of the pointer, by avoiding parallax, as it swings to the left. Then read the extreme position (say, 14) of the subsequent swing to the right. Again read the next swing to the left (say 5).

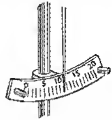


Fig. 114(d)

Thus three readings, one to the right and two to the left, have been taken from which the R.P. for empty pans can be calculated in the following way. Take the mean of the two left-hand readings, *i.e.* the first and the third readings. Then the mean of this mean and the right-hand reading, *i.e.* the second reading, will give the mean R.P. for empty pans.

For greater accuracy five consecutive readings (two to the right and three to the left) should be taken. The R.P. for the empty pans is found as above a number of times and therefrom the mean R.P. (say, x) for the empty pans is obtained.

The reason for taking an odd number of observations is that the arc over which the pointer swings continually grows less due to friction and air resistance, and thus if only two observations, say, one to the left and then another to the right, or two to the left and two to the right, are taken, the position of rest obtained by taking the arithmetic mean of these two will be too far to the left. The mean of any odd number of observations, obtained as above, will represent the true position of rest more or, accurately.

Next place the body to be weighed on the left-hand pan and try to get its weight (w) by adding wts. on the right-hand pan until the pointer oscillates within the scale. Let the mean R.P. for the loaded pans be y.

Next find out the mean R.P. (z) when the wt. w on the right-hand pan is increased by 1 milligram or any such small weight.

Now, it is necessary to calculate out what weight must be added to or subtracted from w in order to reduce the R.P. from y to x.

Calculation.—

True wt. $= w + \dfrac{0{\cdot}001}{y-z} \times (y-x)$ gm., when $y > x$.

If y is less than x, true wt. $= w - \dfrac{0{\cdot}001}{y-z} \times (x-y)$ gm.

Note.—As the sensibility of a balance varies with the load, it should be calculated everytime a body is weighed. *The sensibility of a balance is defined as the change of the resting point due to a change of some definite weight, usually one milligram, in one of the pans.*

195. The Method of Double Weighing:—The true weight of a body can be determined with the help of a false balance by any one of the following two methods of **Double Weighing**—

(i) The Method of Substitution *(Borda's Method)*.—Place the body to be weighed on the left-hand pan and counterpoise it by sand (or any other convenient substance) in the right-hand pan. Then remove the body and replace it by known weights to balance the sand. Since the body and the weights both balance the sand under exactly the same conditions, they must be equal.

(ii) Gauss's Method.—Let a and b be the lengths of the arms. Place a body of true weight W in the left-hand pan, and let its apparent weight be W_1.

Then, taking moments of the force on each side,

$$W \times a = W_1 \times b \qquad (1)$$

Now put the body in the right-hand pan, and let W_2 be its apparent weight, then $W_2 \times a = W \times b$ (2)

From (1) and (2), $\frac{W}{W_2} = \frac{W_1}{W}$, or, $W^2 = W_1 \times W_2$;

$$\text{or, } W = \sqrt{W_1 \times W_2} \qquad (3)$$

Thus, *the true weight is the geometrical mean of the two apparent weights.*

Ratio of the arms.—

From eq (1), $\frac{a}{b} = \frac{W_1}{W}$, and from eq (2), $\frac{a}{b} = \frac{W}{W_2}$.

$$\therefore \frac{a^2}{b^2} = \frac{W_1}{W} \times \frac{W}{W_2}, \text{ or, } \frac{a}{b} = \sqrt{\frac{W_1}{W_2}} \qquad (4)$$

196. A False Balance:—By using a false balance with unequal arms, a tradesman will defraud himself if he weighs out a substance (to be given to a customer), in equal quantities, by using alternately each of the scale pans. Let W be the true weight of the quantity of a substance which appears to weigh W_1 and W_2 successively by the two scale pans of a balance of which a and b are the lengths of the arms. Here the customer gets $(W_1 + W_2)$ instead of $(W + W)$, i.e. $2W$; and we have, $W_1 + W_2 - 2W = W\frac{a}{b} + W\frac{b}{a} - 2W$ (from eqs 1 and 2 of Art. 195)

$$= W\left(\frac{a^2 + b^2 - 2ab}{ab}\right) = W\frac{(a-b)^2}{ab}.$$

The right-hand side of the equation is always positive whatever be the values of a and b, and so (W_1+W_2) is always greater than $2W$. Thus the tradesman defrauds himself by the amount $W\dfrac{(a-b)^2}{ab}$.

Hence, at the time of purchasing a substance, *a customer should always insist on having half of that substance weighed on one pan and the other half on the other if he doubts the balance.*

Example. *An object is placed in one scale pan, and it is balanced by 20 lbs. The object is then put into the other scale pan, and now it takes 21 lbs. to balance it. When both scale pans are empty, the scales balance. What is the matter with the balance, and what is the true weight of the object?* (Pat. 1934)

Two different weights are required to balance the same object when placed on different pans of the balance, because the arms of the balance are unequal [*vide* Art. 195, *ii*]. The true wt. $=\sqrt{20\times21}=20{\cdot}494$ lbs.

197. The Common (or Roman) Steelyard:— This is a form of balance with unequal arms and is used for rough quick weighing. It consists of a graduated beam AB (Fig. 115) movable about a fixed fulcrum F very near to one of its ends. A known sliding weight E slides over the arm AB. The object W to be weighed is suspended at a hook A and then the beam is made horizontal, that is, the body is balanced, by changing the position of E. It should be noted that the graduations are correct with only a constant weight E, and if this weight is changed, the graduations must be changed correspondingly. If M be the weight of the beam acting at its centre of gravity G, we have, for equilibrium, $W\times AF=(M\times GF)+(E\times BF)$.

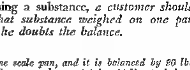


Fig. 115

198. Platform Balance:— The **platform balances** often used for weighing luggages and parcels in Railway Stations work on the principle of a Common Steelyard. It consists essentially of three levers $A_1F_1B_1$, $F_2A_2B_2$, and A_3bF_3 having their fulcrums respectively at F_1, F_2 and F_3 [Fig. 115(*a*)]. There are two knife-edges a, and b, fixed on two separate levers, upon which the platform P of the balance rests. The pressure exerted by any load placed on P is communicated to the end B_2 of the lever $F_2A_2B_2$, which again is attached to the point A_1 of the upper lever by a vertical rod B_2A_1.

In the lever $F_2A_2B_2$, the arm F_2B_1 being much longer than the arm F_2A_2, a very small force is required to balance the force exerted on a platform P and this force is again balanced by the force on the upper lever.

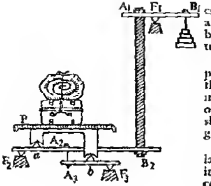


Fig 115(a)—Platform Balance

The upper lever has its fulcrum F_1 very near to A_1 and so a small weight acting at B_1 can balance the force communicated to A_1.

The standard weights suspended at the end B_1 constitute the *effort* in this case for balancing the *load*, and small fraction of weights are measured by sliding small weight along the graduated rod F_1B_1.

Very big balances having larger platforms used for weighing loaded carts or wagons of coals, etc. are called weighbridges.

199. The Spring Balance:—A Spring balance is essentially an instrument for measuring a force So the weight of a body which is a force can be determined with it. It consists of a *spiral spring*, fixed at its top to a metal plate hanging from a ring attached to a rigid support and at the lower end of it is attached a hook for supporting the body to be weighed (Fig 116) An *index* or a pointer, attached to the spring moves along a metal scale which is graduated in grams or pounds with the help of known weights The body to be weighed is suspended from the hook, and *the spring is elongated due to the force with which the body is attracted by the earth* The position of the pointer on the metal scale indicates the weight of the body which is the measure of the force with which it is attracted towards the centre of the earth So by a spring balance the weight of a body at a given place is directly obtained, and this weight will differ at different places as the pull on the spring due to the force of attraction of the earth changes from place to place So a spring balance can give the true weight of a body only at the particular place where it was graduated By a *spring balance* we compare *different weights* while by a *common balance we compare different masses* and not weights

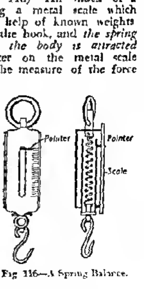


Fig 116—A Spring Balance.

Principle. The principle of the spring balance may be learnt by arranging a spiral 'e of thin steel wire, to move in a groove

between two strips of wood. The upper end of the spring is clamped and the lower end carries a scale pan and a pointer or index, which moves over a millimetre scale attached to the side of the spring. This forms what is called a **spring balance or a spring dynamometer** (force-measurer).

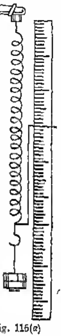


Fig. 116(*a*)

Experiment. *To graduate a dynamometer* [Fig. 116(*a*)], fix it vertically and mark the initial position of the pointer. Add known weights, say 10 gms. at a time, and read the position of the pointer after each addition. Repeat these observations until the spring is extended to nearly twice its original length. Then reverse the process, *i.e.* remove the weights step by step, and note the readings as before. Tabulate the readings. Find the mean index readings for the loads increasing and loads decreasing.

Now plot a curve (Fig. 117) taking weights as abscissæ and the mean index readings as ordinates. *The graph is a straight line.* The mean elongation for any weight is the difference between the corresponding mean index reading and the no-load reading.

Conclusion.—The result of the above experiment shows that (*i*) the amount of *elongation* is proportional to the load applied, and that (*ii*) the spring used is a very elastic material; because, on the removal of the various loads, the index returns to the original position. The first of these is known as **Hooke's Law.***

Experiment. *Determination of an unknown weight.*—Place a small object on the scale pan of the dynamometer and note the position of the index, which is, say, 8·2 cms. Now, by means of the graph, as obtained above, deduce the weight of the object. The weight as indicated by the graph [Fig. 117], is 59 gms.

Note that, unlike a common balance, the spring balance not only measures the absolute weights of bodies, but it can also measure other forces, after its scale has been calibrated by comparison with known forces.

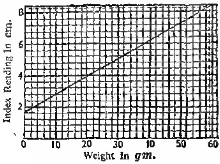


Fig. 117

200. Distinction between Mass and Weight:—The **mass** of a body is the quantity of matter in the body, and the **weight** of a mass is the force with which it is attracted by the earth.

If g be the acceleration due to gravity at any place, the weight of a body of

* This has been treated separately in Art. 203 under 'Elasticity'.

mass m grams at that place is mg dynes (Art. 82) So, at different places the weight of the same body will be different if the value of g be different, but the mass, or the quantity of matter in the body, remains constant. The value of g differs from place to place on the surface of the earth due to—

(a) *The peculiar shape of the earth*.—The earth is flattened at the poles, the polar diameter being less than the equatorial diameter by about 27 miles Evidently the value of g is greater at the poles and so the same body would weigh greater at the poles than at the equator; that is, *the weight of a body increases from the equator to the poles*, or, in other words, the weight of a body increases with the latitude of the place

For example, *the absolute weight of a pound-mass varies from 32·091 poundals at the equator to 32·255 poundals at the poles; and the absolute weight of a gram mass varies from 978·10 dynes, at the equator to 983·11 dynes at the poles.*

(b) *The altitude of the place*.—As the value of g decreases with the increase of altitude, being *inversely* proportional to the square of the distance of the body from the centre of the earth (*vide* Art 98), the weight of a body decreases in higher altitudes, the mass remaining constant For instance, the weight of a 10 lb body, at a height of 50,000 miles, would be about as much as an ounce at the surface of the earth Again, since the value of g is less inside the earth [being *directly* proportional to the distance of a body from the centre of the earth (*vide* Art. 98)], so the weight of a body decreases as it is taken down inside the earth, say, to the bottom of a mine; the greater the depth, the less is the weight Thus the *weight* is *not an essential property of matter*, as a body taken to *the centre of the earth*, where the value of g is zero, would have **no weight** but the *mass* of it will remain *unaltered*

(c) *The rotation of the earth*.—The value of g also differs owing to *the diurnal rotation of the earth about its axis*, due to which every body on the earth's surface also revolves and in order to keep the body in the circular path a certain fraction of the true weight of the body is lost So the observed weight becomes less than its true weight [*vide* Art. 98(c)].

At any place the mass of a body is proportional to its weight This means that if a piece of iron weighs five times as much as a piece of lead, the mass of iron is five times that of the lead piece. Hence when we obtain the weights of various bodies, we also measure their respective masses.

Mass is measured in *grams* or *pounds*, while weight should be measured in units of **force**, **i.e.** *dynes* or *poundals* Ordinarily

however, the two words, ***mass*** and ***weight***, are used as synonymous, because, as stated above, we get only a comparison of the masses by weighing a body by an ordinary balance, and so the weight of a body *at a given place* may be regarded as a measure of its mass, and this has led to the use of the units of mass as units of weight. But we must be **beware of this double meaning of 'weight'.**

201. Detection of the Variation of the Weight of a Body with Change of Place :— The difference in the weight of a body at different places cannot be detected by an ordinary balance, because the body as well as the 'wts.' that are used to weigh it are both equally affected by the variation of g. So, if a weight is balanced at one place, the balancing continues when the same is taken to a different place. The difference in weight can, however, be detected by means of a delicate spring balance, where the body depresses the pointer to different distances, as the force on the body becomes different for different values of g. For instance, if a cubic foot of water is weighed in a spring balance in London and also at the Equator, the indicated weight would be $8\frac{1}{2}$ oz. greater in London than at the Equator (*vide* the table in p. 94). Similarly, it would be $\frac{1}{2}$ oz. greater in Manchester than in London.

Examples. *(1) A body is weighed in a spring balance at a place where $g=980{\cdot}94$, and the reading indicated by the balance is 50 grams. What will the reading be, if the body be taken at a place where $g=981{\cdot}54$?*

Let W_1 and W_2 be the readings of the spring balance, g_1 and g_2 the corresponding values of g; then, if m be the mass of the body, which is the same everywhere, we have,

$$\frac{W_2}{W_1}=\frac{mg_2}{mg_1}. \quad \therefore \quad W_2=\frac{g_2}{g_1}\times W_1=\frac{981{\cdot}54\times 50}{980{\cdot}94}=50{\cdot}031 \text{ gms. (nearly).}$$

(2) If the weight of a man is 160 lbs. on a beam balance at a place where $g=980{\cdot}665$ cm./sec.^{2}, how much would he weigh on an accurate spring balance at the Equator ($g=978{\cdot}1$), and at the Pole ($g=983{\cdot}1$)?

The mass of the man $=\dfrac{160}{980{\cdot}665}$, which remains constant both at the Equator and at the Pole.

If w' be the weight of the mass at the Equator and g' the value of acceleration due to gravity, we have, $w'=mg'=\dfrac{160}{980{\cdot}665}\times 978{\cdot}1=159{\cdot}56$ lb.

Similarly, if w'' be the weight at the Pole, $w''=\dfrac{160}{980{\cdot}665}\times 983{\cdot}1=160{\cdot}39$ lb.

(3) If the mass of the earth is 81·53 times that of the moon, and the diameter of the earth is 3·673 times that of the moon, compare the weight of a body on the surface of the moon with its weight on the surface of the earth.

We know from the law of gravitation that the forces of attraction between two bodies are directly proportional to their masses and inversely proportional to the square of the distance between them.

Let m be the mass of the body, M the mass of the moon, M' the mass of the earth, and d the distance between the moon and the body, when it is on the surface of the moon, i.e. d = the radius of the moon, and d' the radius of the earth, i.e. distance between the body and the earth, when the body is on the surface of the earth. Then, we have the attraction of the moon, $F \propto \frac{mM}{d^2}$

and the attraction of the earth, $F' \propto \frac{mM'}{d'^2}$. $\therefore \frac{F}{F'} = \frac{M}{M'} \times \frac{d'^2}{d^2} = \frac{1}{81 \cdot 63} \times \frac{(3 \cdot 673)^2}{1}$

$= 0 \cdot 1655 = \frac{\textit{weight on the moon}}{\textit{weight on the earth}}$, $\left(\because \frac{d'}{d} = \frac{\textit{radius of earth}}{\textit{radius of moon}}\right)$.

This problem shows that while the mass of the body is the same on the moon as on the earth, its weight on the earth is about 6 times greater than that on the moon.

Questions

1. Define the terms, *velocity ratio*, *mechanical advantage*, and *efficiency*, as applied to machines. (Mysore, 1952, P. U. 1952, E. P. U. 1951, '53)

2. Show that once a body is just ready to slide down an inclined plane, the tangent of the angle of inclination of the plane is equal to the co-efficient of friction. (Del. H S, 1952, Dac. 1942, Nag. U, 1952; Pat, 1927)

3. What is the acceleration of a block sliding down a 30° slope, when the co-efficient of friction is 0.25? (Poona, 1954)
[*Ans.* 9.14 ft./sec.²]

What h.p. is exerted in pulling a 200 lb. log up a 30° slope at the rate of 12 ft./sec. (co-eff. of friction = 0.3)?
[*Ans.* 5.3 h.p.]

4. A body starting from rest slides down an inclined plane whose slope is 30° (co-eff. of friction = 0.2). What is its speed after sliding 76 ft.?
[*Ans.* 40 ft./sec.]

5. What is the mechanical advantage of an inclined plane used as a machine, when $\theta = 30°$, and the force acts horizontally? When it acts along the plane?
[*Ans.* $\sqrt{3}$, 2]

6. State what you mean by 'Limiting friction', and the 'Angle of friction'. (Dac. 1925)

Explain the laws of limiting friction and describe experiments to verify them. (Pat. 1952)

7. (*a*) Define 'machine' and 'mechanical advantage'. (Pat. 1947)

(*b*) Justify the statement 'What is gained in power is lost in speed' by considering two important machines. (Pat. 1947)

8. Give a neat diagram and very brief description to show the working of the second system of pulleys, and deduce the mechanical advantage. (Pat. 1929)

9. What are levers? Give examples of different classes of levers. (Pat. 1921)

10. A uniform beam weighing 72 lb. and 12 ft. long is supported on two props at its ends. Where must a mass of 108 lb. be placed so that the thrust on one prop may be twice as that on the other? (Utkal, 1951)
[*Ans.* At 2⅔ ft. from one end.]

11. Describe a lever of Class III. Calculate its mechanical advantage and show that the principle of work has been satisfied there. (Utkal, 1952)

Give a very brief description of the second system of pulleys; and deduce the mech. advantage. (E. P. U. 1952)

12. Deduce the M.A. of a *wheel and axle* from the general principle of conservation of energy. (Pat. 1929, '31)

13. A screw jack has a pitch of ·05 inch. What weight will it lift (neglecting friction) when a force of 20 lb. is applied at a point on the arm 18 inches from the axis.

[*Ans.* 45216 lb.]

14. Describe a jack-screw and state one of its practical applications with which you are familiar. Neglecting weight and friction of the machine, find out an expression for the mechanical advantage.

A jack-screw having a pitch 0·25 inch is turned with a force of 50 lb.-wt. applied at the end of a hand 3 ft. from the axis of rotation of the screw. Calculate the load which the jack will be able to raise. (C. U. 1956)

[*Ans.* 45216 lb.-wt.]

15. What are the requisites of a good balance? You are given an inaccurate balance; explain how it can be used to obtain accurate results.

The only fault in a balance being the inequality in weights of the scale pans, what is the real weight of a body, which balances, 10 lb. when placed in one scale pan and 12 lb. when placed in the other? (All. 1929; Dac. 1933)

[*Ans.* 11 lb.]

16. What are the requisites of a good balance? A balance with unequal arms is used for weighing. The apparent weights of the same body when placed in the two pans are 158·0 and 158·25 gms. respectively. Find the ratio of the balance arms. (Dac. 1934; *cf.* Pat. 1923, '44; *cf.* All. 1946; C.U. 1950)

[*Ans.* $\sqrt{633}$; $\sqrt{632}$.]

17. Explain with a neat sketch the principle and construction of a physical balance. Why is the method of double weighing adopted in the case of an inaccurate balance? (C. U. 1930; All. 1946)

18. What are the requisites of a good balance? Explain clearly how you would proceed to determine the *true* weight of a body using a balance having unequal arms. (Utkal, 1944; A. B. 1952)

19. How would you determine whether the arms of a balance are of equal length, and how would you eliminate errors due to such an irregularity? (Pat. 1928)

20. A body is placed on the pan of a balance, whose arms are unequal and is found to weigh w_1 gm. It is then removed to the other pan and weighs w_2 gm. Show that the actual weight is $\sqrt{W_1 W_2}$ gm. (U. P. B. 1926, '55)

21. Explain why a balance which is sensitive cannot be stable. (P. U. 1952; *cf.* Guj. U. 1952; *cf.* Bomb. 1955)

22. A tradesman sells his articles weighing equal quantities alternately from the two pans of a balance having unequal arms. If the ratio of the lengths of the two arms be 1·025, what is his percentage loss or gain? (Pat. 1952)
[*Ans.* 0·06% loss of trader.]

23. A body *A* when placed successively in the pans of a faulty balance, appears to weigh 8 lb. and 18 lb.; another body *B* when treated in the same way appears to weigh $5\frac{1}{3}$ lb. and 12 lb. In what respect is the balance false, and what are the real weights of *A* and *B*? (Utkal, 1954)

[*Ans.* Arms unequal; ratio of arms $\frac{3}{2}$.]

24. The turning points of a balance were observed to be successively 13, 8, 11. With the body on the left pan and 24·82 gm. on the right pan, the turning points were 14, 9, 12. On adding 10 more milligrams to the weights, the turning points becomes 10, 5, 8. Calculate the correct weight of the body (Mysore, 1952)
[*Ans.* 24·822 gm.]

25 Sketch a common steelyard. Explain its underlying principle and show how it is graduated (Pat. 1928; E P. U. 1951)

26 Explain the construction and action of a railway platform balance used for weighing heavy parcels and luggages (All 1941; U. P. B 1942)

27. "In a common balance we compare masses of two bodies while from a spring balance we can get the true weight of a body." Explain (C U. 1927, '40, '47; Dac 1939)

28 Draw a neat diagram, showing the essential parts of a spring balance State why a spring balance gives different values for the weight of a body at different places, whereas a common balance gives a uniform value (C. U. 1952)

29. Explain why a very delicate spring balance would show a slight difference in the weight of a body at different place on the earth, though a common balance would give no indication of any difference (Pat 1920, '32; cf. C. U. 1920)

30 Define 'weight' and discuss, as fully as you can, the factors on which it depends Describe experiments to illustrate your answer. (Pat. 1931)

31 Describe experiments by which it can be shown that the mass of a body is proportional to its weight and explain carefully the reasoning by which this conclusion is drawn from the results of experiments.

What is meant, by the statement that weight is not an essential property of matter? (Pat 1932)

CHAPTER VIII

PROPERTIES OF MATTER

202. Constitution of Matter: Molecules and Atoms:—Any given kind of matter is now universally recognised as being made up of a very large number of extremely tiny pieces—pieces which are too small for an ordinary microscope to detect, these pieces are the smallest ones in which the mass of a body may be *sub-divided while still retaining the properties of the original substance* These unitary blocks of a substance are called its *molecules* Each kind of matter has its own distinctive molecule In other words, molecules of the same substance are always alike and those of different substances, different.

A molecule, again, is composed of still smaller particles, called *atoms* which are the *elementary particles* of chemical elements When a molecule is split up, the original matter loses its identity. The different kinds of known molecules, in the last analysis, have revealed the existence of only *ninety-two* distinctive chemical elements All of these have been experimentally isolated with the possible exception of two about which there is at present

some doubt. This material universe comprises an almost infinite variety of substances but the most striking feature is that every one of them, when analysed, is ultimately found to have been built up of any or some of these elements only. The atoms are incapable of free existence but by combining with each other form molecules which exist freely. When chemical reaction takes place between two substances, what happens is that the atoms of one substance combine with the atoms of the other to form the molecules of a new substance.

Formerly, the atoms were supposed to be indivisible and were regarded as the ultimate particles with which all matter is built up. Recent researches have, however, established the existence of particles far smaller than the atom, as *Electrons, Protons, Neutrons*, etc. (*vide* Chapter I, Part VI).

The distances or spaces between consecutive molecules of a body are known as *inter-molecular spaces*, which can decrease or increase producing a change in volume of the body. The inter-molecular spaces are not vacuous, but it is imagined that these spaces are filled with a subtle imponderable fluid, called the ether whose physical existence has, however, not yet been demonstrated.

The molecules of a body are held together in their positions firmly in the case of a solid and less so in a liquid, by their mutual force of attraction, known as *inter-molecular force* of attraction, while in the case of a gas, the inter-molecular force is almost negligible.

The molecules of a solid execute vibration about their mean positions of rest which cannot be easily altered, while in case of liquids they may be altered more easily. In the case of a gas the molecules are in random motion perpetually. The degree of motion in all the three states increases with the increase of temperature.

In a solid, the molecules are very closely packed and so the forces of cohesion far exceed the forces of repulsion and the result is that a solid acquires the ability to preserve a definite *shape* and *volume* and to put up a great resistance to any change in either of them; this explains why a solid possesses high *rigidity* as also high *bulk* (or *volume*) *elasticity* (*vide* Art. 218).

In a liquid, the molecules are packed less closely than in a solid, so cohesion is much smaller and is such that a liquid readily yields to any external force tending to change its shape. A liquid thus has *no definite shape* and takes the shape of the vessel in which it is placed But the cohesion of the molecules is still sufficiently big to enable the liquid to preserve its volume, and quite a large force, though relatively smaller than in the case of a solid, is necessary to change it even a little.

In a gas, the molecules are widely separated from each other so that inter-molecular attraction is almost absent and the molecules move about independent of each other, being only limited by the walls of the containing vessel. So it cannot preserve any definite *shape* or *volume*, but, by spreading out readily fills the whole of the containing vessel, for the same reason, it is incapable of offering any appreciable resistance to any change either in shape or volume.

Fluid.—Liquids and gases are usually classed together as fluids for they can *flow* readily

204. The Physical States and Temperature:—No physical state is permanent for a substance, for it can be made to pass from one state to another under suitable conditions Thus water which is a liquid at the ordinary room temperature, can be converted into solid ice by progressively cooling it, or can be changed into steam, a vapour, by applying heat to it A similar property is common to all substances, *i.e.* a liquid can be solidified by extracting heat from it and can be vaporised by adding heat to it. It is the *temperature* which determines the physical state of any particular substance and stands for the molecular motion in the substance. For a substance, relatively speaking, the gaseous state corresponds to the hottest, the solid state to the coldest and the liquid state to an intermediate temperature That is, with increase of temperature inter-molecular space increases That is accounted for by saying that molecular motion increases with increase of temperature.

205. Molecular Motion in the solid, liquid and gaseous States:—In the solid state, the motion is so restricted that a molecule can only vibrate about a mean position of rest which cannot be altered easily. The amplitude of this vibration increases with the rise of temperature and at a certain temperature, called the *melting point* of the solid, the motion becomes violent enough to enable a molecule

to break away from its confinement and to acquire a motion of translation. That is, at this state of energy it has not only vibratory motion but translatory motion too. In a liquid, a molecule has no mean position of rest and this accounts for the ability of a liquid to flow. At a still higher temperature, when the liquid boils, the molecular motion is so violent that the forces of cohesion cannot prevent the molecules from shooting away from the boundary of the liquid. This is vaporization of the liquid. The molecules at this stage acquire a violent translatory motion to limit which a container closed on all sides becomes necessary. By successive reflections from the boundary wall and mutual collisions, a very chaotic type of motion of the molecules results within the gas.

206. General Properties of Matter :— Certain properties are found to be common to all the three states of matter,—solid, liquid and gas, and are called the *general properties* of matter, while there are other properties which are peculiar to a particular physical state or states only and are referred to as *special properties* of a physical state.

Inertia.—Inanimate matter by itself cannot change its own state whether be it a state of rest at a given position, or configuration, or a state of motion in a straight line. It has no initiative of its own. This property is known as *inertia*. The *inertia* of a body is due to its mass.

Gravitation.—Every particle of matter in this universe attracts every other particle towards itself. The strength of this attraction between two particles is directly proportional to the product of the two interacting masses and inversely proportional to the square of the distance between them. The falling of a fruit to the earth when the former is detached from its stalk is due to the mutual attraction between the earth and the fruit. The mutual attraction between the moon and the ocean water causes the high and ebb tides. The earth rotates round the sun due to the mutual attraction between them.

Cohesion and Adhesion.—*Cohesion* is the force of attraction between molecules of the *same* kind, and *adhesion* is the force of attraction that exists between molecules of *different nature*. Cohesive force keeps the molecules together in a substance and adhesion is the cause of sticking together of two substances, *e.g.* wetting glass by water and other liquids, gluing wood to wood, 'tinning' metals with solder, etc. Cohesion holds together the particles of a crayon, but adhesion holds the chalk to the blackboard.

Impenetrability.—It is the property in virtue of which two bodies cannot occupy the same space at the same time. If a metal ball is immersed in a liquid, the liquid moves away to make room for the ball. When water is poured on sand, it seems, as if the former

penetrates into the latter, but in fact it only fills the pores between the particles of the sand.

Extension.—It is the property in virtue of which every body occupies some definite space. The space which a body occupies is called its volume. The volume may be changed due to changes in temperature, pressure, etc. but cannot be reduced to zero

Divisibility.—It is the property in virtue of which a material body can be sub-divided into extremely minute parts The physical processes of sub-division, such as hammering, sawing, rubbing, filing, etc. can no doubt reduce a lump of matter to a state of fine powder, but even at the last state of sub-division, the grains are very large compared to the molecules which compose them. By *an act of solution*, the particles are dispersed to much finer pieces. In the colloidal state of solution, the dispersion is lesser than in the state of *true solution* and the particles are within the range of vision through a powerful microscope, *viz.* a Zsigmondy's Ultramicroscope When the state of dispersion is such that a particle has a diameter of the order of 10^{-7} cm. or less, we term it a true solution Even greater sub-division of the particles of matter takes place, when a scent or perfume spreads out in air A rose smells for hours without any visible changes in mass, a bit of musk sends out its scent for years together, what unique processes of sub-division are taking place in nature!

Porosity.—All bodies contain *pores* more or less. The pores may be of two types sensible and physical In the case of solids and liquids, sensible pores are very large compared to the intermolecular spaces and so intermolecular forces cannot act across them They are spaces left between one cluster of molecules and another Physical pores are small enough for intermolecular forces to act across them The intermolecular spaces are these pores. Solids contain both types of these pores Often the physical pores are not regularly situated within a body. A piece of chalk, a heap of sand, our skin, earthenware pots, filter paper, leather, wood, sponge, etc. are some instances of very porous solids. The liquids are also porous. When a salt is slowly added to water, the volume of the latter does not increase by the act of solution. How are then the salt grains accommodated? They only go to fill the pores between liquid particles. These pores are the physical pores made up of the intermolecular spaces, and the molecules around such a pore are within the sphere of each other's action With increase of temperature, these spaces expand when more salt can go into solution

The intermolecular spaces in a gas are extreme cases of physical pores A gas can be easily compressed, and one gas easily and rapidly diffuses into another on account of these pores being very large.

Compressibility.—It is the property of a body in virtue of which it can be compressed so as to occupy a smaller volume by application of external pressures. Compression is possible because of the fact that all bodies contain pores. Gases are the most compressible; liquids are only slightly compressible; in the case of solids compressibility varies widely from solid to solid, namely, while rubber is very compressible, glass and diamond can hardly be compressed.

Density and Elasticity.—All material bodies must have some mass and the mass per unit volume of a body is called its *density*. So density is a universal property of all matter.

Elasticity is the property (which all matter possesses more or less in all the three states: solid, liquid and gas) in virtue of which a matter can offer resistance to a force or system of forces which produces a deformation of it either in shape or size or both and can regain its shape and size (if the deformation produced is within a limit for it) as the deforming force is withdrawn.

Both the density and elasticity are of such primary importance that it is claimed that all other properties of matter can be accounted for in terms of these two factors. So both of them have been separately dealt with in the following pages.

ELASTICITY

207. Elasticity :—It is an inherent general property of all kinds of matter: solids, liquids and gases. It is that property in virtue of which a body offers resistance to any change of its size or shape, or both, and can resume its original condition when the deforming force is removed.

A body resumes its original condition after the removal of the deforming force provided the deforming force does not exceed a certain maximum limit, called the *limit of elasticity* or *the elastic limit*. If it exceeds that limit, the body will not completely recover its original size or shape when the deforming force is removed. The force in this case is said to have exceeded the limit of elasticity.

208. Some common Terms used in connection with Elasticity :—

Strain.—When a force or a system of forces acting upon a body produces a relative displacement between its parts, a change in size or shape or in both may take place. The body is then said to be under *strain*. The strain produced in a body is measured in terms of the *change* in some measure of the body, such as its length, or volume and so on, divided by the total measure. Strain is thus the ratio of two like quantities, and is a pure number without dimensions and has no *unit* for it.

Stress.—When a body is strained, internal forces of reaction are automatically set up within the body, which act in the opposite direction, due to which the body tends to return to its original size and shape on withdrawal of the deforming forces. This restoring force is called the *stress*. It is numerically equal to the deforming force, according to Newton's law of reaction, as long as the strain produced is within the elastic limit.

The stress or a component of it, which acts normally to any section of a body, is called a **normal stress** to that section and that stress or a component which acts parallel to any section of the body, is called a tangential stress on that section

The stress intensity or simply the stress *is measured by the force per unit area of a section and, when uniform, is obtained by dividing the total force by the total area over which it acts.*

Perfectly Rigid Body.—A perfectly rigid body is defined to be such that no relative displacement between its parts takes place whatever force is externally applied to it No body is known to be perfectly rigid, though glass, steel, etc. are nearly so

Perfectly Elastic Body.—If a body perfectly recovers its original size and shape, when the deforming force acting on it is withdrawn, it is said to be *perfectly elastic* No such body is known for all values of stress A body, however, behaves as perfectly elastic, when the deforming force does not exceed a certain limiting (maximum) value, called the elastic limit of the body, whose value depends on the nature of the material of the body and the nature of the stress

Elastic Limit.—A body behaves as perfectly elastic only as long as the deforming force acting on it does not exceed a certain maximum value depending on the nature of the substance and the nature of the stress This limiting value of the stress is called the *elastic limit* of the material of the body for that type of stress.* A high elastic limit is possessed by steel and a low one by lead

209. Load-Extension Graph:—The elastic behaviour of a solid, particularly that of a metal, such as mild steel, when subjected to deforming forces ranging from low values to high values exceeding the elastic limit, is well illustrated by what is known as a *load-extension graph* as shown in Fig 118 In obtaining the experimental values for such a graph, a wire of the material may

* Various theories have been suggested as to when the elastic limit is reached by a body. Is it when the stress developed attains a limiting value or the strain a definite value for the substance, etc.?

be taken and elongated by hanging *weights* (called the *load*) from it, as in experiments described in Art. 219.

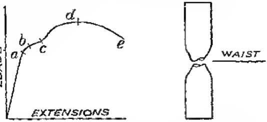


Fig. 118

On commencing to load the specimen and noting for each increment of load the extension (change in length from the original) produced, it is found that a straight line is obtained when the loads and extensions are plotted. This straight line continues up to a point *a* [Fig. 118, left], but beyond *a* the graph becomes slightly curved. Up to the point *a*, the load is proportional to the extension. The next point *b* marks the **elastic limit** for the material, and is so called because if we do not exceed *b*, the material shrinks back to its original length if the load is taken off, and the material has not lost its elastic properties even in the least. The points *a* and *b* may, for all practical purposes, be regarded as one and the same point, as indeed they sometimes are, unless very accurate tests are intended. Very soon after the point *b*, the elastic limit is exceeded, a considerable amount of extension of the material takes place even though the increase of the load on the specimen is quite small. The point *c* which marks the limit of this stage of the specimen is termed the **yield point**. It is often named the *commercial elastic limit* in commercial testing of materials. After passing the point *c*, the material seems to regain its strength somewhat, as it is found that further additions of load are required for further extensions to be produced. The **maximum or ultimate load**, from which the *ultimate stress* is calculated, is reached at the point *d*, and beyond this the specimen relieves itself of load by rapid stretching, whereby a *'waist'* (right diagram, Fig. 118) or *local contraction* develops at some part of the material where finally fracture occurs. The position of the material where it will occur is, however, unpredictable. The waist is quite pronounced in ductile materials, and small for brittle ones.

The part of the extension up to the elastic limit *b* is termed elastic deformation, and the remaining part from *b* to *e* is termed non-elastic or *plastic* deformation.

210. Factor of Safety:—Material with which machines and structures are made are often subjected to stresses, other than

normal stresses, which cannot be always predetermined. To avoid failure of structures, designers therefore choose as a measure of safety a *working stress* for the material, which is much below the *ultimate stress* corresponding to point *d* in Fig. 118, for the material, for design purposes The ratio $\frac{\text{ultimate stress}}{\text{working stress}}$ is termed the factor of safety The working stress so taken must also be less than the elastic limit stress so that no permanent deformation may take place in any part of the material.

211. Different Kinds of Strain :—

(1) Longitudinal (or Tensile) Strain.—When a body is acted on by a stretching (or compressive) force, the fractional *increase (or decrease) in the length* in the direction of the force is called longitudinal or tensile strain The corresponding stress is called longitudinal or tensile stress. Thus, if due to stretching or compression, l = the change in length of a body of length L,

$$\text{the longitudinal (or tensile) strain} = \frac{l}{L}.$$

It being a ratio of two lengths is a pure number having no unit for its measurement. *Only solids can have such strains*

Poisson's Ratio.— When a body is acted on by a stretching force, the extension in the direction of the applied force is always accompanied by a lateral contraction in all directions at right angles to the direction of applied force It is found that this lateral strain is proportional to the direct strain

$$i.e. \quad \frac{\text{lateral strain}}{\text{longitudinal strain}} = \sigma, \text{ a constant, called the Poisson's Ratio,}$$

whose value depends only on the nature of the material in question and not at all on stress applied provided it is within the elastic limit Poisson's Ratio for a stretching force is the same as that for a compressive force in which case there is *lateral expansion*

(2) Volume (or Bulk) Strain.— In such strains there is a change in volume only without any change in shape. This takes place when a body is subjected to a uniform pressure acting normally at every point on its surface The corresponding stress (force per unit area) is called the volume stress If V be the original volume of a body and v, the change produced in the volume,

$$\text{volume strain} = \frac{v}{V}$$

It is a pure number and has no unit for its measurement Volume strain, even for very large deforming forces, is small for solids and liquids, while in the case of gases even a very small force produces a very large volume strain.

(3) Shearing Strain (or Shear).— When the strain produced in a body is such that there is only change in shape or form of it but no change in volume, it is said to be a shearing strain or simply a *shear*. It is a *special property of solids* only because they only have a definite shape of their own.

Suppose a rectangular block, *ABCDEFGH*, [Fig. 118(*a*), left], of a solid has its bottom face *CDEH* fixed to a horizontal platform.

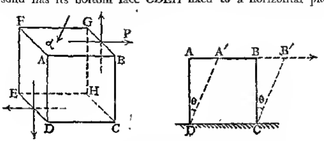


Fig. 118(*a*)

If a force *P* be now applied so as to act uniformly and tangentially over the face (area$=\alpha$) in the direction shown, this face (section *AB*) will be displaced, suppose, to the position represented by the section *A′B′* in Fig. 118(*a*), right, relative to the face *CDEH* represented by the section *CD*, the block assuming a rhombic form. The material of the block suffers a change in shape only without any change in volume. The strain produced in this case is a case of *shear* and is measured by the angle *ADA′* ($=\theta=$the angle *BCB′*), which is called the **angle of shear.** Let *AA′* be x and $AD=b$, then ($\because \theta$ is small),

$$\textit{shearing strain}=\theta=\tan\ \theta=\frac{x}{b}.$$

$$=\frac{\text{relative displacement of two planes of the body}}{\text{distance of separation of the two planes}}$$

$=$relative displacement for planes at unit distance apart $=$*displacement gradient.*

The corresponding stress, which is tangential to the surface is called the shearing stress and is given by P/α.

212. Hooke's Law:—This is the *basic* law of elasticity. It was established in 1678 by Robert Hooke of England.

In the original language the law was stated as *'ut tensio sic vis'*, which means that the stretching is proportional to the force producing it. The law is true for all cases of elastic deformations, provided the deformations are small. Some elastic deformations of different kinds in the case of solids, such as stretching, compressing, bending, twisting, etc. are illustrated below.

atmospheres are required to bring about a volume decrease of 0·1% in copper.

215. Steel more elastic than India-Rubber :— As stress/strain = modulus of elasticity, a large modulus of elasticity, for a body means that a large force is necessary, *i.e.*, a large stress is developed within the body, in order to produce a given strain in it. The modulus of elasticity is by far greater for steel than for India-rubber and so the stress developed, in order to produce a given strain, is far greater in steel than in India-rubber. In scientific definition, a body *A* is said to be more elastic than another body *B*, if the stress developed in the former is greater than that in the latter, when the same strain is produced in either. That being so, steel is far more elastic than India-rubber. For similar reasons, glass is more elastic than steel.

216. Verification of Hooke's Law :— Hooke's law may be easily verified in various ways of which one method is by spring-balance. By placing different weights on the pan and noting the corresponding elongations, a ***graph*** can be plotted with ***load*** and extension. The law will be verified, if the graph is a straight line [*vide Art* 100]. It should be noted that the elongation is proportional to the load within the elastic limit.

217. (i) Young's Modulus :— According to Hooke's law applied to longitudinal elasticity, longitudinal stress divided by longitudinal strain is a constant quantity for a solid within the elastic limit. This constant, which is the co-efficient of longitudinal (tensile) elasticity, is called Young's modulus in honour of Thomas Young of England.

Thus, if F be the force which acting in the direction of a length L of a wire of cross-section A stretches it by a small length l, then the stress=force per unit area $=F/A=F/\pi r^2$, where r is the radius of the wire, and the longitudinal strain = elongation per unit length $=l/L$.

$$\therefore \quad Y \text{ (Young's Modulus)} = \frac{F/\pi r^2}{l/L} = \frac{FL}{\pi r^2 l} \text{ dynes per sq. cm.}$$

(or lbs.-weight or tons-weight per sq. inch)

(ii) Bulk Modulus.— It is the co-efficient of bulk (*i.e.* volume) elasticity. If V be the volume of a body which is increased or diminished by an amount v when subjected to a uniform pressure p (stretching or compressive) acting from all sides on the body, the bulk (or volume) strain $=v/V$, and the bulk stress $=p$.

$$\therefore \quad \text{Bulk modulus} = p \div \frac{v}{V} = \frac{pV}{v} \quad \text{dynes/cm.}^2, \text{ or lbs.-weight}$$

or tons-weight per inch².

keep the wire taut and free from kinks. This may be called the initial load. Take the scale and the vernier reading at this load. This is the initial reading.

Then increase the load by ½ kgm., and again note the reading. Go on increasing the load by steps of ½ kgm., and note the reading for each load up to the maximum permissible load. This is the point beyond which Hooke's law does not hold. Now reduce the load by equal amounts (i.e. by ½ kgm.) till the initial load is reached, noting the reading in each case. Take the mean of the readings for increasing and decreasing load for each load. This mean gives the probable real reading corresponding to that load. The two sets of readings should closely agree, but if they differ appreciably, it is possible that the wire has been stretched beyond the elastic limit, in which case the experiment should be repeated with a new wire.

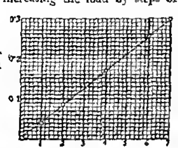


Fig. 122

Measure the diameter of the wire very accurately with a micrometer screw-gauge at several places along the length of the wire (from the point of support to the zero of the vernier) and in doing so readings should be taken twice in right-angled directions at each place. Find out the mean radius r from the above.

Tabulate the readings against the corresponding loads and find out the elongations for the various loads by subtracting the reading corresponding to each load from the initial reading. Then plot a curve with loads as abscissæ and the corresponding elongations as ordinates (Fig. 122). The graph should be a straight line passing through the origin, meaning thereby that the elongation is zero for zero load. Hooke's law is verified, if the graph is a straight line.

From the graph find out the elongation l corresponding to any suitable load, say m grams. Measure the length L of the unstretched experimental wire from the point of suspension up to the point where the vernier is attached. Then,

$$\text{Young's Modulus} = \frac{F/\pi r^2}{l/L} = \frac{mg/\pi r^2}{l/L} \text{ dynes/cm}^2.$$

Note.—(a) As the wires hang from the same support, any yield of the support will affect both the wires similarly and so there will be no relative motion of V over S due to this cause.

(*b*) As the two wires are made of the same material, any variation of temperature will affect both the wires by equal amounts and so the readings will not be affected.

(ii) Searle's Method.—This method and the 'Vernier Method' are exactly identical except in the process of measurement of the extension in length. A straight vernier is used in the 'Vernier Method' to measure the extension of the wire when stretched, while in Searle's apparatus a screw-gauge is adapted for the same purpose. The accuracy reached in this latter method is greater since a screw-gauge is more accurate than a straight vernier.

In Searle's apparatus (Fig. 123) each of the two wires, the *comparison wire A* and the *experimental wire B*, carries a brass rectangle from the lower ends of which weights can be hung. A spirit level *L* is put across from one rectangle to the other. It turns freely round a hinge *G* at one end and at the other rests on the point of an accurately cut vertical screw *C* of small pitch. working in the same vertical line as the experimental wire *B*. The screw carries at its lower end a cylindrical head *H* whose circular edge is uniformly divided and forms a circular scale. The pitch of the screw is usually 1 mm. and the circular scale is divided into 100 divisions. As the head of the screw is turned, the circular scale moves across a short vertical scale *S*. Thus, if the head is turned through 1 circular scale division, the point of the screw moves upwards or downwards through 0·01 mm.

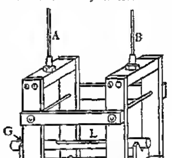


Fig. 123—Searle's Apparatus.

The method of use of the apparatus is as follows: The air-bubble in the spirit level is brought to the middle of it by turning the cylindrical head and a reading is taken by the help of the linear and the circular scales. Now, when a given weight is hung from the experimental wire *B*, it extends downwards and the spirit level will be disturbed. Slowly and carefully the screw is turned to raise its point till finally the bubble is brought to the middle again. The number of circular scale divisions through which the head of the screw is turned, is used to calculate the extension of the wire under a given load.

Examples. (1) *A rubber cord of 0·2 cm. radius is loaded with 13 kgms. weight. Length of 50 cms is found to be extended to 51 cms. Calculate the Young's modulus of rubber.*

Here the pulling force $F = 13 \times 1000 \times 981 = 12{,}753{,}000$ dynes

Stress $= 12{,}753{,}000 \div \pi \times (0{\cdot}2)^2$; Strain $= (51 - 50) \div 50 = 0{\cdot}02$;

$$\therefore Y = \frac{12{,}753{,}000}{\pi \times (0{\cdot}2)^2 \times 0{\cdot}02} \text{ dynes per sq. cm} = 0{\cdot}5 \times 10^{10} \text{ dynes per sq. cm.}$$

(2) *A mass of 20 kgms. is suspended from a vertical wire 600·5 cms. long and 1 sq. mm. in cross section. When the load is removed the wire is found to be shortened by 0·5 cm. Find Young's modulus for the material of the wire.* (C. U. 1933)

Pulling force $F = 20 \times 1000 \times 981$ dynes.

$\therefore$ Stress $= F/$ (area of cross section) $= (20 \times 1000 \times 981) \div (0{\cdot}01)$

$= 1{\cdot}962 \times 10^{9}$ dynes per sq. cm

Strain $= l/L = \dfrac{0{\cdot}5}{(600{\cdot}5 - 0{\cdot}5)} = \dfrac{1}{1200}$, $\therefore$ Young's modulus $Y = \dfrac{1{\cdot}962 \times 10^{9}}{1/1200}$

$= 1200 \times 1{\cdot}962 \times 10^{9} = 2{\cdot}3544 \times 10^{12}$ dynes per sq. cm

220. Properties peculiar to Solids :—

Ductility.—It is the property of a solid in virtue of which it can be drawn into fine wires, the finer such wires can be, the greater is the ductility of the material. Quartz and platinum are so ductile that wires having diameter as small as 0·01 mm can be drawn out of them. Ductility increases with temperature.

Malleability.—It is the property in virtue of which metals can be hammered into thin leaves. Gold, silver, lead, etc. are good instances of malleable substances. Lead is a malleable metal but not ductile, as it cannot be drawn into fine wires.

Of all pure metals, gold is the most malleable, so much so that even 30,000 such leaves to the inch can be had. Solids become more malleable when hot and this makes the rolling of metals into sheets possible.

Rigidity.—It is the property of a solid in virtue of which it can resist externally impressed forces tending to change its shape. So by virtue of this quality a solid keeps its own form, unless subjected to a force exceeding its elastic limit. Rigidity of a solid decreases with the increase of temperature.

Tenacity.—It is the property of a solid in virtue of which, when in the form of wires, it can support a weight without breaking. The weight required to break a wire is called its *breaking weight*, and is the measure of the *tenacity* or *tensile strength* of the material of the wire.

Wrought-iron has more tenacity than cast iron; and the steel pianoforte wire is the most tenacious of the three.

Hardness.—It is the property of a solid in virtue of which it offers resistance to being scratched by others. Diamond is the hardest known substance, while glass and steel are harder than many substances. The hardness of a solid decreases with increase of temperature.

There is no meaning for the absolute hardness of a substance. It is a relative property. A scale of hardness can be prepared by arranging all solids according to their relative hardness; one such scale is **Mohr's scale of hardness,** in which a solid anywhere in the scale is more hard than that follows it, but less hard than that precedes it.

Hardness must arise from the elastic property of a solid, which, it must be noted, varies considerably with the previous *'history of the material'*. Take, for instance, *'tempered steel'* which is nothing but ordinary carbon steel only heated to redness and then *suddenly* cooled by plunging into water. The process is called **tempering,** due to which steel becomes harder though a little more brittle too. But if the red-hot metal is allowed to cool *slowly,* the metal becomes much softer, the technique being called, **'annealing'.**

Hardness Testing Machine.— One method of testing the hardness of a material is by the **Brinnel Hardness Tester,** in which a steel ball is pressed under a given force upon a plane surface of the material whereby a depression in the form of a cup is formed in the body. The area of indentation is taken as a measure of the hardness of the material.

Brittleness.– It is the property of a solid in virtue of which it can be broken into pieces by mechanical shocks such as by stroking, hammering, etc. Glass, Porcelain, China clay, though very hard, are brittle. All solids are not brittle but very hard solids generally are. By tempering, when solids are hardened, their brittleness increases too. Thus tempered steel, though very hard, is brittle. Molten glass beads when suddenly cooled by water are rendered so brittle that they can be crushed to fine powder at the mildest blow. **Ruperts drops** are such drops. To reduce the brittleness of substances like glass, steel, etc. annealing, which is a process of slow cooling, is adopted. Shock-proof glasswares are now-a-days manufactured by allowing molten glass to cool very slowly over days together.

221. Properties peculiar to Fluids:—

Diffussion. *The phenomenon of inter-mixing of two liquids or two gases, sometimes even in opposition to gravity, is called diffusion.*

Expt. 1.—Keep a strong potash permanganate solution (a coloured liquid), at the bottom of a glass-tumbler. Pour water (by the sides of the tumbler) slowly and carefully, without disturbing the solution. It will be observed that the coloured solution, though heavier than water, gradually works up and finally after some time, spreads out uniformly throughout the whole mass.

Similarly, if a few crystals of copper sulphate are placed at the bottom of a glass-tumbler filled with water, the characteristic blue colour of the sulphate is observed to rise slowly, showing the diffusion of the sulphate molecules upwards. The phenomenon cannot be argued to be due to buoyancy for the sulphate is heavier than water.

Expt. 2.—Take a jar filled with a light gas like hydrogen, the jar being closed by a lid. Take another jar filled with a heavy gas like carbon-dioxide, also closed by a lid. Invert the former jar over the latter and take away the lids. Wait for sometime and it will be found that an intimate mixture of the two gases has been formed, as may be proved by analysis. The travel of the hydrogen molecules downwards and of the heavy carbon-dioxide molecules upwards are contrary to the principle of gravity and are characteristic of diffusion.

222. A Simple Explanation of Diffusion :— Molecules of all fluids, according to the Kinetic Theory (*vide* Chapter IV, Part II) are in spontaneous motion in all possible directions irrespective of gravity and as such inter-mixture between two fluids can take place spontaneously. In gases the molecular motion is much more vigorous than in liquids and so gaseous diffusion is much quicker than liquid diffusion.

223. Viscosity :—A rod held fixed at one end and twisted by the other can resume its original shape when the turning force (which is a tangential force and is called a shearing force), is withdrawn. This is possible due to an inherent property of a solid known as its rigidity. All solids can stand shearing forces, though up to a maximum limiting value only.

But this limiting value is very high for the solids. The fluid differ in this respect from the solids. They cannot stand any shearing forces and so they have no definite shape of their own. This difference in behaviour arises from the fact that the inter-acting forces between molecules, known as cohesive forces, in a liquid are very much less than in a solid, while they are negligible in the case of gases.

If water in a pot, after stirring, is left for sometime, the motion of the water subsides. This is a very common observation. What stops the motion? An enquiry into the question reveals that in a fluid, whether liquid or gas, when any relative motion between parts of the fluid is caused, internal forces are set up in the fluid which oppose the relative motion between the parts in the same way as the forces of friction operate when a block of wood is dragged along the ground. In short, *a less quickly moving layer of the fluid exerts a retarding force on the more quickly moving layer*, the motion of the latter being thereby reduced while that of the former accelerated and in this way the relative motion between the two gradually decreases till finally it stops altogether with respect to the walls of the stationary pot in which the water is contained.

The property in virtue of which retarding forces are called into play within a fluid when any relative motion between its parts occurs, is an inherent property of all fluids, differing only in degree from one fluid to another, and is termed the **viscosity of fluids.**

Co-efficient of Viscosity.— A measure of the viscosity of a fluid is given by what is termed the co-efficient of viscosity. *The co-efficient of viscosity is defined as that tangential force applied per unit area which will maintain a unit relative velocity between layers of a fluid at unit distance apart.*

Consider two layers AB and CD in a fluid, distant d apart (Fig. 124), both moving forward in steady motion such that while still remaining parallel to each other, the layer AB moves faster than the layer CD, the former having a relative velocity v with respect to the latter. A driving force, parallel to AB and directed from A to B, will be necessary to maintain this flow. On account of the viscosity of the fluid, the driving force acting on AB will be opposed by force F, called the viscous force, exerted by the layer CD on AB. The layer AB will also exert a force F on CD tending to accelerate the motion of the latter. According to Newton,

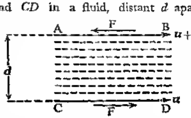


Fig. 124

the stress $\frac{F}{a}$, where a=area of either layer, is proportional to the velocity gradient, $\frac{v}{d}$ (*i.e.* the relative velocity per unit distance), or $F/a=\eta\frac{v}{d}$, where η is a constant called the co-efficient of viscosity of the fluid whose value depends on the nature of the fluid and its temperature.

That is, the viscosity co-efficient, $\eta=\frac{F/a}{v/d}$ =viscous stress intensity per unit relative velocity.

224. Viscosity is a Relative Term :— When water is poured into a funnel, it runs out quickly but glycerine or thick oil does so slowly and treacle much more slowly. Ordinarily, the liquids like water which flow readily are termed *mobile*, while those of the treacle type which do not flow so readily are termed *viscous*. This does not mean that water has no viscosity. Its viscosity is only small. For treacle, it is very much greater. That is, these terms are used in our common language in the relative sense. It should be remembered

that the gases, as a class, are much less viscous than the liquids, while a particular gas may be more viscous than another.

Viscosity and Kinetic Friction.—They have a good deal of similarity between them though they differ in one important respect. The latter is independent of the magnitude of the relative motion of the contacting bodies, while the former is not. The forces due to viscosity are proportional to the relative motion upon a velocity, called the **critical velocity** (whose value is fixed for a fluid) according to Prof. Osborne Reynolds (*vide* Art. 226)

Viscosity may be regarded as Fugitive Elasticity.—A liquid may be regarded as capable of exerting and sustaining a certain amount of shearing stress (which is quite small) for a short time after which the shear breaks down only to appear again. The idea is due to Maxwell. He regards viscosity as the limiting case of an elastic solid when the material of the solid breaks down under shear. It is from this standpoint that viscosity is often referred to as fugitive elasticity.

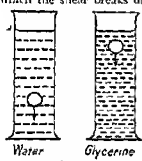


Fig. 125

225. Demonstration of Viscosity:—

(1) For a Liquid.—Two identical weights are dropped at the same time, one into water and the other into glycerine (Fig. 125). In water the weight descends more quickly than in glycerine showing that the viscous drag in glycerine is greater than in water.

(2) For a Gas.—A card-board disc *A* suspended from a rigid support by a thread attached at its centre is held in air just above but not touching a wooden disc *B* as shown in Fig. 126(*a*). When the disc *B* is rapidly rotated, the upper disc also is urged into rotation in the same direction as that of the lower disc. This is possible only because air has viscosity.

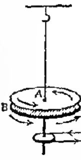


Fig. 126(*a*)

226. Stream-line Motion and Turbulent Motion:—If the path of every moving particle of a fluid coincides with the line of motion of the fluid as a whole [Fig. 126(*b*), left], the motion is said to be a *stream-line motion.*

If the motion of the particles of a fluid are disorderly, *i.e.* in directions also other than the line of motion of the fluid as a whole, the motion is said to be *turbulent* [Fig. 126(*b*), right]

In stream-line motion, under a given pressure gradient the flow of a liquid through a narrow tube is decided mainly by its viscosity, whereas in turbulent motion

it is solely governed by its density and very little by viscosity. According to Prof. Osborne Reynolds the motion of a fluid changes from stream-line motion to turbulent motion, if a certain velocity, whose value is fixed for it for a given temperature, called its *critical velocity*, is exceeded.

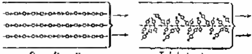


Fig. 126(*b*)

227. Nature of Flow of a Liquid in some important Cases:—

(i) Slow Steady Flow of Water in a River.—Slow steady flow here means stream-line motion. In such motion, it is found from flow-measurements that the speed of motion is maximum (v_m) at the top surface of the water in the river and reduces gradually with the depth below, and finally to almost zero speed v_0 at the bottom or bed of the river (Fig. 127). The idea is that the whole mass of moving water may be taken as consisting of a very large number of thin parallel layers in which each upper layer slides over that below it. Because of the adhesive forces, the rigid bed of the river almost prevents the bottommost layer of the water from moving; this almost stationary layer owing to interacting forces, called cohesive forces, tries to hold back the layer above it with a force which is less than in the previous layer. This layer again tries to hold back the layer above it; all the way up the resistance to motion of a layer diminishes. So the speed of motion of the liquid is maximum at the top and reduces downwards. The mechanism of flow as stated above shows how viscosity actually acts in determining the type of flow.

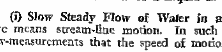


Fig. 127

(ii) Flow of a Liquid through a Narrow Tube.— In stream-line motion of a liquid through a narrow tube (Fig. 128), the speed of flow is maximum (v_m) along the axis of the tube and reduces gradually radially outwards, falling almost to zero at the wall of the tube. Here the whole cylindrical mass of the moving liquid may be thought of as consisting of a very large number, of successive thin cylinders co-axial with the tube, the cylinders in contact

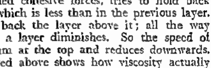


Fig. 128

with the wall being held almost stationary by adhesive forces while each inner cylinder continually slipping away relative to the one just outside it more and more quickly as it nears the axis of the tube.

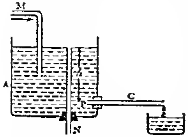


Fig. 129

228. Determination of the Co-efficient of Viscosity of Water :—

Poiseuille's Method.— A long capillary tube C, say of a length l, and internal radius r, is fixed horizontally at a depth h below the level of water in a large vessel A (Fig. 129). The level of water in the vessel A is kept constant by the inlet pipe M which supplies water to it from a raised reservoir and the outlet pipe N which drains out the excess water when the level exceeds its top. If the velocity of flow does not exceed the *critical velocity*, the volume V of water collected at the end of the capillary tube in t seconds will be given by the following equation due to Poiseuille.

$V=\frac{\pi p r^4}{8 l \eta}$, where η = co-efficient of viscosity of water, p = hydrostatic pressure at the level of the capillary tube = $h\rho g$, where ρ = density of water.

229. Practical Importance of Viscosity :— The nature of the viscous resistance offered by sea-water to a ship in motion, that of air to a car or aeroplane in motion etc. are important factors governing the design of such crafts. The quality of the fountain pen ink depends largely on its viscosity. Viscosity of lubricants is a decisive factor in its use. The normal circulation of blood through our veins and arteries is dependent on the viscosity of the blood. Thus, viscosity plays a very important part in various ways.

230. Properties peculiar to Liquids :—

Osmosis.— The process of diffusion is strikingly modified if two liquids are separated from each other by certain membranes. The following simple experiments will illustrate this fact :—

Expt. I.— A pig's bladder is filled with alcohol and placed in water. The bladder gradually swells in size and finally bursts. Conversely, if the bladder is filled with water and placed in alcohol, the volume of liquid in the bladder gradually decreases.

Expt. II.—Suppose a thistle funnel, F (Fig. 130), has its wide lower end closed with a parchment paper and is immersed, parchment down into water contained in a bowl. A strong solution of sugar in water is poured into the tube until the liquids stand at the same level both inside and outside. Wait for some time when it will be found that the liquid level in the tube rises and stands at some higher level. A Traube's Copper-ferrocyanide membrane acts better than parchment paper as a partition wall in similar experiments.

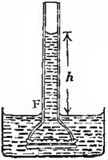


Fig. 130

What happens, in these experiments is as follows: In Expt. I, a pig's bladder is such that water molecules can pass through it while those of alcohol cannot. In Expt. II, a parchment paper is such that molecules of water can pass through it while those of sugar cannot. *A membrane acting in the above way, i.e. transmitting one type of molecules while stopping another when used as a partition wall between the two, is called a semi-permeable membrane and the one-way diffusion, that can take place through it, is known as osmosis.*

In Expt. II, the water molecules hit the parchment paper on both of its sides, but the number of hitting on the solution side is smaller by the number of the sugar molecules present. The result on the whole is that the water-level rises in the tube.

The excess pressure, at equilibrium, corresponding to the difference in level h between the liquid level inside the tube and that of the water outside, is called the osmotic pressure of the solute in the solvent and depends on the concentration of the solution and also on the temperature. The inward diffusion of the water in Expt. II can be stopped, if the solution in the tube F is subjected to a downward pressure, say by a piston; the pressure, so exerted on the solution side just sufficient to prevent osmosis, will be a true measure of the osmotic pressure of the solution, for in this case the concentration of the solution will not change due to dilution by diffused water. Incidentally, osmotic pressure is not an absolute pressure exerted by any component but is only a difference of pressure which must be maintained between two liquids separated by a semi-permeable membrane such that the escaping tendency of any of them into the other may just be balanced.

Pfeffer, Van't Hoff, Earl of Berkeley, Hartley and others made important studies on the osmotic pressure of a solution, and laws are now available from their work, which govern the osmotic pressure, volume and temperature of a solution. It is found that the mole-

of the large cohesive forces which bind the molecules. What happens then is that the surface becomes depressed until the resultant upward force ($-W$) due to the surface tension T acting as shown in the figure is equal to the downward force W due to the weight of the needle.

The phenomena of insects walking and running on the surface of liquids are also possible due to similar reasons.

(2) Spreading of Oil on Water.—Take a little oil, mustard seed or preferably kerosene, and drop it on water. It is pulled in all directions until it spreads over the entire surface. This is because the suface tension of oil is much less than that of water; the greater tension of the water stretches the oil in all directions.

Take some camphor shavings and simply put them on a water surface. They are smartly turned or moved hither and thither in different directions. The fact is that at each pointed end each flake readily goes into solution in the water and this reduces the surface tension at that end more than at any other, resulting in a motion of the flake.

(3) Soap-bubble.—Force air into a soap-bubble carefully when it will expand. Remove the mouth from the pipe-end, the bubble will contract forcing the gas out. This happens because due to surface tension the surface of a liquid behaves as a stretched membrane having a tendency to contract.

(4) Camel Hair Brush Expt.—Dip a camel hair brush into a liquid. When the brush is taken out, the hairs are all found to be drawn together as if the hairs are now connected by a stretched membrane.

235. Spherical Shapes of Liquid Drops:—On account of surface tension the skin of a liquid tends to contract in area and to attain a shape in which the exposed area is minimum for a given volume, *i.e.* it takes on a spherical shape, for, a sphere has the least surface area for a given volume. The effect of gravity on a liquid is to make the liquid flat, for, at this condition the centre of gravity of the liquid will be at the lowest. In small masses of liquids, usually the effect of the surface tension predominates over that of gravity, while in large masses the effect is the reverse. The spherical shape of soap-bubbles, rain-drops, etc. illustrates the effects of surface tension in small masses of liquids, while in tanks and ponds the water assumes a flat surface illustrating the effect of gravity. Because of its large surface tension, mercury, when spilt on a floor, takes on the shape of small pellets in defiance of gravity.

236. Part played by Cohesion and Adhesion:—When the mutual attraction between the molecules of a liquid (*cohesion*) contained in a vessel is less than their attraction to the sides (*adhesion*), the liquid wets the side of the vessel as in the case of water in a glass-vessel, but if the attraction of adhesion is less than that of cohesion, as with *mercury* in a glass-vessel, the liquid does not wet

glass; so mercury sprinkled on a glass surface separates out into spherical drops, whereas water or oil easily spreads over a glass surface.

237. The Angle of Contact:—When a plate is plunged vertically in a liquid, the liquid is drawn a little up the wall when the liquid wets it, as in the case of water, alcohol, copper sulphate solution, ether, etc. (Fig. 133, left), while the liquid is depressed a little when it does not wet the wall as in the case of mercury etc. (Fig. 133, right). The section of the liquid surface near the plate is a continuous curve and is known as the *capillary curve*. Consider a point C where the capillary curve meets the solid surface. The angle ACB in the liquid, which AC, the tangent to the capillary curve at C, makes with the solid surface BC, is called the *angle of contact* between the liquid and the solid. It is an *acute angle* when the liquid wets the solid and is *obtuse* when the liquid does not wet it (Fig. 133). The angle of contact of water with glass in air is very small and can be taken as zero.

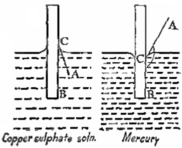
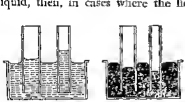


Fig. 133

238. Surface Tensions at 20°C., and Angles of Contact (Liquid—glass)

Liquid	S. T. (dynes/cm.)	Angle of Contact
Water-air	73·0	8° to 9°
Soap solution-air	30 (approx.)	...
Paraffin oil-air	26·4	26°
Mercury-air	465·0	130° (approx.)

239. Capillarity:—If a glass tube of small bore is dipped in a liquid, then, in cases where the liquid wets glass, as in the case with water, the internal level of the liquid will be higher than the level outside [Fig. 134(a)] but with mercury, which does not wet glass, the interior surface is *below* the exterior surface [Fig. 134 (b)]. The surface in the case of water in glass is *concave* upwards, but for mercury in glass, it is *convex* upwards.


(a) (b)
Fig. 134

These results are said to be due to what is known as *capillarity*, which is a consequence of surface tension of the liquid and smallness of the bore of the tube. It arises out of the fact that the molecular attraction of glass for water, *i.e.* the force of adhesion between the solid and the liquid, is greater than the attraction (*i.e.* the force of cohesion) of water for water, and that the force of adhesion between glass and mercury is less than the force of cohesion between mercury and mercury. The elevation or depression of the liquid in the tube is inversely proportional to the diameter of the tube; so the capillary effect can be clearly shown only in the case of very narrow tubes, called **capillary tubes.**

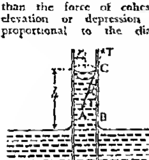


Fig 135

The rise of oil in wicks of lamps, the soaking up of ink by blotting paper, the retaining of water in a piece of sponge, the rapid absorption of liquid by a lump of sugar, the wetting of a towel when one end of it is allowed to stand in water, are all *instances of capillarity*

240. Height of a Liquid (Capillary Rise of the Liquid) in a Tube:—Let a capillary tube of radius r be dipped into the liquid and the liquid rises in the tube until it stands at a height (Fig. 135). The surface of the column at the top assumes the shape of a spherical cup with its concavity turned upwards. Let the height of the column be h measured from the level of the surface of the liquid outside the tube up to the lower meniscus of the cup. Let the angle of contact ($\angle ACB$) between the liquid and the wall be α. A force T due to surface tension acts along the tangent to the liquid surface in the direction CA at each point of contact C of the liquid with the wall. According to Newton's third law, this force sets up an equal reaction in the opposite direction, as shown by the dotted line. The component of this reaction in the vertically upward direction $= T\cos\alpha$. Since the liquid surface in the tube makes a circle of contact with the wall of the tube, the total vertical force upwards $= 2\pi r \times (T\cos\alpha)$. This force lifts up the liquid in the tube. The mass of the raised liquid in the position of equilibrium $= \{(h+r)\pi r^2 - \frac{\frac{4}{3}\pi r^3}{2}\}\rho$, where ρ = density of the liquid. For equilibrium,

$$2\pi r\, T\cos\alpha = \{(h+r)\pi r^2 - \frac{\frac{4}{3}\pi r^3}{2}\}\rho g = \pi r^2(h + \frac{r}{3})\rho g,$$ where g = acceleration due to gravity.

$$\therefore \quad T = \frac{r.\rho.g\left(h + \frac{r}{3}\right)}{2 \cos \alpha}$$

For water, alcohol, chloroform, etc. $\alpha = 0$, approximately. Neglecting $r/3$ compared to h,

$$T = \frac{r.\rho g.h}{2} \text{ , approximately} \quad \ldots \quad \ldots \quad (1)$$

241. Jurin's Law :— The elevation or depression of a liquid in a capillary tube is inversely proportional to the radius of the tube at the place of contact. This is known as Jurin's Law of capillarity. This at once follows from equation (1) above, for T, ρ and g are constants for a given liquid at a given place, *i.e.* according to Jurin's Law, $h \times r =$ constant for a given liquid at a given place.

242. Robert Hooke (1635—1703) :— An Oxonian experimental physicist. For some years he was a research assistant to Robert Boyle. He had a remarkable talent at Mechanics and Drawing. His principal work in Physics relates to the wave-theory of light, universal gravitation, atmospheric pressure, and elasticity of solids. *"Ut tensio sic vis"*—the basic law of elasticity bears his name. We owe to him the first balance wheel of the watch. In 1662 when the Royal Society was formed he was appointed "Curator of Experiments" and became its secretary in 1677. His researches cover a wide range of subjects but he concentrated on few of them. He was temperamentally irritant and made virulent attacks on many contemporary scientists, including Newton, alleging that many works published by them were due to him.


Thomas Young

243. Thomas Young (1773—1829) :— An English scientist and linguist. He successfully deciphered many Egyptian inscriptions. He studied medicine extensively and acted as Professor of Physics at the Royal Institution. His main works relate to medicine, the wave-theory of light, contribution to mechanics of solids, and mechanism of sight and vision.

Questions

1. State Hooke's law and explain what is meant by *stress, strain, and coefficient of elasticity*. Classify the various types of strain and write down the names of the corresponding coefficients of elasticity. (Gau. 1955)

2. Upon what factors does the stretch of a wire depend? Can you connect them by a law? What do you mean by elongation, Young's modulus, and tensile strength? How would you determine Young's modulus for a steel wire?

3. Find the stretching force on a steel wire 2 metres long, 1 mm. in diameter, when it is stretched by 1 mm.
(Young's modulus for steel $= 2 \times 10^{12}$ dynes/cm^2)
[*Ans.* 7.85×10^{6} dynes.] (C. U. 1957)

4. A copper wire 2 metres long and 0.5 mm. in diameter supports a mass of 3 kgm. It is stretched 2.38 mm.
Calculate Young's modulus. (Poona, 1954)
[*Ans.* 12.6×10^{11} dynes/cm.^{2}]

5. A force of 100 kgm. is exerted on a piston sliding in a tube filled with water. The column of water compressed by the piston is 2 metres long and 1 cm. in diameter. How far does the piston move in compressing the water?
[*Ans.* 1.32 cm.]

6. What force is required to stretch a steel wire of 1 sq. cm. cross section to double its length? Young's modulus of steel $= 2 \times 10^{11}$ dynes/cm^2.
(U. P. B. 1942)
[*Ans.* 2×10^{11} dynes]

Discuss the practicability of the above in the light of the *load extension* graph.

7. Tell how you may, by the use of Hooke's Law and a 20 lb. weight make the scale for a 32 lbs. spring balance. (C. U. 1936)

8. A wire of 0.4 cm. diameter is loaded with 25 kgms.-wt. A length of 100 cms. is found to be extended to 102 cms. Calculate the Young's Modulus of the wire. (All. 1946; C. U. 1963)
[*Ans.* 9×10^{9} dynes per sq. cm.]

9. Calculate the depression of a mercury column in a glass tube whose inner diameter is 0.058 cm.
[*Ans.* 1.55 cm.] (s. t. of mercury $=465$ dynes/cm.)

10. How high does water rise in Capillary glass tube whose inner diameter is 0.044 cm., if the angle of contact is negligible?
[*Ans.* 6.7 cm.] (s. t. of water $=73$ dynes/cm.)

CHAPTER IX

HYDROSTATICS

PRESSURE IN LIQUIDS

244. Hydrostatics:—*Hydrostatics* deals with liquids at rest under the action of forces within them or on the sides of the containing vessel, and the phenomena that arise out of them.

A perfect liquid is a substance which has no shape of its own and takes up the shape of the containing vessel. It is absolutely incompressible and is incapable of offering any external or internal friction. No such liquid, which fulfills the theoretical considerations completely, is actually known. But in *hydrostatics* whenever liquid is referred to, it is taken as a perfect liquid.

245. Liquid Pressure:—A liquid contained in a vessel always exerts pressure on the walls and on the bottom of the vessel. The existence of liquid pressure can be known from the following simple observation: Take a vessel with a hole on its wall and pour some liquid into it. The liquid will flow out through the hole when the former reaches the level of the latter. To stop the outflow a thin plate of equal area may be put on the hole. The plate will remain at rest only when some force from outside is applied to it. This shows that a liquid exerts pressure on the wall of the container.

Jets of water that squirt out from water pipes in the municipal streets from holes in the pipe walls are due to liquid pressure.

Pressure at a Point in a Liquid.—*Pressure at* a point in a liquid is the *thrust (force) exerted by the liquid per unit area* surrounding the point. That is, *pressure* $P = \frac{\textit{total force}}{\textit{total area}}$, which is the same as *the force per unit area.*

Consider a cylindrical column of liquid of height h, the area of cross-section of the cylinder being A (Fig. 136). The weight of this column of liquid is the total thrust upon the base. Therefore the total thrust upon the base $= A\,h\,\rho\,g$, where ρ (pronounced *'rho'*)=density of the liquid, and g=acc. due to gravity at the place. $\therefore$ Pressure exerted by the liquid column $= \frac{A\,h\,\rho\,g}{A} = h\,\rho\,g$. That is, *the pressure at a point in a liquid is proportional to its depth*, ρ and g being constants.

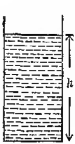


Fig. 136

If the face of the membrane of the thistle funnel be turned to different directions, upwards, downwards, sideways, etc., the mean depth of it being not altered, the index will still remain stationary, showing *equality of liquid pressure in all directions at the same level.*

246. The Free Surface of a Liquid at rest is always horizontal :— (i) If possible let the surface $A'B'$ be not horizontal (Fig. 139). Consider two points A and B in the liquid at the same horizontal level vertically below the points A' and B'.

The pressure at A due to the liquid is $P+\rho gh_1$, and that at B is $P+\rho gh_2$, where ρ is the density of the liquid, and h_1 and h_2 are the depths of the liquid at A and B respectively, and P, the atmospheric pressure. Since h_2 is greater than h_1, the pressure at B is greater than that at A. So the liquid particles will move from B towards A and equilibrium will be lost. This flow in the direction B to A will continue as long as the pressure at B and A are not equalised. That is, for equilibrium the pressure at B must be equal to that at A and so h_1 must be equal to h_2. Since B and A are on the same horizontal plane, B' and A' must also be on another horizontal plane at an upper level. That is, the free surface of a liquid at rest must be horizontal.

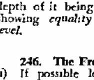


Fig. 139

(ii) From the above principle it follows also that if a liquid be poured into a series of connected vessels of varied shapes, the liquid, when at rest, will stand at the same level in all the vessels (Fig. 140). So in a tea-pot tea stands at the same level in the spout as in the vessel itself. This is commonly expressed by saying that **in a communicating vessel a liquid finds its own level everywhere.**


Fig. 140

(iii) If several liquids which do not mix with one another are placed in the same vessel, they will arrange themselves one above another in the order of their densities, the heaviest of them being at the bottom and the lightest at the top. It will be found that the surface of separation is horizontal between any two of them.

In Fig. 140(*a*), a tall jar is seen containing three liquids: mercury, water and kerosene in steady equilibrium. Mercury being the heaviest occupies the lowest position, and kerosene being the lightest occupies the topmost position, water going in between. That is, *liquids at rest contained in a vessel lie in the order of decreasing density from the bottom upwards, the surface of separation between any two of them being always horizontal.*

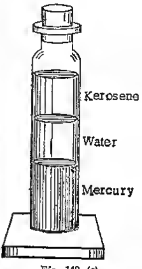


Fig. 140 (*a*)

247. Some Illustrations of Equilibrium of Liquids:—

(1) The Spirit Level.—The instrument is based on the principle explained in Art. 246 and is used to test whether a surface is horizontal or not. It consists of a slightly curved glass tube filled with alcohol, except for a small bubble of air, which naturally occupies the highest part of the tube (Fig. 141). This glass tube is fixed in a brass mount. The air-bubble occupies exactly the middle position of the tube if the instrument is placed on a perfectly horizontal surface, and the bubble will move to a different position if the surface is not horizontal.


Fig. 141—A Spirit Level.

(2) City Water Supply.—The principle that water, or any other liquids, finds its own level everywhere in connected vessels is applied in supplying water to the different houses of a city. In order that every house may have an adequate supply of water at a considerable pressure, water obtained from the source of supply, say a river or a well, is pumped up by suitable pumps to a large reservoir placed at the highest place in the neighbourhood, or on lofty water towers specially erected for the purpose. The water from the reservoir is carried to different sites by means of water-mains and branch-pipes. The pressure of the water supply depends upon the vertical height—called the **"head of water"**—of the water surface in the

reservoir above the point of supply. Therefore water supply should be available up to a height equal to that of the reservoir. In practice, however, the water does not rise as high as the water surface in the reservoir. This is due to loss of pressure on account of internal friction within the pipes.

For water supply in Calcutta, water is filtered and stored in tanks at a height of about 100 ft. at the Palta station which is about 20 miles north of Calcutta. As a considerable loss in pressure of the water takes place in transit along the pipes, a huge reservoir has been erected at a height of about 100 ft. at Tala which is just north of the city, where water is again pumped up and stored for distribution in the city.

(3) Artesian Well.— Within the earth's crust there are layers of *clay*, *slate*, etc. which are impervious to water, and also other layers of sand, gravel, etc. which are pervious. These layers are generally concave in structure. Where a porous layer of sand etc. is included between two impervious layers, a channel or water-bed is formed where rain water percolates and ultimately collects at the bottom of the concave bed. There may be similar water-beds at different depths of crust. Some of these beds again may be in communication with outlaying rivers or lakes so that they are like water contained in *U*-tube. When a boring is made up to such *U*-source, water gushes out and rises up to the head of the water in the source (Fig. 142). In the province of Artois in France the first well of this type was bored and hence the name Artesian well. An Artesian well 2,000 ft. deep bored in the desert of *Sahara* supplies considerable water even there. Wells which give out hot water are known as **hot springs**.

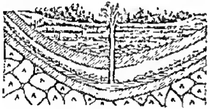


Fig. 142—The Artesian Well

(4) Tube-wells.—The principle, which is utilised in the case of a tube-well, is the same as that of the Artesian well, but in this case, the underground water-beds which are fed by outlying rivers and lakes are much less deep. As soon as a boring is made anywhere below the surface of the earth reaching any of these water-beds, water gushes forth upwards with a tendency to find its own level, which is the level of rivers, etc. or some such source whose level

is below the earth's surface at the place. A pump is, therefore, generally required to raise the water up to the surface of the earth. Thus in a tube-well, unlike in an Artesian well, the water does not automatically come to the surface and so a pump is required.

Example. *Neglecting the loss of pressure in the transit, calculate what head of water is necessary to produce a pressure of 250 lbs. per sq. inch in the street mains.*

1 cu. of water weighs 62·5 lbs. $\therefore$ For a head of water 1 ft. high, the pressure per sq. foot equals 62·5 lbs. $\therefore$ Pressure per sq. inch $= \frac{62{\cdot}5}{144} = 0{\cdot}434$ lbs.

$\therefore$ To maintain a pressure of 0·434 lb. per sq. inch. a column of water 1 foot high is necessary.

Hence to maintain a pressure of 200 lbs. per sq. inch, the height (head) of water necessary $= \frac{200}{0{\cdot}434} = 460{\cdot}8$ ft. (approx.).

That is, the water in the reservoir should stand 460·8 ft. above the point in question.

248. The Lateral Pressure of a Liquid :—A liquid at rest exerts pressure on the sides of the containing vessel. This is known as *lateral pressure.*

Fig. 143 shows a vessel floating on water, having a tubular outlet provided with a stop-cock fitted at one side near the bottom. Fill the vessel with water and open the stop-cock. Water flows out from the tap and the vessel is seen to move backwards, *i.e.* in a direction *opposite* to that of the water jet. This is due to the fact that a liquid exerts *lateral pressure.*

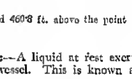


Fig. 143

Explanation.—It will be seen from the next two articles that the magnitude of the lateral pressure depends on the depth of the level at which the pressure is considered and acts at right angles to any surface in contact. When the liquid is at rest (*i.e.* when the stop-cock is not opened), the lateral pressures at the two ends of a diameter of the vessel at the level of the tap are equal, but being oppositely directed cancel each other, and so the vessel is stationary. When the cock is opened, the lateral pressure there is released on account of the water coming out through it. But the lateral pressure at the opposite end of the wall remains as before. This unbalanced pressure makes the vessel move opposite to the issuing water.

249. The pressure of a liquid at any point on the wall of a vessel acts in a direction perpendicular to the wall :—

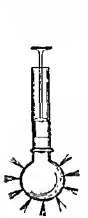


Fig. 144

Expt. 1. Take a hollow globe (perforated all over) fitted with a syringe as shown in Fig. 144 Remove the piston, fill the globe and a part of the barrel of the syringe with water. Re-insert the piston and slowly push it inward when water will be found to spurt out radially from the globe (*i.e.* in a direction perpendicular to the wall of the vessel) with equal force.

This shows that pressure is transmitted equally in all directions by a liquid, although the pressure is exerted on it in a particular direction, and on the wall of the containing vessel it acts perpendicularly.

This fact is also shown when the wall of a pipe containing a liquid at rest is pierced by a small hole. A thin jet squirts out at right angles to the surface of the pipe.

Expt. 2. Take a spherical vessel, as shown in Fig. 144(*a*), fitted with several tubular outlets distributed all around radially directed, each outlet having a closely-fitting piston capable of moving outwards or inwards The vessel is filled up with water. Suppose one of the pistons, *A*, is pushed inwards by applying a force *F*. It will be found that all the other pistons are pushed outwards equally. This shows that the pressure exerted by the piston *A* inwards on the mass of the water is transmitted by it to all the other pistons in the different directions and at right angles to the surface in contact.

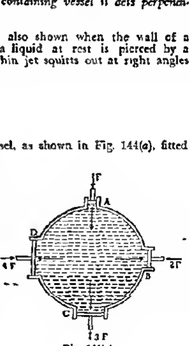


Fig 144(*a*)

Examples. *(1) A plate 10 metres square is placed horizontally 1 metre below the surface of water, when the height of the mercury barometer is 760 mm. What will be the total thrust on the plate? (The density of mercury=13·6). (C. U. 1911)*

1 metre=100 cms.; 10 metres square=10 metres$\times$10 metres
=1000 cms.$\times$1000 cms.=10^6 sq. cms.

The pressure at a point 100 cms. below the surface of water=atmospheric pressure+the pressure due to a column of water of height 100 cms.=pressure due to $(76\times13·6+100)$ cms. height of water=$1133·6\times981$ dynes ($\because$ 1 gm.-wt.=981 dynes). This is the force exerted on unit area of the plate.

The total thrust on the plate=$1133·6\times981\times10^6$ dynes.
=$1·112\times10^{12}$ dynes.

(2) A U-tube open at one end and closed at the other is partially filled with mercury (density 13·6). The closed end of the tube contains some air and the mercury in the open limb 30 cms. higher than it does in the closed limb. Find in c.g.s. units the intensity of pressure of the air in the closed end of the tube (Barometric pressure=76 cms.) (C. U. 1910)

The pressure of the enclosed air =Pressure due to (76+30) cms. of mercury
=$(106\times13·6\times981)$ dynes=$1·41\times10^6$ dynes

(3) At what depth below the surface of water will the pressure be equal to two atmospheres, if the atmospheric pressure be 1 megadyne (10^6 dynes) per sq. cm.? (g=981 cms./sec.^{2}). (C. U. 1931)

Let h cms. be the required depth at which the pressure is equal to 2 megadynes.

$\therefore$ The pressure due to h cms. height of water=1 megadyne=10^6 dynes.

The pressure due to 1 cm. height of water (*i.e.* the wt. of 1 c.c. of water) =1 gm.-wt.=981 dynes.

$\therefore$ The pressure due to h cms. height of water=$h\times981$ dynes=10^6 dynes.

$\therefore$ $h=\frac{10^6}{981}=1019·36$ cms.

250. The Pressure at any particular depth depends on the depth and does not depend on the shape of the vessel :—

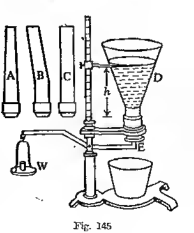


Fig. 145

Expt. 1.— The area of cross-section of the base of all the four vessels A, B, C, D (Fig. 145) known as *Pascal's vases* is equal, but the vessels are of different shapes and containing capacities. They can be screwed on to a platform carried horizontally by a vertical stand which is also provided with a horizontal pointer intended to mark the level of any liquid contained in the screwed vase. This stand also supports a fulcrum. A plate E attached to one end of a lever, the middle of which rests on the fulcrum, is pressed against the bottom of the vase by adding counterpoising

weights on a scale pan hung from the other end of the lever. Placing a suitable weight W on the scale pan, water is poured into the vase until the supporting plate E just yields, and water escapes. Noting the height h of the water by the pointer, the experiment is repeated with the other vessels, the weight on the scale pan being kept the same in every case. It will be found that water begins to escape when it attains the same height in every case, proving that *pressure depends only on the depth, and not on the size or shape of the vessel*, i.e. the pressure, is independent of the quantity of water contained in the vessel, but depends only on the depth of the water. The same is true for any liquid.

The fact illustrated in the above expt. is known as the **Hydrostatic Paradox.**

Explanation.—The result appears at first to be puzzling but a moment's consideration will show that there is no real inconsistency. Suppose there are two vessels, (*a*) and (*b*), in Fig. 146 of different shapes and capacities. They are filled with water up to the same level. Though the amount of water in the two vessels is different, the pressure exerted on the base of the vessel is the same in the two cases.

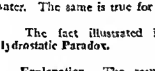


Fig. 146

This is because, the sides of the vessel exert pressure on the liquid at right angles to the surface. This pressure is represented by P in the two vessels, (*a*) and (*b*), and can be resolved into two components, V acting vertically and H acting horizontally.

In the vessel (*a*), which contains a larger quantity of water, all the vertical components like V acting upwards serve to support some of the water on the sloping side. In the vessel (*b*), containing a smaller quantity of water, the slope of the side being opposite, the vertical component V is acting downwards, which is transmitted to the base. Due to this, the total pressure on the base of vessel (*b*) is the same as that of vessel (*a*) and is equal to that of a vessel having vertical sides of equal height. *This explains that the pressure depends only on the depth and not on the shape or size of the vessel.*

Expt. 2.—That the pressure of a liquid at a point depends on the depth of the point and not on the shape of the vessel containing the liquid is also shown by the following simple and interesting experiment—**the bursting of a cask.**

A stout cask A (Fig. 147) is completely filled with water. The quantity of water in the cask is quite large but the cask does not burst. A long narrow tube T is then fixed vertically through the top of the cask and water is gradually poured into the tube when the pressure of water inside the cask increases gradually with the rise of the level of water in the tube; when the level reaches a certain height the pressure inside becomes so great that the cask bursts though the actual quantity of water added is very small.

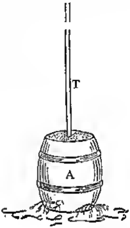


Fig. 147

This experiment was first carried out by Blaise Pascal (1623-1662) taking water in a narrow tube about 30 ft. high. The pressure exerted by such a column of water at a level near the bottom of the cask will be about 15 lbs./in². This pressure will be transmitted in all directions at the same level with equal force and acting perpendicularly to each sq. inch area of the inside wall of the cask may be sufficient to burst the cask, if the same is not sufficiently strongly built.

251. The upward pressure at any depth in a liquid is equal to the downward pressure :—

Expt.— Take a glass cylinder with both ends open. A thin disc of tin is held tightly against the lower end by a string passing through its centre (Fig. 148). On lowering the whole into water and loosening the string, it will be found that the tin disc does not fall. This is due to the vertical upward thrust exerted by the water underneath the disc.

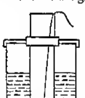


Fig. 148

Now, carefully pour water inside the cylinder and note that the disc remains in its place so long as the level of water inside is less than that at the outside, but the disc falls down by its own weight when the level of water inside and outside the cylinder is the same.

This proves that *the upward pressure, or the buoyancy, at any depth, is equal to the downward pressure.*

252. Pascal's Law :—*The pressure exerted anywhere in a mass of confined liquid is transmitted undiminished in all directions throughout the mass so as to act with equal force on every unit area of the*

containing vessel in a direction at right angles to the surface of the vessel exposed to the liquid.

Expt. 1.—Take a stout glass flask fitted with a closely fitted piston at the neck. There are four tubes, bent upwards and attached to the flask, as shown in Fig. 149 Put a little mercury into the bend of each of these tubes Then each of these tubes serves as a manometer (or pressure-measurer).

Remove the piston and fill the flask with water, and then apply pressure by re-inserting the piston. The pressure is transmitted in all directions.

On pushing the piston, the mercury will be seen to rise to the same height in all the tubes showing that the pressure exerted is the same in every case.

Fig 149

If each of the openings has got the same area, then the total force exerted (i.e. pressure × area) will also be equal in every case. If the area of one of the openings be twice that of another, the total force (here total force = 2 × area × press.) exerted there will also be twice, but the pressure, *i.e.* the force per unit area will be the same and so the manometer will indicate the same difference of level.

Expt. 2.—Refer to the experiment described in Art. 249 as Expt. 2 Suppose the piston A is of unit cross-section and the other pistons B, C, D, have sectional areas of 2, 3 and 4 units It will be found that when a force F is applied on the piston A pushing it inwards, forces of magnitude $2F$, $3F$ and $4F$ will be required to stop the pistons B, C and D from being pushed outwards. This shows that the force exerted by A in a given direction is transmitted with equal force per unit area in the different directions in which the pistons B, C and D are situated, and so the expt. verifies **Pascal's law.**

253. The Principle of Multiplication of Force :— Consider two cylinders A and B (Fig. 150) of different areas fitted with pistons and communicating with each other through a pipe. Now, if a pressure P be applied on the piston in A, an equal pressure P will be transmitted to the piston in B. Remember that *it is the pressure*

which is transmitted and not the total force. The pressure is the force per unit area. Hence the areas of the pistons must be taken into account in considering the transmitted force. So every unit area of the piston in B will be pressed upwards with the same force as exerted on a unit area of the piston in A.

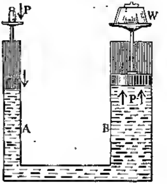


Fig. 150

Thus, if the diameter of B is four times the diameter of A, the area of cross-section (assumed circular in the two cases) of B will be sixteen times that of A. The pressure on the piston in B will be the same as that applied by the piston in A, but since the total force is the product of pressure and area, the upward force W on the platform will be sixteen times the force on the piston in A, or, in other words, if α and β be the areas of the small and large pistons respectively, and f the force applied by the piston in A, then the force F on the piston in B will be given by,

$F = \frac{f}{\alpha} \times \beta.$

254. The Hydraulic Press (Bramah's Press):—A schematic diagram of the hydraulic press is shown in Fig. 151.

Construction.—The machine essentially consists of two parts: a water pump whose piston Q works in a narrow metallic cylinder A and a thick ram R acting as a piston in a wide cylinder B, the two cylinders being connected by a stout metallic tube D. The strength of the cylinders to stand large internal pressures is usually increased by shaping the bases hemispherically but not shown in that way in the figure. The piston Q is connected to some point K in the middle of a lever L by which it is worked. The lever has its fulcrum at one end F and at the other end of it an effort P_1 is applied. A valve V_1 separates the cylinder A from small tank T which is almost full of water. It allows only an one-way passage of the water from the tank to the cylinder. Another one-way valve V_2 opening from the side of the cylinder A towards the cylinder B separates the latter cylinder from the former. On account of this valve water cannot flow back from cylinder B to cylinder A. The top of the ram R forms a platform on which any material intended for compression is placed and, as the ram is raised upwards, the material is compressed against a fixed girder G which is supported on strong pillars.

To raise the ram R which is the pressure-piston, the pump-piston Q is worked up and down a number of times by the help of the lever

connected to it. As the material is compressed, the pressure of water within the machine increases and so, to prevent damage to the machine on account of excessive pressure a safety valve V_3 is fitted in the tube D which connects the cylinder A to the cylinder B. This blows off when the internal pressure exceeds a certain limiting value, whereby some water escapes and the pressure drops down to the normal.

In order that the ram R may again return to its normal position by its own weight after a compression is over, there is an arrangement of a side-tube H connecting the pipe D to the tank T and the side tube is provided with a stop-cock C by opening which the water from the cylinder B can be made to pass back into the tank. To make the ram R work water-tight, a leather packing J, shown also separately as (a) at the top of Fig 151, having the form of an in-

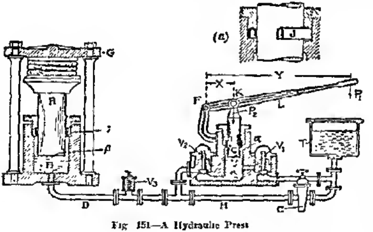


Fig 151—A Hydraulic Press

verted cup is so inserted around the piston in an annular recess in the body of the cylinder B that water, when under pressure, passes into the annular space inside the cup Consequently, the greater the water pressure the tighter the water presses against the ram R and the better become the joint. To make the leather impervious to water, it is previously soaked in oil A similar packing may also be used around the smaller piston Q Such packing to make the joint water-tight, which may be made in some other ways as well, now-a-days, was first devised by the engineer Bramah and so the press is sometimes called after him.

Principle of Action.—The principle of multiplication of force (inherent in Pascal's law) by transmission of fluid pressure is used in the hydraulic press. As the piston Q is raised by the lever L, the pressure inside the cylinder A decreases and so water enters into it from the tank T by lifting the valve V_1. During the down-stroke when the piston is lowered, the pressure inside increases and closes the valve V_1 and the water is forced into the cylinder B through the connecting pipe D lifting the valve V_2. That is, during an up-stroke a quantity of water is drawn inside the cylinder A and during the following down-stroke the water is forced into the cylinder B. The thrust generated on the piston Q due to any small effort applied at the free end of the lever is transmitted to the water in B and produces on the ram R a huge upward thrust which is as many times larger as the cross-section of R is greater than that of Q.

Suppose P_1 is the effort applied at the free end of the lever at a distance Y from the fulcrum F_1 and P_2, the thrust generated on the piston Q which is distant, say, X from the fulcrum. Let the cross-sections of Q and R be α and β respectively. Then, for the lever, the mechanical advantage, m = the force ratio, $\frac{P_2}{P_1}=\frac{Y}{X}$.

That is, P_2, the thrust generated on the piston $Q=\frac{Y}{X}\times P_1$ ... (1)

The pressure exerted on the water $=P_2/\alpha$. This pressure is transmitted undiminished throughout the water across any surface exposed to the liquid, according to Pascal's law. This will, therefore, cause an upward thrust P_3 on the ram R given by,

$$P_3=\frac{P_2}{\alpha}\times\beta=P_1\times\frac{Y}{X}\times\frac{\beta}{\alpha} \quad \ldots \quad (2)$$

$$= \text{effort at lever} \times \text{mechanical advantage of lever} \times \frac{\text{cross-section of ram}}{\text{cross-section of piston}}.$$

Mechanical Advantage of the machine as a whole :—

$$= \frac{\text{thrust generated}}{\text{effort applied}} = \frac{P_3}{P_1}=\frac{Y}{X}\times\frac{\beta}{\alpha} \quad \ldots \text{ from (2)}$$

$$= \text{mechanical advantage of lever} \times \frac{\text{cross-section of ram}}{\text{cross-section of piston}}.$$

Principle of Conservation of Energy applied to the Machine.—If the ram is raised through a vertical distance l_3, and the piston in A pushed down through l_2 then

$l_2\times\alpha=l_3\times\beta$, since the decrease of volume of water in A is equal to the increase in volume of the water in B, assuming water to be incompressible.

So, $l_2/l_3=\beta/\alpha$; but $\beta/\alpha=P_3/P_2$, according to the Pascal's law; that is, $P_3\times l_3=P_2\times l_2$.

In other words, work done by the ram, $(P_3 \times l_3)$
$=$work done $(P_2 \times l_2)$ on the smaller piston

$$= \left(P_1 \times \frac{Y}{X}\right) \times l_2, \text{ from (1)}. = P_1 \times \left(l_2 \times \frac{Y}{X}\right) = P_1 \times l_1,$$

from the geometry of the lever, where l_1 is the vertical distance through which the point of application of the effort P_1 is pushed down
$=$work done on the lever.

Thus, the principle of conservation of energy is obeyed by the machine, as it must. So no gain of work is envisaged.

P_3 is greater than P_1 in a ratio in which l_1 is greater than l_3. This is sometimes expressed in popular language as "What is gained in power is lost in speed." More accurately this fact may be stated as "Mechanical advantage is always gained at a proportionate diminution of speed."

Example. *A Bramah Press has a piston whose cross section is 144 sq. in. The cross section of the pump is 2 sq. in. The shorter arm of the lever working the pump is 1 foot and the longer one is 4 feet in length. Calculate the total force obtained when an effort of 175 lbs is applied to the end of the longer arm.*
(*C. U.* 1949)

By the principle of the lever we have $175 \times 4 = P_2 \times 1$, where P_2 is the weight or load, or resistance of the pump $\therefore\ P_2 = \frac{175 \times 4}{1} = 700$ lbs

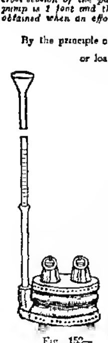


Fig. 152—Hydrostatic Bellows.

That is, the pressure has been increased from 175 to 700 lbs

Now, according to the principle of the hydraulic press, we have $\frac{P_3}{700} = \frac{144}{2}$, where P_3 is the total force.

$\therefore\ P_3 = \frac{700 \times 144}{2} = 50{,}400$ lbs.-wt

255. Hydrostatic Bellows:—Fig. 152 represents an apparatus known as the *hydrostatic bellows*. This is another example of the multiplication of force by the transmission of fluid pressure

The apparatus consists of a stout leather bellows attached to a long narrow vertical tube. The leather bladder and a part of the tube are filled with water. A heavy weight placed on the platform of the bladder will be supported simply by the weight of the column of water in the attached narrow tube.

A man standing on the platform can also be balanced in the above way, if the tube is sufficiently long, and the area of the platform adequate. This may appear quite paradoxial considering the heavy weight of a man being balanced by the weight of a narrow column of water of

small quantity. But it can be easily explained by the principle of multiplication of force.

Suppose the vertical tube is 2 metres long and the area of the platform of the bellows is about 500 sq. cms.; then since the pressure that can be exerted by a column of water 200 cms. high is 200 gms.-wt., the upward thrust on the platform of the bellows by the principle of multiplication of force, will be $500 \times 200 = 100$ kgms.-wt. which is sufficient to balance the weight of an average man.

256. Other Examples of Pascal's Principle:—Another example of the Pascal's principle is **hydraulic lift** which is now-a-days commonly seen in big towns in automobile repairing stations by which automobiles are lifted up to a suitable height above the ground level for the convenience of the repairing workers. The **hydraulic chairs** used by the Dentists also work on the same principle.

257. Blaise Pascal (1623—1662):—A French mathematician, physicist and religious thinker. He ranks with Galileo and Stevin, as one of the founders of the Science of Hydrostatics, Hydrodynamics and Pneumatics. He is one of those great men who showed signs of uncommon scientific powers in early childhood. He is a successor of Galileo, a contemporary of Torricelli and a forerunner of Guericke in establishing the connection between atmospheric pressure and the weight of air. At the age of twelve he began to master Euclid and at sixteen wrote eighteen essays on *Conic Sections* which are of permanent value. He was a teacher of mathematics in a Polytechnic School where he investigated the properties of fluids. In 1646 he established the Law of fluid pressures known as the Pascal's Law; and invented the Hydraulic press. It is said that he only applied in a new way here what Stevin had previously discovered. After the discovery of the atmospheric pressure by Torricelli, it appeared to him that it is actually the weight of the air that exerts the pressure and holds up the mercury column. So he undertook experiments to prove the same and in 1648 proved beyond doubt that the pressure diminishes as we go upwards in the air-ocean just as it does in the case of a liquid, which also Stevin had stated earlier. That Torricelli's mercury column is not drawn up by '*the vacuum*' as Aristotle thought but is pushed up by the weight of air, as already demonstrated by Galileo, was confirmed finally as a result of Pascal's work. He was the first to make a thrilling demonstration of the fact that a narrow vertical column of water contained in a long tube fixed to the top of a wooden barrel can exert so much pressure on its walls that the barrel may burst. At that time it was called a paradox. The *hydrostatic bellows* (Art. 255) is based on this principle. He devoted the last ten years of his short life to religious thinking and died at the age of thirty-nine at Paris.

Questions

1. How would you prove experimentally that a liquid exerts pressure in all directions? (C. U. 1911, '14, '27)

2. The density of sea water is 1·025. Find the pressure at the depth of 10 ft. below the surface in pounds per square foot, given that one cubic foot of water weighs 62·5 lbs (C. U. 1927)

[*Ans.* 640·625 lbs. per sq. ft.]

3. Define intensity of pressure at a point in a liquid. Prove that the difference of pressure P between the surface of a liquid and a point in the liquid z cms below the surface is given by $P = g \cdot d \cdot z$, when d is the density of the liquid and g is the acceleration due to gravity.

(C. U. 1910; Pat. 1938, *cf.* M. U. 1950; Anna. U. 1951, *cf.* Gau. 1955)

[*Hints.*—Intensity of pressure at a point is the force per unit area surrounding that point. If p be the atmospheric pressure, i.e. force of air exerted on unit area, then the force due to the atmosphere on an area A of liquid surface $= p \times A$. The force due to the liquid column of the same area A and height z cms $= Agdz$. ∴ Total force on an area A in the liquid z cms below the surface $= pA + Agdz$.

∴ The force on unit area = pressure $= p + g\,d\,z$, and the pressure on the surface $= p$

∴ The difference of pressure $P = (p + gdz) - p = g\,d\,z$.]

4. State Pascal's law regarding the transmission of pressure in a liquid and define intensity of pressure at a point in a liquid. (Gau. 1955)

5. A rectangular tank 6 ft. deep, 8 ft. broad and 10 ft. long is filled with water. Calculate the thrust on each of the sides and on the base (1 cu. ft. of water weighs 62·5 lbs.) (Pat. 1939)

[*Ans.* On the base—960,000 poundals, on each of the shorter sides = 288,000 poundals, on each of the longer sides = 360,000 poundals.]

6. What is the total force on a submerged rectangular area 12×16 cm. when it is inclined at 30° to the horizontal and its upper edge of 12 cm. is 20 cm. below the surface of water in a jar.

[*Ans.* $4·5 \times 10^5$ dynes.]

7. A tall vessel, provided with a tap at the side near the bottom is filled with water and made to float upright on a thick plate of cork. Explain what will happen when the tap is opened. (C. U. 1914)

8. The neck and bottom of a bottle are ½ inch and 4 inches in diameter respectively. If, when the bottle is full of oil, the cork in the neck is pressed in with a force of 1 lb. wt., what force is exerted on the bottom of the bottle?
[*Ans.* 64 lbs. wt.] (Pat. 1944)

9. Draw a neat diagram of the hydraulic press, and give a brief description of it with an explanation of the action.

(Dac. 1934; Gau. 1949; C. U. 1950; Del. 1951; Pat. 1952, Utkal 1954; Vis. U. 1955)

What is the mechanical advantage in such a machine? Does it violate the principle of conservation of energy? Justify your statement. (C. U. 19 9)

10. In a Bramah's Press, the areas of the two plungers are ½ sq. inch and 0 sq. inch respectively. The pump-plunger is worked by a lever whose are 2 inches and 28 inches. If the end of the lever is raised and ered by 1 foot at every stroke, find the number of strokes required to raise e press plunger by 1 inch. (Utkal, 1951)

[*Ans.* 140/3 times.]

11. State Pascal's principle of transmission of fluid pressure and apply it to secure multiplication of force.

Describe a Bramah's Press with a neat diagram. What is the mechanical advantage in such a machine? (C. U. 1957)

12. A force of 50 kgms. is applied to the smaller piston of a hydraulic machine. Neglecting friction, find the force exerted on the large piston, the diameters of the pistons being 2 and 10 cms. respectively. (Pat. 1922; P. U. 1925)

[*Ans.* 1250 kgms. wt.]

13. The area of the small piston of a Hydraulic Press is one sq. ft. and that of the large piston twenty sq. ft. How much wt. can be raised on the large piston by a force of 200 lbs. acting on the small piston? (C. U. 1946)

[*Ans.* 4,000 lbs.]

CHAPTER X

ARCHIMEDES' PRINCIPLE

258. Archimedes' Principle :— *A body, immersed partly or wholly in a fluid at rest, appears to lose a part of its weight, the apparent loss being equal to the weight of the fluid displaced.*

Verification.—The above principle can be verified in the case of a liquid by a *Hydrostatic balance*, which is simply an ordinary balance by which the weight of a body immersed in a liquid can be conveniently measured. (In a special form of this balance, the suspending frame of the left-hand pan is storter than that of the other pan. This pan has a hook attached to its bottom. The body to be weighed is hung from the hook.) A wooden bridge C (Fig. 153) is placed on the floor across the left-hand pan of the balance in order that a beaker containing a liquid may be placed on it and the body to be weighed is hung into the liquid contained in this beaker.

Expt.—A solid metal cylinder, A, is suspended from a hook fixed at the bottom of a hollow cylinder or bucket B into which the solid cylinder A exactly fits. So the internal volume of the bucket is the same as the volume of the solid cylinder. The whole thing is suspended from the left-hand arm of the balance and counterpoised. The solid cylinder is then totally immersed in the liquid contained in a beaker D which rests on a small wooden bridge C placed across the left-hand pan free from it, when the ordinary form of the hydrostatic balance is used as shown in the figure. The equilibrium is now disturbed as the solid cylinder, now immersed in water, has lost a part of its weight due to the upward thrust, *i.e.* the **buoyancy of the water.**

Fig. 153

Now fill the bucket B completely with the liquid and the balance will be restored again, showing that the solid cylinder lost a part of its weight equal to the weight of its own volume of

the liquid (which is the same as the weight of the displaced liquid); or, in other words, *the upward thrust on the cylinder is equal to the weight of the liquid displaced by it.*

This verifies the principle of Archimedes in the case of a liquid. For verification in the case of a gas, see Art. 262

Apparent Loss.—It should be noted that *the loss* in weight of the cylinder *A is only an apparent one and not true,* for really the beaker with the liquid in it together with the cylinder placed on the *scale-pan would* weigh the same whether the cylinder is placed outside or inside the liquid in the beaker as explained in the case (2) on downward thrust (*vide* Art. 265). When the cylinder is inside the liquid, it experiences an upward thrust exerted by the liquid (causing the apparent loss of weight), which tends to raise the arm to the balance, and the cylinder in turn exerts at the same time a *reaction* which is a downward force *of equal magnitude* on the liquid (according to Newton's Third Law of Motion) Thus the balance is not disturbed.

***259.* Buoyancy**:—The bouyancy of a fluid may be defined as the resultant upward thrust experienced by a body when immersed in the fluid When standing or lying in water, you must have noticed that water tends to raise you or buoy you up. The result of the buoyancy of water can also be observed, if a lead pencil (or any other thing which floats) is pushed into water and then let go, when the solid will be seen to float up through the water

Theoretical Proof of the Value of Buoyancy.—Consider a solid rectangular block *ABCD* inside a liquid (Fig 154). The liquid presses on the block all over The horizontal pressures on the two pairs of opposite vertical surfaces counteract each other as they are of equal magnitude and correspondingly act in the same horizontal line. The top surface *AB* is pressed downwards by the weight of the column of liquid *AEFB* The bottom surface *CD*, which is at a depth *CF* below the surface, is pressed upwards by the weight of the column of liquid *EDCF*. It is clear that the upward force exceeds the downward force by the weight of the column of liquid *ADCB*, which is the quantity of liquid displaced by the block, *i.e.* the upward thrust exerted by the liquid is equal to the weight of the displaced liquid.

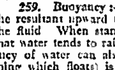


Fig. 154

Mathematical Proof.—Let *EA* and *ED*, i.e. the depths of *AB* and $CD = h$ and h' respectively; area of the faces *AB* and $CD = A$; density of the liquid $= d$; acceleration due to gravity$=g$.

$\therefore$ The total *downward* force on the face $AB = Ahdg$;
and the total force on *CD* acting *vertically upwards* $= Ah'dg$.

$\therefore$ The resultant thrust on the block exerted by the liquid acting

vertically upwards $= A(h'-h)dg$. But $A(h'-h)$ is the *volume* of the block: so the resultant upward thrust is equal to the weight of the volume of the liquid displaced by the block. This upward thrust is called the buoyancy of the liquid.

Besides the buoyancy, there is another force acting on the body, which is the weight of the body acting vertically downwards. If W be this weight, the resultant force acting on the body is $\{W - A(h'-h)dg\}$; that is, *on account of immersion the body loses a part of its weight equal to the weight of the liquid displaced by it.*

260. Practical Applications of Archimedes' Principle:—

(1) Determination of Volume of a Solid.—The volume of a solid of any shape (which is heavier than and insoluble in water) can be easily determined by the following method: Let the wt. of the body in air $= W_1$ gm. Let its wt. when suspended in water with a hydrostatic balance (Fig. 153) $= W_2$ gm. Loss of wt. in water $= W_1 - W_2 =$ wt. of water displaced. The volume of this displaced water is equal to the volume of the solid.

Now the volume of $(W_1 - W_2)$ gm. of water $= (W_1 - W_2)/d$ c.c., where d gm. per c.c. is the density of the water taken.

$\therefore$ Volume of the body $= (W_1 - W_2)/d$ c.c. If the weights are given in pounds, the volume of the body $= (W_1 - W_2)/62 \cdot 5$ cu. ft., as the density of water is 62·5 lbs. per cu. ft.

(2) Determination of Density of a Solid.—As density is mass per unit volume, density of the solid $= \dfrac{\text{mass}}{\text{volume}} = W_1 \div \dfrac{W_1 - W_2}{d}$

$= \dfrac{W_1 \times d}{W_1 - W_2} = \dfrac{W_1}{W_1 - W_2}$ gm. per c.c. (taking the density of water $d = 1$ in C.G.S. units). In F.P.S. units, the density of the solid $= \dfrac{W_1}{W_1 - W_2} \times 62 \cdot 5$ lbs. per cu. ft.

261. History:—The principle of Archimedes is also known as the **law of buoyancy**. It was discovered by Archimedes (287—212 B.C.), a celebrated mathematician and philosopher born at Syracuse in Sicily. The story of Hiero's crown in connection with the discovery of this law has been very well known. Hiero, the king of Syracuse, wished to be certain that the crown made for him was of pure gold, and he asked Archimedes to ascertain this. This job was not an easy one, for the crown must not in any way be damaged. Archimedes was puzzled at first but one day while he was taking his bath in a tub of water, he felt a loss of weight of his body and the idea crossed his mind that a body immersed in a liquid loses a part of its weight. Subsequently, he found that the loss of weight is equal to the weight of the displaced liquid. This enabled him to find the volume of the crown and therefrom the density of the material. It is so said that from the tub of water he jumped up in ecstasy of joy and rushed out into the street,

naked, crying "*Eureka! Eureka!*", i.e. I have found out, I have found out.

262. The Principle of Archimedes is also true for Gases:— *A body will apparently weigh less in air than it would in vacuo*, for the air exerts an upward thrust equal to the weight of the displaced air, but the weight of the displaced air is so small that ordinarily the loss in weight is not taken into account.

Expt.—That air, or any other gas, exerts an upward thrust on a body immersed in it can be demonstrated by the **baroscope** (Fig 155). The arrangement is as follows: A large sphere of cork *M* is suspended from one arm of a small balance and is equipoised by brass wts *W* placed on the other arm. The whole system is then placed under the receiver *JR* of an air-pump. On drawing out air from within the receiver by means of a pump, the arm carrying the cork sphere is seen to sink down. The cork sphere owing to its larger volume displaces a greater volume of air than the brass pieces do and so the up thrust, or the buoyancy of air, is also greater for the cork sphere. As the apparent wts. of both in air are the same while the buoyancy on the cork sphere is greater, the true wt. of the cork sphere must be greater. That is why, in the absence of air, the cork sphere sinks down. If, however, the two are equipoised first in vacuum and then air is introduced, the cork will go up and the *weights* sink down.

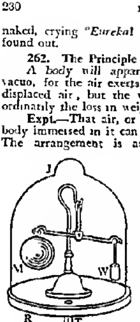


Fig 155

263. True Weight of a body: Buoyancy Correction:— In very accurate weighings it is necessary to take account of the air displaced by the body in order to reduce the weighing to vacuum.

Let W = true wt. of the body, *i.e.* its weight in vacuum;

W_1 = true wt. of the counterpoising weights;

d = density of the body, d_1 = density of the material of the wts;

ρ = density of air

Then the volume of the body $= W/d$, and the volume of the counterposing wts. $= W_1/d_1$. So the wt. of the air displaced by the body $= \rho.\ W/d$, and that by the wts. $= \rho.\ W_1/d_1$.

Hence, for equilibrium, we have,

$$W - \rho.\ W/d = W_1 - \rho.\ W_1/d_1;$$

$$W = W_1 \frac{(1-\rho/d_1)}{(1-\rho/d)} = W_1 + W_1 \rho \left(\frac{1}{d} - \frac{1}{d_1}\right),$$

since ρ is small in comparison with d or d_1.

Example. *The wt. of a body in air is 30·5 gms. The density of the body is 0·76 gm/c.c., that of brass wts. is 8·4 gms/c.c., and that of air is 0·001293 gm/c.c. Calculate the true wt. of the body.*

True wt., $W = W_1 + W_1 \cdot \rho \left(\frac{1}{d} - \frac{1}{d_1}\right)$

$$= 30{\cdot}5 + 30{\cdot}5 \times 0{\cdot}001293 \left(\frac{1}{0{\cdot}76} - \frac{1}{8{\cdot}4}\right)$$

$$= 30{\cdot}54704 \text{ gms.}$$

Hence the true wt. is greater than the apparent wt. by 0·04704 gms.

264. Which is heavier, a lb. of Cotton or a lb. of Lead ?— *Prima facie*, one would be inclined to think that a lb. of both should be equally heavy. But one should remember that a lb. of cotton occupies, owing to lower density, a much larger volume than a lb. of lead and so the buoyancy of air on the former is much greater. As a result the former suffers a greater loss of weight in air. So, *if their apparent weights in air* are equal, the true weight of a lb. of cotton, *i.e.* in vacuum, is bound to be greater than that of lead. If their true weights, *i.e.* weights in vacuum, are one lb. each, a lb. of cotton will weigh less in air than a lb. of lead.

265. Two Interesting Cases on Downward Thrust :—The following interesting cases should be noted carefully regarding the downward thrust on a liquid by an immersed body :—

(1) A beaker containing water ($\frac{2}{3}$ full) is placed on one pan of a balance and counterpoised (Fig. 156). Now a body L of known volume, say v c.c., suspended by a thread from an external support (not from the balance beam), is allowed to sink into the water. What effect has this on the balance?

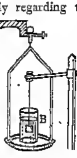


Fig. 156

It will be found that the arm of the beam on the side of the pan will be tilted down. To restore balance, the weight on the other pan will have to be increased by v gm.

The body is held by the support and its weight cannot add any weight to the side. Why is the side weighted more then? The phenomenon, though paradoxial, can be explained thus: The body when dipped in water experiences an upward thrust equal to the weight of the water displaced by it (v gm.). According to Newton's Third law of Motion, the body in its turn exerts an equal (v gm.-wt.) and opposite force (reaction of buoyancy) on the water contained in the beaker. This latter force accounts for the excess weight responsible for the tilting down of the arm. This excess weight is v gms.-wt. ; so an equal weight added on the other pan restores the balance.

(2) A beaker containing water is placed on the left pan of a balance and a body is also placed on the same pan outside the beaker and the two are counterpoised. Now the body is suspended from the left hook of the balance and is allowed to sink into the water. It will be found that equilibrium will not be disturbed in this case. The

phenomenon appears puzzling, for the natural expectation is that the body being immersed in water will lose some weight due to which the equilibrium should be disturbed. But a little reflection will show that the explanation of the result is simple. The reaction of the buoyancy, which is equal and opposite to the buoyancy, acts on the water downwards. The total weight on this side therefore remains the same, the buoyancy and its reaction cancelling each other's effects being equal in magnitude but opposite in direction. So the equilibrium cannot be disturbed.

266. Immersed and Floating Bodies :—

Let W represent the weight of a body immersed in a liquid. It will displace its own volume of the liquid of weight, say, W'.

Then W' is the upward thrust or buoyancy, which will act in opposite direction to W which is acting downwards.

(1) If $W > W'$, the body will sink.

(2) If $W = W'$, the body will float being wholly immersed anywhere in the liquid.

(3) If $W < W'$, the body will float being partly immersed in the liquid, the weight of the displaced liquid, in this case, will be equal to the weight of the whole body; that is,

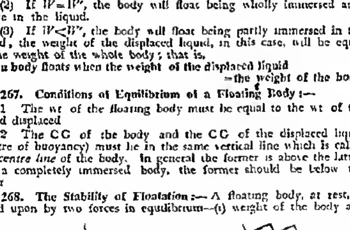
a body floats when the weight of the displaced liquid
= the weight of the body.

267. Conditions of Equilibrium of a Floating Body :—

1 The wt of the floating body must be equal to the wt of the liquid displaced

2 The C G of the body and the C G of the displaced liquid (centre of buoyancy) must lie in the same vertical line which is called *the centre line* of the body. In general the former is above the latter. For a completely immersed body, the former should be below the latter

268. The Stability of Floatation :— A floating body, at rest, is acted upon by two forces in equilibrium—(i) weight of the body acting



(a) Fig. 157 (b)

vertically downwards through the centre of gravity G, and (ii) the of the displaced liquid acting vertically upwards through B,

the C.G. of the displaced liquid, otherwise known as the centre of buoyancy. As the body is at rest, these forces must act in the same line as shown in Fig. 157(a). The line joining the points B and G of the floating body is called its *centre line*.

When a body is inclined on account of any external forces acting on it, the shape of the displaced water changes and the centre of buoyancy shifts to the leaning side. Now, the forces of weight and buoyancy no longer act in the same vertical line but form a couple. This couple may or may not restore the body to its position of equilibrium.

(*i*) If the vertical through the new centre of buoyancy B' cuts the line BG (called the **centre line**) above G, the couple will tend to restore the body to its position of equilibrium [Fig. 157(*b*)].

(*ii*) If the vertical through the new centre of buoyancy B' cuts the line BG below G, then the couple will tend to overturn the body.

In the case of a ship where the inclination θ is not more than 15°, the intersection of the vertical through B' with the line BG is practically a fixed point M known as its *meta-centre*. Thus, in short, *if M is above G, then the ship is stable and if below, it is unstable.*

[N.B. The C.G. of a ship is kept below the meta-centre by loading the bottom of the ship with ballast and thereby, the stability of the ship is increased. Restoring (or upsetting) moment $= W \times GM \times \sin\ \theta$.]

269. The Meta-centre:— If a body floating in equilibrium in liquid leans on one side, the C.G. of the body and the centre of buoyancy of the liquid are both displaced in the direction in which the body leans. The point, where the vertical line through the new position of the centre of buoyancy intersects the *centre line* of the body (*i.e.* the line joining the C.G. of the body and the C.G. of displaced liquid when the body floats in equilibrium), is called the meta-centre of the body.

270. Densities of Immersed and Floating Bodies:— Let the density of a liquid be d_1, in which a body of density d_2 and volume V is placed. Then when the body is totally immersed, the mass of liquid displaced $= d_1 \times V$. The mass of the body $= d_2 \times V$. Hence (*vide* Art. 266),

(1) if $(d_2 \times V) > (d_1 \times V)$, i.e. if $d_2 > d_1$, the body will sink, as a piece of stone or iron sinks in water.

(2) if $d_2 = d_1$, the body will float being wholly immersed anywhere in the liquid. Olive oil is lighter than water but heavier than alcohol, but by mixing alcohol with water in equal quantities, the density of the mixture becomes the same as that of Olive oil, when a drop of Olive oil will float anywhere in the mixture;

(3) if $d_2 < d_1$, the body will float partially immersed. A piece of wood floats on water and iron floats on mercury. When a body of

density smaller than that of a liquid is placed on the liquid, it sinks until the weight of the displaced volume of the liquid becomes equal to the weight of the body, when the body sinks no further and keeps floating. In this case, if v be the volume of the liquid displaced by the immersed part of the body, $d_1v=d_2V$; or, $\frac{v}{V}=\frac{d_1}{d_2}$;

i.e. $\frac{\text{volume of the immersed part}}{\text{total volume}}=\frac{\text{density of the body}}{\text{density of the liquid}}$.

271. Illustrations of the Principle of Buoyancy of Liquids:—

(1) Why Ice floats on water?— It is known that 1 gm of ice at 0°C. occupies 1/0·92 or 1·09 c.c., the density of ice being 0·92 gm/c.c. but 1 gm of water at 0°C occupies very nearly 1 c.c. Hence 1 c.c of water at 0°C. becomes 1·09 c.c when turned into ice at the same temperature, that is, when water freezes into ice, it increases in volume by about 9 per cent, i.e. *11 volumes of water at 0°C. becomes about 12 volumes of ice at the same temperature.*

Hence the density of ice will be diminished in the same proportion. So, from the above relation we get

$\frac{\text{volume of ice under water}}{\text{total volume of ice}}=\frac{11}{12}$, i.e. ice will float on water with $\frac{11}{12}$ of its volume below the surface and $\frac{1}{12}$ above it.

Note. A body which floats in one liquid may sink in another which is lighter. Thus iron floats on mercury but sinks in water, oil floats on water but sinks in alcohol, wax floats on water but sinks in ether, etc.

(2) Why an Iron Ship floats on Water?— It is a well-known fact that a solid block of iron readily sinks in water, because the density of iron is greater than that of water; but the mystery of why an iron ship floats on water lies in its construction, namely in its hollow shape. When the ship enters water, the volume of water displaced is much greater than the volume of actual iron immersed and, as a solid cannot displace more than its own weight of a liquid the ship sinks in water until the weight of the displaced water is equal to the weight of the ship. That is, the ship is immersed to such a depth that the weight of the ship with its contents (*i.e.* the engines, cargo, passengers, etc.) is balanced by the upward thrust or the force of buoyancy of the displaced water.

272. The Carrying Capacity of a Ship:— The carrying capacity of a ship is determined by the tonnage which is found by taking the difference of the weights of water displaced by the empty ship and the fully loaded ship. The weight of a big ship with its contents often comes up to 65,000 tons, *i.e.* 65,000 tons of water will be

displaced by the vessel when afloat. It should be remembered that the depth of immersion of a ship is less in sea-water than in fresh water because the density of sea-water is a little greater than that of fresh water, and so, in order to obtain the same upthrust, a smaller volume of sea-water must be displaced. Thus a ship can carry more cargo on sea-water than on fresh water. Now-a-days, according to law, every ship must bear a mark called **the Plimsoll line,** showing the limit up to which it is permitted to immerse in sea-water of normal density.

273. The Plimsoll Line:—This is a mark recorded on the side of a ship showing the limit of its immersion in sea-water in lawful loading. The letters **L.R.** (which stand for Lloyd's Register) are often used to indicate this line and they signify that this safe-loading line is considered reasonable for the particular ship by the Lloyd's Insurance Company and the fact is recorded in Lloyd's Register of shipping. The line is named after Sammuel Plimsoll (1824—1898), a Bristol M.P. who initiated the law in the Parliament to stop the over-loading of ships. The enactment of such a law was considered necessary at the time for it was found that dishonest owners often sent to sea old vessels loaded very heavily after insuring them for large sums and profited by the disasters that followed. The sailors often called such ships, 'Coffin ships'.

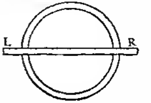


Fig. 158

It is relevant here to take note of *two expressions* which are very much in use in this connection. A ship *'drawing 30 ft. of water'*, means that 30 ft. is the distance from its keel to the water-surface. *'Water line area'* means the area enclosed by a line drawn round the ship along the water-surface. This cross-section is not the same all the way down, for a ship tapers towards the keel. The change in the 'water line area' however, is not much for some distance above or below the Plimsoll line and so is not often taken into consideration.

Example. *A sea-going ship (without cargo) draws 20 ft. of water. If its water line area is 15,000 sq. ft. what load will make it draw 22 ft. of water? (Sp. gr. of sea-water = 1·25).*

Extra volume of water to be displaced

$= 15{,}000\ (22-20) = 15{,}000 \times 2$ cu. ft.

Weight of extra water to be displaced $= 15{,}000 \times 2 \times 62{\cdot}5$ lbs.

$\therefore$ The weight of sea-water to be displaced $= 15{,}000 \times 2 \times 62{\cdot}5 \times 1{\cdot}25$ lbs.

$$\therefore \text{ Load} = \frac{15{,}000 \times 2 \times 62{\cdot}5 \times 1{\cdot}25}{2240} \text{ tons.}$$

$= 1046{\cdot}3$ tons (approximately).

274. The Floating Dock :— A floating dock (Fig. 159) contains air chambers in its base. When the same are full of water, the dock sinks to a line such as AB, say, and the vessel floats, as shown in the figure. As the water is gradually pumped out of the chambers, the dock rises until finally the floor of the dock is clear of water. The upthrust due to the water displaced balances now the total weight of dock and ship together.

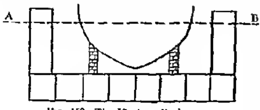


Fig 159—The Floating Dock.

Example. *The weight of a big liner is given as 64,000 tons What must be the volume of a floating dock which will be able to support it ? (Sp. gr. of sea water = 1 025)*

The volume of the dock must be equal to the volume of sea water weighing 64,000 tons, i e., (64,000 × 2,240) lbs

1 cu ft of pure water weighs 62 5 lbs.

∴ The wt of 1 cu ft of sea water = 62·5 × 1 025 lbs

$$\therefore \text{ Volume required} = \frac{64{,}000 \times 2{,}240}{62{\cdot}5 \times 1{\cdot}025} = 2{,}237{,}814{\cdot}6 \text{ cu ft (approximately)}$$

275. The Principle of a Life-belt :— It is known that a piece of marble can be made to float when tied to a suitable piece of cork. Thus bodies heavier than water can be made to float by being tied up to lighter bodies of suitable size This embodies the principle of the *life-belts,* which are found in steamers and ships

276. Swimming :— It is an art of moving in water keeping the head out of the surface of water Though the human body is lighter than water of the same volume and will float, the head is heavier and tends to sink in water The secret of swimming, therefore, lies in keeping the head out of water by the movement of limbs It is much *easier to swim in salt water* than in fresh water, because the density of salt water being greater, less force is required to prevent the body from sinking.

277. The Cartesian Diver :— This is a hydrostatic toy invented by Descartes *The principles of equilibrium of a body floating in a liquid, transmission of fluid pressure, and compressibility of gases are demonstrated by it.*

The diver is usually a small hollow doll having a tubular tail communicating with the inside and open at the end (Fig. 160) In some cases the doll is solid and is attached to a hollow ball having a

small opening at the bottom (shown on the right of the jar), so that the two together can float in equilibrium.

The diver is kept in a tall jar which is nearly full of water. The top of the jar is closed air-tight by means of a sheet of rubber. The diver is partly filled with air and partly with water, the total mass being slightly less than the mass of water displaced and so the diver floats partly immersed in the water.

On pressing the rubber sheet by means of the fingers, the diver is seen to sink down and on releasing the pressure to rise up again. By keeping the pressure on the rubber sheet constant, the diver may be kept stationary at any depth.

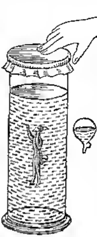


Fig. 160

Explanation.—When the rubber sheet is pressed, the volume of the air below is diminished whereby its pressure is increased. This pressure is transmitted through the water to the air inside the diver. As a result, the volume of the enclosed air is reduced and so an additional quantity of water enters into the diver through the opening whereby the diver is rendered heavier than the displaced water and so it sinks. When the pressure on the rubber sheet is released, the air inside the diver expands driving out the additional water and the diver is rendered lighter and so it rises up again.

If it were possible to make the diver sink to such a depth that the liquid pressure at that depth is too great for the inside air to expand adequately on the release of pressure, the diver will not rise up again. This aspect of the problem has been mathematically investigated in the worked-out Example No. 9 at the end of Chapter XII.

N.B. Most *fishes* have an *air-bladder below* the spine, which they can compress or dilate at pleasure and thus can either sink or rise up in water.

278. The Submarine:—It is a small sly vessel commonly used by the military navy. It can float on the surface of the sea like an

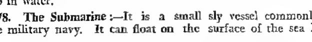


Fig. 161—A Submarine.

ordinary ship or sink when necessary and reappear on the surface

again. The principle on which it works is similar to that of the Cartesian Diver. The vessel is supplied with large ballast tanks T (Fig. 161) both in the stern and bow, which can be filled with water. When water is taken into the tanks (which are provided with trap-doors), the weight of the boat is so increased as to make the vessel sink, and the water is pumped out of the tanks by pumps worked by compressed air, the ship is made so light that it rises to the surface. Thus by emptying or filling the tanks, the mass of the ship is so varied and controlled that the ship is made to rise or sink as desired.

The act of filling or emptying the tanks is done very quickly. Moreover, the ship can be kept steady at any depth by the help of a vertical rudder R and other horizontal rudders not shown in the figure. A *Conning tower C*, in which a periscope is fitted, always projects above the surface of the water so that objects lying on the surface of the water may be viewed from within the boat.

279. The Density of Ice :— The density of ice can be determined by preparing a mixture of water and alcohol in such a proportion that when a piece of ice is placed in it, the ice will neither sink nor float but will remain anywhere within the liquid being completely immersed [*vide* Art. 170(2)]. The density of ice is then equal to that of the liquid mixture, which can be found out by means of a hydrometer (*vide* Art. 291). Its value is about 0.92 gm. per c.c.

280. The Density of Wood, Wax, etc. by Floatation :— The density of a solid having some regular form can be determined by the method of floatation if the solid is lighter than and insoluble in water. Take a cylindrical block of wood B whose length is l cm. and whose area of cross section is a sq. cm. (Fig. 162).

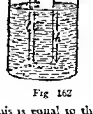


Fig. 162

∴ The volume of the block $= la$ c.c. and the weight of the block $= la \times d$ grams, where d is the density of wood. ... (1)

Float the block vertically in water and measure the depth (h cm.) to which it sinks. Then the volume of water displaced $= ha$ c.c.

∴ The weight of displaced water $= ha$ grams (∵ the density of water is 1 gram per c.c.) and this is equal to the weight of the block according to the law of floatation, i.e. $lad = ha$

$$\text{or,} \qquad d = \frac{h}{l} = \frac{\text{length immersed in water}}{\text{total length}}.$$

N.B. *The method applies to other materials also, such as wax, etc. which are not affected by water and can be cut in regular forms such as cube or cylinder. Only a metre scale is sufficient and a balance is unnecessary in this method.*

Examples. (1) *A hollow spherical ball has an internal diameter of 10 cms. and an external diameter of 12 cms. It is found just to float on water. Find the*

density of the material of the ball. (The volume of a sphere varies as the cube of the diameter.) (*C. U. 1928*; *Dec. 1933*)

Let V=volume of the sphere, and d=diameter of the sphere.

Then $V \propto d^3$; $\therefore$ $V=Kd^3$, where K is a constant.

The internal volume of the hollow ball$=K(10)^3=1000\ K$ c.c.

and the external volume$=K(12)^3=1728\ K$ c.c.

$\therefore$ The volume occupied by the actual material of the ball $=1728\ K-1000\ K=728\ K$ c.c.

As the ball is found just to float in water, the mass of the ball=the mass of the displaced water=volume of the displaced water$\times$density of water $=1728\ K\times 1=1728\ K$ gms.

$\therefore$ The density of the material of the ball

$$=\frac{\text{mass of the ball}}{\text{volume occupied by the actual material of the ball}}$$

$$=\frac{1728\ K}{728\ K}=2\cdot37 \text{ gms. per c.c.}$$

(2) *Given a body A which weighs 7·55 gms. in air, 5·17 gms. in water and 6·35 gms. in another liquid B; calculate from these data the density of the body A and that of the liquid B.*

Wt. of A in air$=7\cdot55$ gms.; wt. of A in water$=5\cdot17$ gms.

$\therefore$ Wt. of the same volume of water$=(7\cdot55-5\cdot17)=2\cdot38$ gms.

Hence, volume of $A=2\cdot38$ c.c. $\therefore$ Density of $A=\frac{7\cdot55}{2\cdot38}=3\cdot17$ gms. per c.c.

Again, loss of wt. of A and $B=(7\cdot55-6\cdot35)=1\cdot20$ gms.

Hence, 1·20 gms. is the wt. of B whose volume is the same as that of A which is 2·38 c.c. $\therefore$ Density of $B=\frac{1\cdot20}{2\cdot38}=0\cdot5$ gm. per c.c.

(3) *A sphere of iron is placed in a vessel containing mercury and water. Find out the ratio of the volume of the sphere immersed in water to that immersed in mercury.* (*Density of mercury=13·6; density of iron=7·8; density of water=1.*)

Let V_1 c.c. be the volume of the sphere immersed in mercury, and V_2 c.c. the volume immersed in water.

Then, the wt. of displaced mercury$=V_1\times 13\cdot6$ and that of displaced water $=V_2\times 1$. Now, wt. of displaced mercury+wt. of displaced water=wt. of the iron sphere, *i.e.* $V_1\times 13\cdot6\times V_2=(V_1+V_2)\times 7\cdot8$; or, $V_1\ (13\cdot6-7\cdot8)=V_2\ (7\cdot8-1)$.

Hence, $\frac{V_2}{V_1}=\frac{13\cdot6-7\cdot8}{7\cdot8-1}=\frac{29}{34}$.

(4) *A body of density δ is dropped gently on the surface of a layer of liquid of depth d and density δ' (δ' being less than δ). Show that it will reach the bottom of the liquid after a time, $\sqrt{\frac{2d\delta}{g(\delta-\delta')}}$, g being the acceleration due to gravity.* (*Pat. 1951*)

If m be the mass of the body, the volume of the body$=m/\delta$, which is also the volume of the displaced liquid. So the weight of the displaced liquid $=\left(\frac{m}{\delta}\times\delta'\right)g$, which is the upthrust acting on the body.

The force tending to bring the body down in the liquid$=mg$, the weight of the body, and the upthrust on it is $mg\ (\delta'/\delta)$. Hence the resultant downward force$=mg\ (1-\delta'/\delta)$=mass $(m)\times$acceleration (f) with which it is going down the liquid.

$$\therefore\ f=g\left(\frac{\delta-\delta'}{\delta}\right).$$

But we know that if d be the distance travelled in time, t, $d=\frac{1}{2}ft^2$

$$=\frac{1}{2}\,g\left(\frac{\delta-\delta'}{\delta}\right)t^2$$

$$\text{or,}\qquad t^2=\frac{2d\delta}{g(\delta-\delta')}\,;\quad\therefore\quad t=\sqrt{\frac{2d\delta}{g(\delta-\delta')}}.$$

(5) *A toy man weighs 150 gms. The density of the man is 1·12, that of cork 0·24 and that of water 1. What weight of cork must be added to the man that he may just float in water?*

Let W be the wt. of cork required, the volume of cork $=W/0{\cdot}24$ c.c.

Volume of man $=\frac{150}{1{\cdot}12}$ c.c. $\therefore$ Their total volume $=\left(\frac{150}{1{\cdot}12}+\frac{W}{0{\cdot}24}\right)$ c.c.

The man and cork will just float in water when their total weight is equal to the weight of water displaced by them. Hence the volume of displaced water $=(150+W)/1$ c.c. and this is equal to the total volume of man and cork.

$$\therefore\quad \frac{150}{1{\cdot}12}+\frac{W}{0{\cdot}24}=150+W;\quad\text{or}\quad W=5{\cdot}075\text{ gms.}$$

(6) *A piece of metal weighing 20 gms. has equal apparent weight with a piece of glass when suspended from the pans of a balance and immersed in water. If the water is replaced by alcohol (density 0·9), 0·84 gm. must be added to the pan from which the metal is suspended in order to restore equilibrium. Find the weight of the glass.*

Let m = mass of glass, ρ = sp. gr. of glass and d = sp. gr. of the metal.

For equilibrium in water we have $m-\frac{m}{\rho}=20-\frac{20}{d}$ (1)

and for equilibrium in alcohol $m-\left(\frac{m}{\rho}\times0{\cdot}9\right)=20-\left(\frac{20}{d}\times0{\cdot}9\right)+0{\cdot}84$ (2)

Multiplying (1) by 0·9 and subtracting it from (2)

$m\times0{\cdot}1=20\times0{\cdot}1+0{\cdot}84,\qquad m=28{\cdot}4$ gms.

(7) (*a*) *Find a mathematical expression for the density of a mixture, when the densities and the masses of the components are known.*

(*b*) *Calculate the quantity of pure gold in 100 gms. of an alloy of gold and copper of density 16. (Density of gold = 19, and that of copper = 9.)* *(Dec. 1930, '32)*

(*a*) Let m_1 and m_2 be the masses of the components and d_1 and d_2 their respective densities, and let d be the density of the mixture.

Then, the volume of the mixture $=\frac{m_1+m_2}{d}=\left(\frac{m_1}{d_1}+\frac{m_2}{d_2}\right)$ = volume of the components, whence d can be calculated.

(*b*) If m gms. be the mass of pure gold in the alloy, the mass of copper $=(100-m)$ gms. Now, volume of the alloy $=\frac{100}{16}$; vol. of gold $=\frac{m}{19}$, and vol. of copper $=\frac{100-m}{9}$. $\therefore\ \frac{100}{16}=\frac{m}{19}+\frac{100-m}{9}$, whence $m=83{\cdot}12$ gms.

(8) *A solid displaces $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of its volume respectively when it floats in three different liquids. Find the volume it displaces when it floats in a mixture formed of equal volumes of the aforesaid three liquids.* *(Pat. 1945)*

Let V be the volume and d the density of the solid, let d_1, d_2, and d_3 be the densities of the three liquids.

When a body floats, its volume$\times$its density$=$the volume immersed$\times$density of the liquid. $\therefore$ We have, (i) $Vd=V/2\times d_1$; or, $d_1=2d$, (ii) $Vd=V/3\times d_2$ or, $d_2=3d$, (iii) $Vd=V/4\times d_3$; or, $d_3=4d$.

The mixture is formed by taking, say, v c.c. of each liquid, then the total volume$=3v$ c.c. and its total mass$=v(d_1+d_2+d_3)=V(2d+3d+4d)=9vd$.

$\therefore$ Density of the mixture$=\dfrac{\text{mass}}{\text{volume}}=\dfrac{9vd}{3v}=3d$.

Now, if x c.c. be the volume of the mixture displaced, we have $Vd=x\times 3d$. Or, $x=V/3$, i.e. it displaces $\frac{1}{3}$ of its volume.

281. The Principle of a Hydrometer :— Various methods are given in Art. 287 for the accurate determination of the specific gravity of a liquid. For commercial purposes we need a method which should be simple and quick, although the results may not be very accurate. For such purposes instruments, called *Hydrometers*, are used. They depend for their action upon the principle of floatation; the principle is, "when a body floats in a liquid, the weight of the body is equal to the weight of the displaced liquid".

There are two types of hydrometers in use. One is called the **Variable Immersion** (or constant weight) type, ordinarily known as the common hydrometer, and the other, the **Constant Immersion** (or Variable weight) type, known as the Nicholson's type. In the former case, the portion of the hydrometer immersed depends on the density of the liquid; the immersion is greater, the less the density of the liquid. In the latter case, the principle used is to get the hydrometer immersed up to a fixed mark of it into the liquid, whatever is the liquid used. In commercial practice, however, the *variable Immersion type* is used.

282. The Principle of the Variable Immersion Hydrometer :— The principle of this type of hydrometer may be understood by taking an ordinary flat-bottomed uniform test-tube T (Fig. 163) and loading it with a suitable amount of sand, or lead shot, so that it floats vertically in a liquid. Paste inside the test-tube a strip of millimetre squared paper, marked off in centimetres measured from the bottom, and close the tube with a cork. Now float the tube in a jar of water and observe the depth immersed d_1. Take out the tube, wipe it dry and float it in a jar containing a different liquid. Again observe the depth immersed d_2.

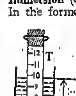


Fig. 163

Let W be the weight of the hydrometer, *i.e.* the test-tube with load, a its area of cross-section, and ρ the density of the liquid; then, since the weight of the hydrometer is equal to the weight of the displaced liquid in each case,

we have $W=a\times d_1\times 1=a\times d_2\times \rho$, or $\rho/1$ (or sp. gr.)$=d_1/d_2$.

Since d_1/d_2 is a ratio, the depths can be measured in inches or in centimetres.

The experiment can be varied with different amounts of lead shot in the tubes and a graph can be drawn with d_1 and d_2. [For the actual type of variable immersion hydrometer and its use *vide* Art. 297(2)]

Examples. (1) *The density of sea water is 1·025 gms. per c.c. and the density of ice is 0·917 gm. per c.c. Find what portion of an ice-berg is visible above the water surface, when it is in sea-water, and when in fresh water* (Utkal, 1951)

Let v be the volume of ice immersed, and V the total volume of ice.

Then $(V-v)$ is the volume which is visible above the surface of the sea.

We have, from Art. 270, $\frac{v}{V}=\frac{\text{density of ice}}{\text{density of sea-water}}=\frac{0·917}{1·025}$;

or, $$1-\frac{v}{V}=\frac{V-v}{V}=\frac{1·025-0·917}{1·025}=\frac{0·108}{1·025}=\frac{1}{9·5}.$$

Therefore the portion of the ice-berg which is above the water surface is 1/9·5 of the total volume.

In fresh water, the density of which is 1 gm. per c.c., we have

$$\frac{V-v}{V}=\frac{1-0·917}{1}=\frac{0·083}{1}=\frac{1}{12}\ \text{(approx.)}$$

(2) *A variable immersion hydrometer is prepared by taking a test tube 15 cms. long and 3 cms. wide. The test-tube which is assumed to have a uniform cross section is weighed with a few lead shots to make it float upright. A narrow piece of graph paper is pushed into the test tube to serve as a scale. The tube is then placed in glycerine of specific gravity 1·25 and then it is placed in water. The scale reading which increases upwards, is 1·6 cms. for the level of the glycerine surface and 2·8 cms. for the level of the water surface. The scale reading, when the test-tube is placed in a solution of copper sulphate, is 2·5 cms. What is the specific gravity of the latter?*

Let V be the volume of the portion of the test tube below the zero mark and V' the volume of 1 cm. of the tube.

Then in glycerine the immersed volume $=(V+1·6V')$ c.c.

∴ The upthrust in glycerine = wt. of this volume $=1·25\,(V+1·6V')$.

Similarly, the upthrust in water $=1\times(V+2·8\,V')$.

Since each upthrust = wt. of the test-tube, $V+2·8\,V'=1·25\times(V+1·6\,V')$,

or, $0·25\,V=0·8\,V'$; or, $V=3·2V'$. Again, if S be the sp. gr. of the copper sulphate solution, $S\,(V+2·5V')=(V+2·8V')$.

or, $$S=\frac{V+2·8V'}{V+2·5V'}=\frac{3·2V'+2·8V'}{3·2V'+2·5V'}=\frac{6}{5·7};\quad\therefore\ S=1·05$$

[Note that this example explains the principle of preparing and graduating a variable immersion type of hydrometer.]

(3) *The stem of a common hydrometer is cylindrical and the highest graduation corresponds to a specific gravity 1·0 and the lowest to 1·3. What specific gravity corresponds to a point exactly midway between these divisions?* (Pat. 1931)

Let l be the total length of the stem from the lowest to the highest graduation, a the area of cross-section of the stem, V the volume of the bulb up to the lowest graduation and W the weight of the hydrometer. Then,

$(V+la)\times 1=W$ and $V\times 1·3=W$ (1)

∴ $V+la=1·3V$; or, $0·3V=la$ (2)

Again, $(V+l/2\times a)S=W$, where S is the required sp. gr.

From (1) and (2), $(V+\frac{0\cdot3}{2}V)S=1\cdot3V$; or, $S=\frac{2\cdot6}{2\cdot3}=1\cdot13$.

283. Archimedes (287—212 B.C.) :—A mathematician and inventor of immortal name. He was born at Syracuse in Sicily. Son of a mathematician-astronomer, he was a close associate of Hiero, King of Syracuse. He may be regarded as an ideal scientific worker, always occupied with thinking on his problems. During the Roman invasion of Syracuse in 212 B.C., it is said the soldiers entered his premises and challenged him. At that time he was brooding over a geometrical figure drawn on sand before him and he had not time to reply. Just before he was slain, he called out to the soldiers, "Kill me but spare my figure." Once Hiero ordered a crown to be made for him in pure gold. When the crown was presented, he requested Archimedes to test if it was made of pure gold (of course without causing any damage to it). This put Archimedes to restless thinking. One day while he was bathing in a tub of water, it is said he felt a loss of weight of his body as the water was displaced. At once the idea crossed his mind that a body immersed in a liquid loses a part of its weight. Subsequently, he found this loss in weight to be equal to the weight of the displaced water. This enabled him to find the volume of the crown and calculate its density and compare it with that of a piece of pure gold. It is so said that he jumped up from the tub in ecstasy of joy and rushed out into the street, naked, crying, *"Eureka! Eureka!"* (*i.e.* I have found out, I have found out).

The following statement on the lever is another famous story told of him. He said, "Give me a place to stand on and I will move the earth." In testifying to the truth in it in presence of Hiero, he applied one end of a lever to a ship and while the other end was lightly pressed upon by Hiero himself, the ship moved into the water.

His name is connected with many inventions in Machines, Mechanics and Mathematics. The pulley, the windlass, the Archimedian screw, hydraulic and compressed air machines are some of them. He is said to have used the concave mirror for the first time to focus the sun's rays for generating heat at a point. Besides the principle of buoyancy and his work on floating bodies he also discovered how the circumference of a circle could be calculated and his method gave the number later designated by the Greek letter π. He also developed the Conic Sections and the concept of infinity is due to him.

Questions

1. Explain how Archimedes' principle may be used to distinguish a metal from its alloy. (C. U. 1922, '26; *cf.* Pat. 1928, '32)

[Hints.—Determine the density of the alloy and compare its value with that of the pure metal.]

2. Why is it easier to lift a heavy stone under water than in air?
(C. U. 1937)

3. A beaker containing water weighs 300 gms., and a piece of metal whose volume is 10 c.c. and mass 88 gms. is immersed in the water, being suspended by a fine thread. Find (a) the upward force which must be applied to the thread to support the metal, and (b) the upward force necessary to support the beaker.
[*Ans.* (a) 78 gms.-wt., (b) 310 gms.-wt.]

4. A flask when full of water weighs 75 gms.; when full of mercury of density 13.6 gms. per c.c. it weighs 705 gms. and when full of sulphuric acid it weighs 117 gms. Find the density of the acid. (C. U. 1952)
[*Ans.* 1.83 gms. per c.c.]

5. Describe how you will determine experimentally the density of a metal in the form of a long wire of about 5 metres in length. (Pat. 1922)

[Hints.—Measure the diameter and hence the radius of the wire by a screw gauge and measure the length. Volume$=\pi r^2 l$. Weigh it in a balance by turning the wire into a coil of several turns. Then density=mass/volume.

The volume can also be determined by the method by displacement of water.]

6. When two equal volumes of two substances are mixed together, the sp. gr. of the mixture is 4. But when equal weights of the same substances are mixed together, the sp. gr. of the mixture is 3. Find the sp. gravities of the two substances. (Utkal, 1954)
[*Ans.* 6 and 2]

7. The densities of three liquids are in the ratio of 1 : 2 : 3. What will be the relative densities of mixtures made by combining (a) equal volumes, (b) equal weights. (Gau. 1953, C. U. 1954)
[*Ans.* 11 : 9]

8. A block of wood of rectangular section and 6 cm. deep floats in water. If its density is 0·6 gm./cm.^{3}, how far below the surface is its lower face?

What weight placed on the upper surface of the block is needed to sink it to a depth of 5 cm., if its area is 120 cm.^{2}?
[*Ans.* 3·6 cm., 168 gm.]

9. Under what conditions do bodies float or sink in a liquid? A piece of iron weighing 272 gms. floats in mercury of density 13.6 with $\frac{5}{8}$ths of its volume immersed. Determine the volume and density of iron.
(C. U. 1930, cf. Dac. 1927, '29)

[Hints.—Let the volume of the iron piece be x c.c. Then $\frac{5}{8}x$ c.c.=volume of iron piece immersed in mercury=volume of mercury displaced by the iron piece.

Then $\frac{5}{8}x=\frac{272}{13.6}$, or, $x=32$ c.c., and density $=\frac{\text{mass}}{\text{volume}}=\frac{272}{32}=8.5$ gms. per c.c.]

10. Discuss the stability of equilibrium of a floating body. Apply your results to the case of a uniform sphere of wood floating on water. (Pat. 1947)

11. State the conditions of equilibrium of a floating body and explain what is meant by *metacentre*. Discuss, in general terms, the question of stability of floatation. Why is the hold of a ship generally loaded with ballast?

A flat boat is 20 ft. by 30 ft. in area. How much will it be lowered when carrying a 1-ton automobile?
[*Ans.* 0·72 inch approx.]

12. What is meant by 'buoyancy'? Explain why an iron ship floats in water.
(C. U. 1928, '37; Pat. 1932; Dac. 1933)

13. Describe the 'Cartesian diver' and explain how it acts. Do you know of any modern appliance which is based on this principle? (C. U. 1930, '46)

14. The specific gravity of ice is 0·918 and that of sea-water is 1·03. What is the total volume of an ice-berg which floats with 700 cubic yards exposed? (C. U. 1932, Pat. 1935)

[**Hints.**—Let V cubic yards be the total volume of the ice-berg. ∴ Volume under water $= (V-700)$ cu. yards. The mass of the ice-berg $= (V\times27)\times62{\cdot}5\times0{\cdot}918$ lbs.]

The mass of the sea-water displaced $=(V-700)\times27\times62{\cdot}5\times1{\cdot}03$ lbs. According to the law of floatation, the mass of the floating body=mass of the displaced liquid. ∴ $(V\times27)\times62{\cdot}5\times0{\cdot}918=(V-700)\times27\times62{\cdot}5\times1{\cdot}03$; or, $V=6437{\cdot}5$ cu. yds.]

15. You are provided with a hollow glass tube of uniform cross-section with a bulb blown at its lower end, and other necessary materials. State how you will proceed to construct a common hydrometer and explain how you will graduate it. [*See also* Art. 287(2).] (Pat. 1944)

16. Two bodies are in equilibrium when suspended in water from the arms of a balance. The mass of one body is 28 gms. and its density 5·6 gms./c.c.; if the mass of the other is 36 gms., what is the density? (Pat. 1928)
[*Ans.* 2·77 gms./c.c.]

17. *1 c.c. of lead (sp. gr. 11·4) and 21 c.c. of wood (sp. gr. 0·5) are fixed together.* Show whether the combination will float or sink in water. (C. U. 1933)

[**Hints.**—Mass of 1 c.c. of lead=11·4 gms.; mass of 21 c.c. of wood=$21\times0{\cdot}5$ $=10{\cdot}5$ gms. ∴ The total mass of the combination $=11{\cdot}4+10{\cdot}5=21{\cdot}9$ gms.

Their total volume $=21+1=22$ c.c. So the combination floats with 21·9 c.c. of it being immersed in water and keeping the rest (*i.e.* 0·1 c.c.) above the surface of water.]

18. Show that a hollow sphere of radius R made of metal of sp. gr. S will float on water, if the thickness of its wall is *less than* $R/3S$. (Nag. U. 1952)

CHAPTER XI

SPECIFIC GRAVITY

284. Density and Specific Gravity :— The density *of a substance is its mass per unit volume*, i.e. its

$$\text{density} = \frac{\text{mass*}}{\text{Volume}}.$$

The **specific gravity** *of a substance is the ratio of the weight of any volume of the substance to the weight of an equal volume of water at 4°C.*

$$\text{Sp. gr. of a substance} = \frac{\text{Wt. of } V \text{ c.c. of the substance}}{\text{Wt. of } V \text{ c.c. of water at } 4^\circ\text{C.}}$$

$$= \frac{\text{Mass of } V \text{ c.c. of the substance}}{\text{Mass of } V \text{ c.c. of water at } 4^\circ\text{C.}}$$

$$= \frac{\text{Mass of unit volume of the substance}}{\text{Mass of unit volume of water at } 4^\circ\text{C.}}$$

$$= \frac{\text{Density of the substance}}{\text{Density of water at } 4^\circ\text{C}}$$

* If the mass of a body is uniformly distributed in the volume, the density of a part of it is the same as the density of the body as a whole. Ordinarily, the density of a substance is experimentally determined by taking a portion of it assuming the density to be uniform. To test if the density is uniform, experiments may be carried out by taking samples from the different portions of the substance. If they slightly vary, the average density is obtained by finding the arithmetic mean value of the several densities determined.

So, the specific gravity of a substance is really a *relative density*, *i.e.* its density relative to that of water at 4°C.

Note. (*i*) *Specific gravity is expressed as a ratio*: it expresses the number of times a substance is heavier than an equal volume of water at 4°C. So it is a pure **number** while density, which is the mass per unit volume, is not a mere number. *Density must be expressed in some unit, say, in grams per cubic centimetre, or in pounds or ounces per cubic foot.*

(*ii*) We may speak of *mass* instead of *weight* in defining specific gravity, because the ratio of two masses is equal to the ratio of their weights at the same place.

285. Relation between Density and Specific Gravity in the Two System of units:

(*a*) In C.G.S. units, the density of water at 4°C = the mass of 1 c.c. of water at 4°C = 1 gram per c.c.

Since 1 c.c. water weighs 1 gm., the volume of a substance in c.c. is *numerically* equal to its mass in grams. *So the density of a substance may be written in C.G.S. units as follows:*

$$\text{Density} = \frac{\text{Mass of the substance}}{\text{Volume of the substance}}$$

$$= \frac{\text{Mass of the substance}}{\text{Mass of an equal volume of water}}, \text{ numerically.}$$

But the ratio of the masses of two bodies is the same as the ratio of their weights. Hence, when measured in C.G.S. units,

$$\text{Density} = \frac{\text{weight of a body}}{\text{weight on an equal volume of water}}$$

$$= \text{specific gravity of the body.}$$

Therefore, the density of a substance in C.G.S. units is numerically [illegible] For example, the density of lead in C[illegible] c.c. and the sp. gr. of lead = [illegible] per c.c. / [illegible] per c.c. = [illegible]6

(*b*) In F.P.S. units, the densi[illegible] of water at 4°C = the mass of 1 cu. ft. of water at 4°C = 62.5 lb[illegible]. [illegible] Since sp. gr. of a substance = $\frac{\text{density of the substance}}{\text{density of water at 4°C.}}$, the density of a substance = density of water at 4°C. × sp. gr. of the substance.

So, the density of a substance in F.P.S. units (lbs. per cu. ft.) is numerically equal to 62·5 × sp. gr. of the substance.

For example: (*i*) the density of lead in F.P.S. units is 709 pounds per cu. ft. and the sp. gr. of lead $= \frac{709 \text{ lbs. per cu. ft.}}{62.5 \text{ lbs. per cu. ft.}} = 11.3.$

(*ii*) The density of iron in C.G.S. units is 7.8 gms. per c.c. and the sp. gr. of iron $= \frac{\text{mass of 1 c.c. of iron}}{\text{mass of 1 c.c. of water}} = \frac{7.8}{1} = 7.8.$

And since the density of iron in F.P.S. units is 487·5 lbs. per cu. ft. then the sp. gr. of iron $= \frac{\text{mass of 1 cu. ft. of iron}}{\text{mass of 1 cu. ft. of water}} = \frac{487 \cdot 5}{62 \cdot 5} = 7 \cdot 8$.

(c) In C.G.S. units, sp. gr. $= \frac{\text{density of the substance}}{\text{density of water at } 4^\circ\text{C.}}$

$$= \frac{\rho \text{ gms./c.c.}}{1 \text{ gm./c.c.}} = \rho .$$

In F.P.S. units, sp. gr. $= \frac{\text{density of the substance}}{\text{density of water at } 4^\circ\text{C.}}$

$$= \frac{62 \cdot 5 \times \rho \text{ lbs./cu. ft.}}{62 \cdot 5 \text{ lbs/cu. ft.}} = \rho .$$

Thus sp. gr. is the same in both the units.

The relations between the two systems will be clear from the following table:

System	Density	Specific gravity
Metric (or C.G.S.) ...	x gms. per c.c.	x
British (or F.P.S.) ...	$62 \cdot 5 \times x$ lbs. per cu. ft.	x

286. Sp. Gr. of Solids :—

(1) By Direct Measurement.— In the case of a solid having some regular form (*e.g.* rectangular, spherical or cylindrical), the volume of the solid can be calculated by measuring its linear dimension. The body is then weighed. Let the weight of the body be W gm., and let its volume be V c.c., then
density of the body $= W/V$ gm. per c.c.,
and sp. gr. " $= W/V$.

(2) By the Hydrostatic Balance :—

(a) Solid heavier than water.—

Let the weight of the solid in air $= W_1$ gm. and the wt. in water $= W_2$ gm.

To take the weight of the body in water, it is suspended by means of a fine thread from the hook of the left pan and made to sink completely in water contained in a beaker (Fig. 164). The beaker is placed on a small wooden bridge, which is put across the pan in such a way that the bridge, or the beaker, does not touch any part of the pan of the balance.

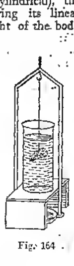


Fig. 164

The weight of the same volume of water as that of the solid $= (W_1 - W_2)$ gm.

$\therefore$ Sp. gr. $= \dfrac{\text{wt. of the body in air}}{\text{wt. of an equal volume of water}} = \dfrac{W_1}{W_1 - W_2}$.

(b) Solid lighter than water.—

Let the weight of the solid in air $= W_1$ gm

Take another heavy body, called a *sinker*, such that the two tied together may sink in water.

Let the weight of the solid and sinker both in water $= W_2$ gm,

and the weight of the sinker alone in water $= W_3$ gm.

$\therefore$ The weight of solid in air + the weight of sinker in water − the weight of solid and sinker in water = upward thrust by water = the weight of water whose volume is the same as that of the solid $= (W_1 + W_3 - W_2)$gm.

Hence, $$\text{Sp gr} = \frac{W_1}{W_1 + W_3 - W_2}.$$

Otherwise thus:—

wt. of the solid in air $= W_1$

wt. of solid in air + sinker in water $= W_3$

wt. of solid and sinker both in water $= W_2$

$\therefore$ wt. of water displaced by solid $= W_3 - W_2$.

Hence, $$\text{Sp gr} = \frac{W_1}{W_3 - W_2}.$$

(c) Solid soluble in water.—The specific gravity of a solid soluble in water can be found by immersing the solid in a liquid of known specific gravity in which the solid is insoluble

Determine the specific gravity of the solid relative to the liquid *Then the actual specific gravity of the solid will be obtained by multiplying this value with the specific gravity of the liquid.* For we have,

$$\begin{aligned}\text{Sp gr of the solid} &= \frac{\text{weight of solid in air}}{\text{weight of the same volume of water}}\\ &= \frac{\text{weight of solid in air}}{\text{weight of the same volume of liquid}}\\ &\times \frac{\text{weight of the same volume of liquid}}{\text{weight of the same volume of water}}\\ &= \frac{\text{weight of solid in air}}{\text{weight of the same volume of liquid}}\\ &\times \text{sp gr. of the liquid} = \frac{W_1}{W_1 - W_2} \times \rho.\end{aligned}$$

where W_1 = wt. of solid in air; W_2 = wt. of solid in the given liquid; ρ = sp. gr. of the liquid.

(3) By the Specific Gravity Bottle.—It is a glass bottle fitted with a ground glass stopper having a narrow central bore. The bottle is filled to the top of the neck with any liquid, and the surplus liquid overflows through the hole in the stopper when the stopper is pushed into its position (Fig. 165). Shake the bottle to remove air bubbles. The bottle holds a definite quantity of liquid. *This bottle is used to find out the specific gravity of a solid in the form of powder, or small fragments, and of liquids also.*

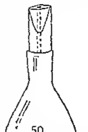


Fig. 165—The Specific Gravity Bottle.

Let the weight of the empty bottle $= W_1$ gm.

The weight of the bottle+powder put inside $= W_2$ gm.

$\therefore$ The weight of the powder$=(W_2 - W_1)$ gm.

The weight of the bottle+powder+water to fill the rest of the bottle$= W_3$ gm.

Now pour out all the contents of the bottle and fill it up with pure water taking care to remove any air bubbles from inside.

Let the weight of the bottle when full of water$=W_4$ gm.

Then the weight of an equal volume of water as that of the powder

$$=(W_4 - W_1)-(W_3 - W_2) \text{ gm.}$$

Hence, $$\text{sp. gr.}=\frac{W_2 - W_1}{(W_4 - W_1)-(W_3 - W_2)}.$$

N.B. To determine the specific gravity of a powder soluble in water, a liquid is taken in which the solid does not dissolve or chemically act. *Then, the sp. gr. so found is multiplied by the sp. gr. of the liquid at the observed temperature.*

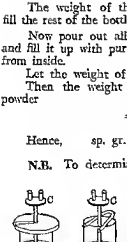


Fig. 166—The Nicholson's Hydrometer.

(4) By the Nicholson's Hydrometer.—This is a *constant immersion type* hydrometer.

It consists of a cylindrical hollow vessel *A* to which is attached a thin stem *B* at the top of which there is a small scale-pan *C* (Fig. 166). Below the vessel is attached, by the curved metallic hook *D*, a conical pan which is so weighted with lead shots or mercury that the hydrometer may float vertically in a liquid. There is a scratch mark on the stem up to which the instrument is always made to sink in a liquid. The hydrometer is placed in

water contained in a glass cylinder. A slotted cardboard (left-hand figure) or a bent wire (right-hand figure), is so placed across the mouth of the cylinder that the upper pan is arrested before sinking into water in the cylinder. All the joints in the hydrometer must be made air-tight. Weights are placed on the upper pan of the hydrometer to make it sink up to the mark on the stem. Let the weight required be W_1 gm.

Remove the weights and place the solid on the upper pan. Add weights again on the upper pan to make the instrument sink up to the mark. Let it be W_2 gm. Then the weight of the body in air $=(W_1-W_2)$ gm.

Now remove the weight and place the body in the lower pan which is in water.

Again, find the weights necessary to bring the hydrometer up to the mark. Let this weight be W_3 gm.

Then the weight of the body in water $=(W_1-W_3)$ gm.

The weight of displaced water $=(W_1-W_2)-(W_1-W_3)$ gm
$=(W_3-W_2)$ gm.

$$\therefore \quad \text{Sp. gr} = \frac{W_1-W_2}{W_3-W_2}.$$

(**Note.**—It is evident that the method depends on Archimedes' principle. If the solid be lighter than water, tie it to the lower pan and proceed exactly as above.)

(5) By Method of Floatation—[*Vide* Art 280]

287. Specific Gravity of Liquids:

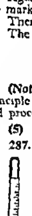


Fig 167—The Common Hydrometer

(1) By the Hydrostatic Balance.—

Let the weight of a solid body, which is heavier than the liquid but which is not chemically acted upon by it $=W_1$ gm., and the weight of the solid when immersed in water $=W_2$ gm.; and that when immersed in the liquid $=W_3$ gm.

The (W_1-W_3) represents the weight of a volume of liquid equal to the volume of the solid; and (W_1-W_2) is the weight of the same volume of water.

$$\therefore \quad \text{Sp. gr} = \frac{W_1-W_3}{W_1-W_2}.$$

(2) By the Common (or Variable Immersion) Hydrometer.—

Description.—This is a glass instrument (Fig. 167) which floats vertically in different liquids with a part of the stem above the surface of the liquid. In order that the instrument may float vertically, the small lower bulb B is weighed with mercury or lead shots. The weight of the liquid displaced by the hydrometer is equal to the weight of the hydrometer itself, which is always constant. But mass=volume×density; hence

mass being constant, volume is inversely proportional to density. So the volume of the liquid displaced increases as the density of the liquid diminishes; hence it sinks deeper into a lighter liquid than in a heavier one. The stem S can thus be graduated so that the specific gravity of a liquid can be read off directly. The number of the division on the scale fixed in the tube, which is in level with the surface of the liquid, gives the specific gravity of the liquid.

In Fig. 168, a common hydrometer used for testing the sp. gr. of accumulator acids, etc. is shown. A quantity of the liquid is drawn up in the outer casing by dipping the lower end of the hydrometer into the liquid and then pressing the rubber bulb when some air will be forced out. On releasing the pressure, the atmospheric pressure will raise the liquid into the casing so as to enable the hydrometer to float.

Fig. 168

Graduation of a Common Hydrometer.—To graduate the instrument, float it in water and put a mark on the stem which is in line with the surface of the liquid, and similarly put another mark on the stem when it is floated in another liquid of known density (d). Let the lengths of the stem exposed above the surface of the liquid in the two cases be l_1 and l_2 respectively. Then, if W be the weight and V the volume of the instrument, and a the area of cross-section of the stem, we have,

$W=(V-l_1a)\times 1=(V-l_2a)\times d,$ the density of water being 1.

$$\therefore \quad l_1=\frac{1}{a}(V-W), \text{ and } l_2=\frac{1}{a}\left(V-\frac{W}{d}\right);$$

$$\text{or, } (l_2-l_1)=\frac{W}{a}\left(1-\frac{1}{d}\right).$$

Similarly, if l be the length of the stem exposed in a liquid of density d', we have,

$$(l-l_1)=\frac{W}{a}\left(1-\frac{1}{d'}\right). \quad \therefore \frac{(l-l_1)}{(l_2-l_1)}=\frac{1-1/d'}{1-1/d}.$$

For different values of d' the corresponding value of l can be calculated from the above relation and the instrument can thus be graduated.

It is so graduated that when the hydrometer is floated in water, the scale reading is 1000, which means a sp. gr.$=1\cdot000$. In another liquid it might be 1210, *i.e.* the sp. gr.$=1\cdot210$.

Commercial Hydrometers.—The variable immersion type of hydrometer is generally used in different industries for finding the densities of liquids, and these hydrometers are named according to the

use to which each is put; for example: it is called a lactometer when it is used to find the sp. gr. of milk (which is generally between 1·029 and 1·033), an alcoholometer when used to find the density of alcohol, and a saccharometer to determine the sugar content of a solution.

The determination of density by means of a lactometer, however, is not a conclusive test of the *purity of the milk*; for, the density of skimmed milk is greater than that of unskimmed milk; so by adding water to skimmed milk, the density can be brought to its normal value. So the amount of fat should be determined along with the density in order to test the quality of milk.

(3) By Nicholson's Hydrometer.—

In this experiment the principle that a floating body displaces its own weight of the liquid in which it is floated is utilised by immersing the hydrometer each time up to the same index mark in the liquid and in water.

Let the weight of the hydrometer be W_1 gm.

It is then floated in the liquid contained in a glass cylinder and weights are added on the upper pan to make it sink up to the index mark. Let this weight be W_2 gm.

∴ The total weight of the displaced liquid $=(W_1+W_2)$ gm.

Similarly, let the weight required on the upper pan to bring it up to the index mark when placed in water $=W_3$ gm.

∴ The weight of the displaced water whose volume is the same as that of the displaced liquid $=(W_1+W_3)$ gm. ∴ The volume of the displaced liquid $=(W_1+W_3)$ c.c.

$$\text{Sp. gr} = \frac{W_1+W_2}{W_1+W_3}.$$

Alternative method (without using a Balance).— A piece of solid is taken which is not soluble in the liquid and also will not react chemically with it.

Let the weight required on the upper pan to sink the hydrometer in water up to the index mark, when the solid is placed on the upper pan be W_1.

The solid is then placed in the lower pan and let the wt. required to sink the instrument up to the mark $=W_2$.

Then $(W_2-W_1)=$ wt. of the same volume of water as that of the solid $=$ volume of the solid (∵ Sp. gr. of water $=1$).

Similarly, let W_3 and W_4 be the corresponding weights when the above operations are repeated in the given liquid; then

$(W_4-W_3)=$ wt. of the same volume of the liquid as that of the solid.

$$\therefore \text{ Sp. gr. of liquid} = \frac{W_4-W_3}{W_2-W_1}.$$

(4) By Specific Gravity Bottle.—

Let the wt. of the empty bottle $= W_1$ gm.

It is then filled completely with water and weighed. Let this weight be W_2 gm.

The bottle is emptied out and carefully dried. It is then filled with the liquid. Let the weight be W_3 gm. Then,

$$\text{Sp. gr.} = \frac{W_3 - W_1}{W_2 - W_1}.$$

(5) By Balancing Columns *(U-tube)*.—The densities of two different liquids, which do not **mix**, nor have any chemical action with each other, can be determined by pouring them one after another in a U-tube.

Take a U-tube of glass and pour first the heavier of the two liquids taken (say, mercury), and note that the liquid (mercury) attains the same level in both the limbs (Fig. 169). Now carefully pour some other liquid, say, water into the left-hand limb. The weight of water pushes the mercury down in the left-hand limb and up in the right-hand limb. Let C be the common surface of separation of mercury and water. Consider the horizontal level AC. The pressures at these two points A and C must be equal because the liquids are at rest, and so the two columns AD and CL are called *balancing columns*.

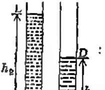


Fig. 169—The Balancing Columns.

Now, pressure at A = force exerted on unit area at A
$= P +$ wt. of the column AD of 1 sq. cm. base
$= P +$ volume of the column AD of 1 sq. cm. base × density × g
$= P + h_1 \times \rho_1 \times g$; where P = atmospheric pressure, $h_1 = AD$, ρ_1 = density of mercury and g is the acceleration due to the gravity.

Similarly, pressure at $C = P = h_2\rho_2 g$, where $h_2 = CL$, and ρ_2 = density of water.

$$\therefore \quad P + h_2\rho_2 g = P + h_1\rho_1 g\,;\; h_2\rho_2 = h_1\rho_1\,;$$

$$\text{or,} \quad \frac{h_1}{h_2} = \frac{\rho_2}{\rho_1}.$$

That is, *the height of the balancing columns are inversely proportional to the densities of the liquids.*

In this case, ρ_1/ρ_2 is the ratio of the density of the liquid (mercury) to the density of water, *i.e.* it is the sp. gr. of the liquid.

(6) By Hare's Apparatus.—The above U-tube method can be applied when the ***liquids do not mix up***, but when two liquids mix up

they must be kept separate, and in that case the following method, which is merely a modification of the U-tube method described above, can be adopted. By this method the relative densities of two liquids can be determined by *balancing* two liquid columns against each other.

The Hare's apparatus consists of two parallel vertical tubes M and N connected at the top by a three-way tube A fitted with a piece of India-rubber tubing B and a clip P (Fig. 170) So it is merely an inverted U-tube with a side tube at the top. The lower end of each tube is dipped in a liquid contained in a beaker x or y. The liquids are drawn up to different heights when sucked through C and they are kept steady by means of the clip P Generally water is taken as one of the liquids.

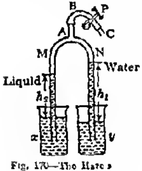


Fig. 170—The Hare's Apparatus

Let h_1 and h_2 be the heights of the liquid columns, having densities ρ_1 and ρ_2 respectively. The height in each case is measured from the surface of the liquid in the beaker up to the lower meniscus of the top of the liquid column. Let,

P = the atmospheric pressure and

p = the pressure of air inside the tube.

The pressure on the liquid in the beaker (y) $=P=g\rho_1 h_1+p$, and that in the beaker (x) $=P=g\rho_2 h_2+p$; and they are equal.

$$\therefore \quad p+g\rho_1 h_1=p+g\rho_2 h_2.$$

$$\text{or,} \quad h_1\rho_1=h_2\rho_2.$$

$$\text{or,} \quad \frac{h_1}{h_2}=\frac{\rho_2}{\rho_1}.$$

That is, *the densities are inversely as the heights of the liquid columns.*

Knowing one of these the other is known

Note.—(i) It is to be noted that though the cross-sections of the tubes do not come into consideration, the tubes should be of moderately wide bore in order to avoid the effects of surface tension. If, however, there is any rise of the liquid column due to capillarity, this should be measured and subtracted from the corresponding height. (ii) Both the tubes need not be of the same bore, as pressure depends only on the vertical height. (iii) It should be tested whether the tubes are vertical. (iv) Take the heights after the liquid columns are steady, which will not be the case if the apparatus is not air-tight. (v) Draw a graph with h_1 and h_2 (which should be a straight line), and calculate h_1/h_2 corresponding to the highest point in the graph, because that will introduce the least error.

Examples. (1) *The cross-sections of two limbs of a U Tube are 10 sq. cms and 1 sq. cm. in area respectively. The lower part of both tubes contains mercury (sp. gr. 13·6). What volume of water must be poured into the wider tube to raise the surface of mercury in the narrow tube by 1 cm?* (Pat. 1928)

The area of cross-section of the wide tube is 10 sq. cms., and that of the narrow tube is 0·01 *sq. cm.*

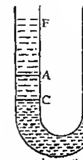


Fig. 171

In Fig. 171, let A and B be the original position of mercury levels in the two limbs and C and E the final positions. Let C and D be in the same horizontal level.

The volume of mercury raised in the smaller tube must be equal to the volume of $(AC \times 10)$ c.c. of water.

$\therefore$ $EB \times$its area$=AC \times$its area;

or, $1 \times 0{\cdot}01 = AC \times 10$; or, $AC = 0{\cdot}01/10$ cm.

But $AC = BD$. $ED = 1 + {\cdot}01/10 = 1{\cdot}001$ cm.

The press. at C=the press. at D.

Since, density of water=1.

$FC \times 1 \times g = 1{\cdot}001 \times 13{\cdot}6 \times g$.

$\therefore$ $FC = 1{\cdot}001 \times 13{\cdot}6 = 13{\cdot}6136$ cm.

$\therefore$ The volume required$=13{\cdot}6136 \times 10 = 136{\cdot}136$ c.c.

(2) Mercury (density 13·6) and a liquid which does not mix with water are placed in the limbs of a U-tube, and the surfaces of the mercury and the liquid are at 3 and 28 cms. respectively from their common surface. Find the density of the liquid. What change, if any, would be produced, if the U-tube is immersed wholly in water so that it enters into both the limbs of the tube? (*Pat. 1938*)

As in Art. 287(5), $P + h_1\rho_1 g = P + h_2\rho_2 g$, where P is the atmospheric pressure; $h_1 = 3$ cms.; $\rho_1 = 13{\cdot}6$; $h_2 = 28$ cms.; and ρ_2 is the density of the liquid.

$\therefore$ $\rho_2 \times 28 = 3 \times 13{\cdot}6$; whence $\rho_2 = \frac{3 \times 13{\cdot}6}{28} = 1{\cdot}457$.

When the U-tube is immersed in water, the height of water in the limb above D (Fig. 169) will be greater than that above L; so the pressure above D being greater, the mercury column will be depressed a little and the liquid column will be raised up.

288. Temperature Correction :— Ordinarily the specific gravity of a substance is determined relative to water at the room temperature, but if true specific gravity is to be obtained it must be relative to water at 4°C. If, however, the water is taken at the room temperature t°C., the true specific gravity of the substance would be given by the product of the actual value of sp. gr. obtained by experiment at t°C., and the *sp. gr. of water* at t°C. For the true sp. gr. at 4°C.

$$= \frac{\text{weight of any volume of a substance}}{\text{weight of an equal volume of water at } t^\circ\text{C.}} \times \frac{\text{weight of the same volume of water at } t^\circ\text{C.}}{\text{weight of the same volume of water at } 4^\circ\text{C.}}$$

=sp. gr. of the substance at t°C. × *sp. gr. of water at* t°C.

(**N.B.**—In the C.G.S. unit, density is numerically equal to sp. gr.)

Examples. *(1) A piece of metal weighs 100 grams in air and 83 grams in water. What would it weigh in a liquid of specific gravity 1·5?* (*C. U. 1915*)

The weight of the volume of displaced water $=100-88=12$ grams.

$\therefore$ Volume of the body = volume of the displaced water $=12$ c.c.

The weight of 12 c.c. of the liquid $=12\times1{\cdot}5=18$ grams. Hence, the apparent weight of the body in the liquid $=100-18=82$ grams.

(2) *A test tube is loaded with shots so that it floats in alcohol immersed to a mark on the tube, the tube and shots weighing 17·1 gms. The tube is then placed in water and shots added to sink it to the same mark; the tube and shots now weigh 20·3 gms. Find the specific gravity of alcohol.* (Pat. 1922)

The wt. of displaced alcohol whose volume is equal to that of the test tube up to the mark $=17{\cdot}1$ gms., and the wt. of the same volume of displaced water $=20{\cdot}3$ gms.

Hence, sp. gr. of alcohol $=\dfrac{17{\cdot}1}{20{\cdot}3}=0{\cdot}84$

(3) *A lump of gold mixed with silver weighs 20 grams. The specific gravity of the lump is 15. Find the quantity of gold in the lump. (Sp. gr. of gold = 19·3, sp. gr. of silver = 10·5.)*

Let W_1 be the weight of gold in the lump, and W_2 that of silver in the lump.

The volume of gold $=W_1/19{\cdot}3$ c.c., the volume of silver $=W_2/10{\cdot}5$ c.c.

The weight of displaced water, when the lump is weighed in water, is $\left(\dfrac{W_1}{19{\cdot}3}+\dfrac{W_2}{10{\cdot}5}\right)$ gms.

The sp. gr. of the lump $=\dfrac{\text{weight of the lump in air}}{\text{weight of displaced water}}$, or, $15=\dfrac{20}{\dfrac{W_1}{19{\cdot}3}+\dfrac{W_2}{10{\cdot}5}}$;

or, $10{\cdot}5\,W_1+19{\cdot}3\,W_2=270{\cdot}2$, and $W_1+W_2=20$ gms., whence $W_1=13{\cdot}16$ gms.

(4) *The crown of Hiero weighed 20 pounds. Archimedes found that immersed in water it lost 1·25 pounds. The crown was made of gold and silver. Find the weight of these metals. (Sp. gr. of gold = 19·3; sp. gr. of silver = 10·5.)* (Dec. 1931)

Let W_1 lbs. be the wt. of gold, W_2 that of silver, then $W_1+W_2=20$ lbs.

The specific gravity of gold is 19·3; hence the density of gold $=(19{\cdot}3\times62{\cdot}5)$ lbs. per cu. ft. (*see* Art. 284). Similarly, the density of the silver $=(10{\cdot}5\times62{\cdot}5)$ lbs. per cu. ft.

The volume of gold $=\dfrac{W_1}{19{\cdot}3\times62{\cdot}5}$ cu. ft., the volume of silver $=\dfrac{W_2}{10{\cdot}5\times62{\cdot}5}$ cu. ft.

$\therefore$ The total volume of the crown $=\left(\dfrac{W_1}{19{\cdot}3}+\dfrac{W_2}{10{\cdot}5}\right)\times\dfrac{1}{62{\cdot}5}$ cu. ft.

Now, the weight of the displaced water $=1{\cdot}25$ lbs. The volume of this water $=(1{\cdot}25/62{\cdot}5)$ cu. ft. and this must be equal to the volume of the crown.

Hence, $\left(\dfrac{W_1}{19{\cdot}3}+\dfrac{W_2}{10{\cdot}5}\right)\times\dfrac{1}{62{\cdot}5}=1{\cdot}25\times\dfrac{1}{62{\cdot}5}$; or, $\dfrac{W_1}{19{\cdot}3}+\dfrac{W_2}{10{\cdot}5}=1{\cdot}25$.

Also, we have, $W_1+W_2=20$. From these two equations we get,

$W_1=15.078$ lbs. and $W_2=(20-15.078)=4.922$ lbs.

(5) The mass of an alloy of copper and lead is 320 gms.; the total volume is 30 c.c. Find the volume of each metal. (Sp. gr. of copper=8·8; sp. gr. of lead=11·3).

Let x c.c.=volume of copper; y c.c.=volume of lead.

$\therefore$ Mass of copper$=x\times 8.8$ gm.; mass of lead$=y\times 11.3$ gm.

Hence, $x\times 8.8+y\times 11.3=320$; and $x+y=30$.

Solving these equations, we get, $x=7.6$ c.c.; $y=22.4$ c.c.

(6) A cylindrical tube one metre long and one centimetre in internal diameter weighs 100 gms. when empty and 150 gms. when filled up with a liquid. Find the specific gravity of the liquid. *(Pat. 1928).*

The wt. of the liquid=150 - 100=50 gms.

The volume of the liquid = internal volume of the cylinder

$$=\frac{22}{7}\times(0.5)^2\times 100=78.57 \text{ c.c.}$$

$\therefore$ The density of the liquid$=\dfrac{50}{78.57}$ gms. per c.c. $=0.636$ gm. per c.c.

But the density of water is 1 gm. per c.c.; so sp. gr. of the liquid $=\dfrac{0.636}{1}=0.636$.

(7) A mixture is made of 7 c.c. of a liquid of specific gravity 1·85 and 5 c.c. of water. The specific gravity of the mixture is found to be 1·615. Determine the amount of contraction. *(C. U. 1937)*

Mass of 7 c.c. of liquid of sp. gr. $1.85=7\times 1.85=12.95$ gms.

Mass of 5 c.c. of water=5 gms. $\therefore$ Mass of the mixture=17·95 gms.

Volume of the mixture$=\dfrac{\text{mass}}{\text{density}}=\dfrac{17.95}{1.615}=11.11$ c.c.

Hence the amount of contraction$=(7+5)-11.11=0.89$ c.c.

(8) A cylinder of iron of specific gravity 7·86 and volume 200 c.c. floats on mercury. Calculate the volume of mercury displaced. Calculate also the volume of mercury displaced by the iron, when water is poured on the top of mercury to cover the iron completely. (Sp. gr. of mercury=13·6.)

If V be the volume of mercury displaced in the first case, we have mass of mercury displaced=mass of iron; or, $V\times 13.6=200\times 7.86$; $\therefore$ $V=115.59$ c.c.

If V' be the volume of mercury displaced in the second case,

the volume of water displaced$=(200-V')$ c.c.

So the mass of water displaced$=(200-V')\times 1$ gm.; and

mass of mercury displaced$=(V'\times 13.6)$ gm.; mass of iron$=200\times 7.86$ gm.

We have mass of mercury displaced+mass of water displaced=mass of iron.

or, $(V'\times 13.6)+(200-V')\times 1=200\times 7.86$;

$\therefore$ $12.6V'=200(7.86-1)=200\times 6.86$; $\therefore$ $V'=108.9$ c.c.

(9) A block of wood of specific gravity 0·85 floats in water. Some kerosene of specific gravity 0·82 is poured on the surface of water until the wooden block is completely immersed. Calculate the fraction of the block lying below the surface of water.

Let V be the volume of the block in the kerosene and V' the volume below the water surface.

So the total volume of the block $=V+V'$. $\therefore$ Wt. of block $=(V+V')\times 0.85$.
The upthrust in kerosene = wt. of V c.c. of kerosene $=0.82\ V$ gm.
The upthrust in water = wt. of V' c.c. of water $=V'$ gm.
$\therefore$ Total upthrust $=(0.82V+V')=$ wt. of the block $=(V+V')\times 0.85$.
$\therefore$ $0.82V+V'=0.85V+0.85V'$; or, $0.03V=0.15V'$.

or, $\frac{V'}{V}=\frac{1}{5}$; or, $\frac{V'}{V'+V}=\frac{1}{1+5}=\frac{1}{6}$.

Hence 1/6 of the block is below the water surface.

(10) A body of specific gravity 2.505 is dropped gently on the surface of a salty lake (sp. gr. 1.025). If the depth of the lake be a quarter of a mile find the time the body takes to reach the bottom. (Pat. 1941)

If m be the mass of the body, the volume of the body $=m/2.505=$ the volume of the displaced liquid.

So, the weight of the displaced liquid $=\left(\frac{m}{2.505}\times 1.025\right)g$

= the buoyancy, or the upthrust, acting on the body, and the force tending to bring the body down in the liquid $=mg$, the wt. of the body.

Hence the resultant downward force

$=mg-\left(\frac{m}{2.505}\times 1.025\right)g=mg\left(1-\frac{1.025}{2.505}\right)=mg\times\frac{1.48}{2.505}$. Therefore, if f be the acceleration with which the body is going down the liquid,

$f=g\times\frac{1.48}{2.505}$. If d be the distance travelled in time t, $d=\frac{1}{2}ft^2$ (the initial velocity u being zero); or, $t^2=\frac{2d}{f}=\frac{2\times(440\times 3)\times 2.505}{1.48\times g}=\frac{4468.37}{g}$

or, $t=\frac{66.8}{\sqrt{g}}$ sec

(11) A cylinder is 2 ft. high and the radius of the base is 3 ft.; its specific gravity is 0.7. It floats with its axis vertical. Find (a) how much of its axis will be under water, (b) the force required to raise it 1 inch (Pat. 1930)

(a) Volume of the cylinder $=\frac{22}{7}\times(3)^2\times 2=\frac{396}{7}=56.571$ cu. ft.

Sp. gr. of the substance of the cylinder $=\frac{\text{wt. of 1 cu. ft. of the substance}}{62.5}=0.7$

$\therefore$ Mass of 1 cu. ft. of the cylinder $=62.5\times 0.7$;
$\therefore$ Mass of the cylinder $=62.5\times 0.7\times 56.571$ lbs;
This is equal to the mass of water displaced by the cylinder.

$\therefore$ Volume of water displaced $=\frac{62.5\times 0.7\times 56.571}{62.5}$

$=0.7\times 56.571$ cu. ft. $\therefore$ volume of the cylinder under water.

Area of the base of the cylinder $=\pi\times 3^2=\frac{22}{7}\times 9=\frac{198}{7}=\frac{56.571}{2}$ sq. ft.

$\therefore$ Length of the axis immersed $=(0.7\times 56.571)\div\frac{56.571}{2}=1.4$ ft.

(b) Force exerted by the cylinder = wt. of the cylinder
$=62.5\times 0.7\times 56.571$ lbs.-wt.

When the cylinder is raised 1 inch, i.e. when $1{\cdot}4-\left(\frac{1}{12}\right)$ or $\frac{7{\cdot}9}{6}$ ft. of its axis is under water, the buoyancy of water=wt. of the water displaced $=62{\cdot}5\times$ vol. of water displaced in cu. ft. $=62{\cdot}5\times$ vol. of cylinder immersed.

$$=62{\cdot}5\times\left(\frac{56{\cdot}571}{2}\times\frac{7{\cdot}9}{6}\right)\text{ lbs.-wt.}$$

But the buoyancy when the cylinder was floating with 1·4 ft. under water $=62{\cdot}5\times56{\cdot}571\times0{\cdot}7$ lbs.-wt. $\therefore$ The force required to raise the cylinder by 1 inch $=(62{\cdot}5\times56{\cdot}571\times0{\cdot}7)-\left(62{\cdot}5\times\frac{56{\cdot}571}{7{\cdot}9}\times\frac{7{\cdot}9}{6}\right)$

$$=62{\cdot}5\times56{\cdot}571\left(0{\cdot}7-\frac{7{\cdot}9}{12}\right)=147{\cdot}32\text{ lbs.-wt.}=(147{\cdot}32\times32)\text{ poundals.}$$

(12) *A cylinder of wood whose specific gravity is 0·25, has another cylinder of metal (specific gravity 8·0) attached to one end. The cylinders are 2 inches in diameter, they have the same axis, and are respectively 20 inches and 1 inch long. If the whole is placed in water, find how much of it will be above the surface.* (*C. U. 1935*)

Volume of wood $=\pi\times1^2\times20=20\pi$ cu. in.; Vol. of metal $=\pi\times1^2\times1=\pi$ cu. in. Their total volume $=20\pi+\pi=21\pi$ cu. in.; sp. gr. of wood $=0{\cdot}25$.

Mass of 1 cu. ft. of wood $=(62{\cdot}5\times0{\cdot}25)$ lbs. Hence mass of 20π cu. in. of wood $=\left(\frac{20\pi}{1728}\times62{\cdot}5\times0{\cdot}25\right)$ lbs. And mass of metal $=\left(\frac{\pi}{1728}\times62{\cdot}5\times8\right)$ lbs.

Their total mass $=\left\{\frac{\pi\times62{\cdot}5}{1728}(5+8)\right\}$ lbs. This is equal to the mass of the displaced water, whose volume $=\frac{\pi\times1^2}{1728}\times h\times62{\cdot}5\left(\frac{\pi}{1728}\times h\times62{\cdot}5\right)$ where h=height in inches under water. $\therefore\ \frac{\pi\times62{\cdot}5}{1728}\times h=\frac{\pi\times62{\cdot}5}{1728}\times13$;

or, $h=13$ inches. Hence, the height above the surface $=21-13=8$ inches.

(13) *A ship with her cargo sinks α inches when she goes into a river from the sea. She discharges her cargo, while still on the river, and rises β inches and on proceeding again to sea she rises by another γ inches. If the sides of the ship be assumed to be vertical to the surface of water, show that the specific gravity of sea-water is $\frac{\beta}{\gamma-\alpha+\beta}$.*

Let x inches=the length of side (of the ship with cargo) immersed when in sea-water before going into the river;

then $x+\alpha$ inches = the length immersed in river;
$x+\alpha-\beta$,, = ,, ,, ,, ,, (without cargo);
$x+\alpha-\beta-\gamma$,, = ,, ,, ,, ,, sea (without cargo).

Now if ρ = density of sea-water and ρ_1 = density of river water, we have wt. of ship+cargo $=\rho x=\rho_1(x+\alpha)$, and

wt. of ship−cargo $=\rho(x+\alpha-\beta-\gamma)=\rho_1(x+\alpha-\beta)$.

So, $\rho x=\rho_1(x+\alpha)$ ... (1)

$\rho(x+\alpha-\beta-\gamma)=\rho_1(x+\alpha-\beta)$ ... (2)

Subtracting (2) from (1), $\rho(\gamma-\alpha+\beta)=\rho_1\beta$. $\therefore\ \frac{\rho}{\rho_1}=\frac{\beta}{\gamma-\alpha+\beta}$.

Questions

1. A piece of iron weighing 275 gms floats in mercury (sp. gr. = 13.59) with 5/9 of its volume immersed. Find the volume and the sp. gr. of iron.

[*Ans.* 36.42 c.c.; 7.55] (C. U. 1946)

2. A piece of wax of volume 22 c.c. floats in water with 2 c.c. above the surface. Find the wt. and the sp. gr. of wax. (C. U. 1947)

[*Ans.* 20 gms.; 0.909]

3. Why in C.G.S. units the values of density and sp. gr. are the same? (C. U. 1947)

4. A lump of 144 gms. of an alloy of two metals of sp. gr. 8 and 12 respectively, is found to weigh 129 gms. when totally immersed in water. Find the proportion by weight of the metals in the alloy. (Pat. 1939)

[Hints.—Let w_1 = wt. of metal A. ∴ Wt. of metal $B = (144 - w_1)$.

Hence $\frac{w_1}{8} + \frac{(144 - w_1)}{12} = \frac{(144 - 129)}{1}$, whence $w_1 = 72$ gms.]

[*Ans.* 1 : 1]

5. A cylinder of iron floats vertically and fully immersed in a vessel containing mercury and water. Find the ratio of the length of the cylinder immersed in water to that immersed in mercury. (Sp. gr. of mercury = 13.6, sp. gr. of iron = 7.78) (Pat. 1935)

[*Ans.* 67 : 113]

6. A piece of cork (sp. gr. 0.25) and a metallic piece (sp. gr. 8.0) are bound together. If the combination neither floats nor sinks in alcohol (sp. gr. 0.8), calculate the ratio of the masses of cork and metal. (U. P. B. 1947)

[*Ans.* 9/22]

7. A solid body floating in water has one-sixth of its volume above the surface. What fraction of its volume will project, if it floats in a liquid of specific gravity 1.2?

[*Ans.* $\frac{11}{36}$]

8. How do you find the specific gravity of a solid lighter than water?

A piece of cork whose weight is 19 grams is attached to a bar of silver weighing 63 grams and the two together just float in water. The specific gravity of silver is 10.5. Find the specific gravity of cork. (C. U. 1935)

[*Ans.* 0.25]

9. Explain how you would determine the specific gravity of an insoluble powder by the specific gravity bottle.

A specific gravity bottle weighs 14.72 grams when empty, 39.74 grams when filled with water, and 44.85 grams when filled with a solution of common salt. What is the specific gravity of the solution? (C. U. 1934)

[*Ans.* 1.204]

10. Describe an experiment to find the specific gravity of a solid soluble in water. (C. U. 1944; Pat. 1949)

11. If the specific gravity of a metal is 19, what will be the weight in water of 20 c.c. of the substance? (C. U. 1917)

[*Ans.* 360 grams.]

12. 60·3 gms. have to be placed on the pan of a hydrometer to sink it up to the mark in water and 6·8 gms. only in alcohol. If the hydrometer weighs 200 gms., what is the specific gravity of alcohol? (C. U. 1951)

[*Ans.* 0·794]

13. Explain clearly how you would determine the specific gravity of a liquid by a Nicholson's hydrometer without using a balance.

14. You are given a specific gravity bottle, enough kerosene and water, heating arrangement and a table of densities of water at various temperatures. How would you find the density of kerosene at 50°C., the room temperature being 30°C. (Pat. 1952)

CHAPTER XII

PNEUMATICS

289. The Earth's Atmosphere :— The gaseous medium which surrounds the earth is called its atmosphere. With this enveloping atmosphere the earth continuously rotates about the *polar axis* while moving along its orbit round the sun, the atmosphere being held bound to the earth by the action of gravity. This gaseous atmosphere is a *mechanical* mixture of several gases and its composition slightly varies from one locality to another. Besides water vapour, it contains about 77% nitrogen, 21% oxygen and 1% argon by weight. The remaining 1% includes traces of carbon dioxide, ammonia, hydrogen, neon, krypton, helium, ozone and xenon. The composite gas, like liquids, transmits pressure and possesses volume elasticity, and unlike liquids, has no free surface, is highly compressible and capable of expansion. A definite volume of it has a definite mass and so it has got some weight.

Densities of Some Gases

Unit	Hydrogen	Helium	Nitrogen	Oxygen	Carbon-dioxide	Air
gm./c.c.	0·00009	0·000178	0·00125	0·00143	0·00198	0·00129
lbs./cu. ft.	0·005	0·011	0·078	0·089 .	0·124	0·08

290. Physics of the Atmosphere :— In meteorology two types of balloons, the *recording balloon* and the *pilot balloon*, and more recently *rockets and artificial satellites* are used for investigations of the upper atmosphere. Recording balloons are Hydrogen-filled and automatic recording instruments such as the barograph, thermometers, etc. are contained in them. They finally burst out as they ascend higher and higher. The meteorograph, as it lands on the ground, is protected from injury by a special device adopted in the

balloon The heights of the balloons are observed by an instrument called the *theodolite.*

The gaseous medium constituting the atmosphere cannot expand indefinitely as it extends upwards, for the expansion is finally limited by the action of gravity. The density decreases with the height of the atmosphere increasing, but nothing is definitely known as to the height to which the atmosphere really extends, though it must have a limiting height. Estimates vary roughly between fifty to several hundred miles for this limiting height. But what is definite is th. [illegible] and probably it is [illegible] Even interstella [illegible] sum, contains traces of matter in the gaseous form, about one atom per c.c.

It is modern custom to give distinctive names to some definite layers of the atmospheric belt depending on their characteristic physical properties which are more or less known now-a-days. The atmosphere is divided into four distinct zones known as the (i) *troposphere,* (ii) *stratosphere,* (iii) *ozonosphere* and (iv) *ionosphere* including *Appleton layer* and *Heaviside layer.* Up to a height of about 6½ miles above the earth's surface, the temperature of the air diminishes steadily as the height increases. This layer of diminishing temperature is known as the troposphere (Fig 172) The layer above this is generally called the stratosphere. Formerly it was thought that the temperature in this region was constant at about— C0°F Recent researches reveal, however, that the temperature in this region increases with height, though very slightly A layer of constant temperature called the tropopause, divides the troposphere from the stratosphere. On the top of stratosphere an **Ozone layer** has been discovered. This layer absorbs the strong ultra-violet rays coming from the sun and this layer is responsible for reflec. ng all sound waves travelling upwards from the earth So beyond the **Ozone layer,** the region is a valley of silence. It has been confirmed now-a-days from studies with radio-waves that beyond the Ozone layer the upper atmosphere is highly electrically conducting, first pointed out by Balfour Stewart and subsequently insisted on by **Kennelly** and Heaviside and is known now-a-days as the **Kennelly-Heaviside layer.** A layer discovered by Appleton above this layer has been named as the **Appleton** layer. These conducting layers are only ionised layers of the atmos-

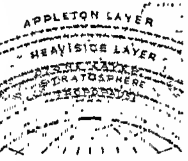


Fig 172—The Ionised Layers of the Atmosphere

phere and the existence of such layers has been proved now-a-days as a result of intensive radio-investigations since the twenties of this century. These ionised layers constituting a spherical belt round the earth collectively form what is often referred to as the **ionosphere.** The heights of these layers are liable to both regular and irregular changes during the night or day, and due to various celestial penomena still obscure to us.

Radio-communication round the globe by the help of short waves, has been possible due to the existence of these conducting layers.

291. The Atmospheric Pressure:—If the whole atmosphere over the surface of the earth is supposed to be divided into a number of layers of air, one above another, then it is evident that the surface of the earth, or any particular layer of air over it has got to bear the weight of the layers above, and is thus exposed to a pressure which is called the **atmospheric pressure.** This pressure at a place will, therefore, be equal to the weight of a column of air of unit cross-section and height equal to that of the atmosphere above that place. The value of this pressure is **15 lbs.-wt. per sq. inch,** or 1,013,981 dynes per sq. cm. approximately on the earth's surface and diminishes upwards gradually.

The following table shows how the atmospheric pressure at different places in India changes with *altitude,* i.e. their heights above the sea-level.

Place	Altitude	Mean Atmospheric Pressure
Calcutta	21 ft.	762·4 mm.
Bombay	33 „	759·3 „
Simla	7233 „	586·5 „
Darjeeling	7425 „	580·2 „

292. Air has Weight:—

Experiments.—(1) Take a fairly large flask fitted with a rubber cork through which passes a glass tube. To this is attached a piece of rubbet tubing provided with a clip. Put a little water in the flask and boil it after opening the clip. After some time close the rubber tubing with the clip and also remove the flame. Weigh the flask when it is cooled. Now open the clip; air rushes in; weigh again. The difference between the weights is the weight of the air that has entered the flask.

(2) The following experiment was done by **Otto Von Guericke** of Germany in 1650 for the first time to prove that *air* has weight.

A glass-globe, about 4 inches in diameter and provided with a stop-cock, is taken (Fig. 173). The globe is exhausted as much as is

A simple Torricellian barometer is placed inside a tall jar (Fig. 177) fitted on the receiver of an air-pump. As the air is slowly pumped out, the mercury column drops and finally, when the jar is well evacuated, the mercury attains almost the same level both inside and outside the tube. On re-admitting air, the mercury is again forced up the tube to the original height finally. If the vacuum above the mercury surface was the cause which drew the mercury up the tube, the column of mercury would not have fallen with the gradual removal of the atmospheric air from inside the jar. It fell because the pressure of the air inside the jar acting on the surface of the mercury in the basin outside the tube was reduced on gradual removal of the air. The column subsequently rose again, as the outside pressure increased on re-admission of air. Thus it is the pressure of the atmosphere which really supports the mercury column in a barometer and *the column is independent of the vacuum above the mercury surface.*

296. The Barometers:— *The barometers* (*baros*, weight) are instruments for measuring the pressure of the atmosphere. In one type of barometer the pressure of the atmosphere is measured by the weight of a column of mercury supported by it. It is called a mercurial barometer. Mercurial barometers are of *two* kinds—**Cistern and Siphon barometers.**

(a) The Fortin's Barometers.—It is a *cistern* type of mercurial barometer. The barometer tube is filled with pure, dry, and air-free mercury and is inverted over a cistern of mercury, *R*, called the reservoir (Fig. 178). The mercury stands in the tube at a certain height. The tube is enclosed within a long brass casing *C* on the front side of the upper part of which there is a rectangular slit through which the upper level of the mercury in the tube can be seen and observed by the help of a small mirror placed on the back side of the tube. The meniscus of the mercury surface is read by a main scale *U*, graduated in inches and centimetres on either side of the slit, with the help of a Vernier *V*, worked by the knob *P* of a rack and pinion arrangement.

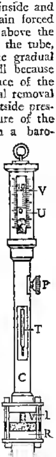


Fig. 178—A Fortin's Barometer.

The cistern has its upper part made of a glass cylinder *F* (Fig. 179) through which the surface of the mercury *M* contained in it can be seen. The glass cylinder is fitted in a box-wood cylinder *K* whose lower end is closed by a flexible leather bag *L* (usually made of Chamois leather). This bag has a wooden bottom *N* against which the point of the base-screw *S* presses. The screw works through the brass casing *E* which surrounds the reservoir. By turning the base-screw, the level of the mercury in the reservoir can be raised or lowered at will and

so that an observed barometric pressure may be compared with the standard barometric pressure as defined in Art. 301.

(1) *Correction for Temperature*—A correction is to be made for the expansion of the metal scale (which is ordinarily supposed to be correct at 0°C.) with rise of temperature. This corrected height is in terms of mercury at the existing temperature. So this again is to be transformed to zero-degree cold mercury.

(2) *Transformation to Sea-Level*—As the value of the acceleration due to gravity diminishes with the height above the sea-level, transformation is necessary so that the observed reading is *reduced to the sea-level*.

(3) *Transformation to 45° Latitude*.—The value of gravity varies from place to place on the earth's surface. It is less at the equator than at the poles. For correcting the above two effects, (2) and (3), the value of gravity at latitude 45° in the sea-level is taken as the standard. *The height reduced to sea-level at latitude 45°* $= H(1-0{\cdot}00257 \cos 2\lambda - 1{\cdot}96s \times 10^{-9})$, where H=observed height reduced to 0°C.; λ=latitude of the place and s=height (in cm.) of the place above the sea-level.

The reading corrected for (1), (2), and (3) would represent the height of the mercury column which would be supported by the existing atmospheric pressure at a standard temperature (*i.e.* 0°C.) and at a standard place (*i.e.* at the sea-level at latitude 45°).

298. Diameter of the Barometer Tube:—The *height* of the mercury column supported by the pressure of the atmosphere is *not affected by the width* of the barometer tube, for, let

a=area of cross-section of the tube; h=vertical height of the column, d=density of mercury; g=acceleration due to gravity.

Then, the wt. of the mercury column$=a\ h\ d\ g$=the upward force due to the atmosphere by which it is supported.

Now, if the area is doubled (*i.e.* $2a$), the upward force due to the atmosphere will act on twice the area, and so is also doubled. This force becomes$=2a\ h\ d\ g$=weight of the mercury column which it has to support. So the force per unit area, *i.e.* the pressure remains the same.

299. Why the Barometric Height Varies? Forecasting of weather:—Some amount of water-vapour is always present in the air. Contrary to the popular belief that moist air is heavy, it is actually lighter than dry air the density of water-vapour being $\frac{5}{8}$ of the density of dry air. Hence when there is a considerable amount of water-vapour present in the air, the density of the atmosphere, and therefore, the pressure exerted by it is less, which causes the mercury column in the barometer to fall slightly. This is the reason of the variations in the height of the barometer. The presence of *much water-vapour* in the air indicates that *rain* is imminent. For this reason, the barometer is used for forecasting the weather. A *low*

barometer reading indicates the presence of much water-vapour in the air, which, again, indicates a fall of *rain* in the near future, and a rapid fall in barometric height is usually accompanied by *stormy conditions*. On the other hand, a *high barometer* indicates *dry weather*. It should be noted, however, that the barometer is, by no means, an infallible guide to forcasting weather conditions.

300. Uses of Barometers:—So we find that a barometer can be used for the following purposes:—(a) *Measurement of the atmospheric pressure*; (b) *forecasting of weather*; (c) *determination of the altitude of a place* (vide Chapter VI, Part. II).

301. The Value of the Atmospheric Pressure:—Ordinarily the mercury column in a barometer may be taken to be 76 cms. (*i.e.* 30 inches) high and so the atmospheric pressure is equal to the weight of a column of mercury 76 cms. in height and 1 sq. cm. in cross-section, or the atmospheric pressure per sq. inch is equal to the weight of a column of mercury 30 inches in height and 1 sq. inch in cross-section. For the calculation of the weight, the mean value of the density of mercury may be taken to be 13·6 gms./c.c. and the value of the acceleration due to gravity, $g=981$ cms./sec.^{2}.

In C.G.S. Units:

Atmospheric pressure = weight of 76 c.c. $(=76\times1)$ of mercury;
$= 76\times13{\cdot}6\times981$ dynes per sq. cm.;
$= 1013961$ dynes per sq. cm.

So, it is approximately equal to one megadyne, i.e. 10^6 *dynes per cm.*^{2}.

The unit pressure used in *meteorology* is 1000,000 (or 10^6) dynes per sq. cm., which is called a **bar**, one thousandth part of which is called a **millibar**. Thus *the value of the atmospheric pressure is* 1013·961 *millibars approximately*.

In F.P.S. Units:

Again, taking 30 inches of mercury as the height of the barometer, atmospheric pressure = weight of $30(=30\times1)$ cubic inches of mercury.

We know that 1 cu. ft. of water weighs 62·5 lbs. So 1 cu. inch will weigh $\frac{62{\cdot}5}{12\times12\times12}$ lbs.

$\therefore$ The weight of 1 cu. inch of mercury $=13{\cdot}6\times\frac{62{\cdot}5}{1728}$ lbs.-wt.

or, *atmospheric pressure* $=30\times13{\cdot}6\times\frac{62{\cdot}5}{1728}$ lbs.-wt. per sq. inch.

$= 14{\cdot}7$ lbs.-wt. per sq. inch
$= 15$ lbs.-wt. per sq. inch (roughly)
$= 15\times32=480$ poundals (roughly).

[So, 30 inches being the height of a mercury barometer, the height of a *water barometer* will be 30 inches × 13·6 = 34 ft. approximately.]

Similarly, to get the height (h) of a *glycerine barometer*, we have, height of water barometer × density of water = height of glycerine barometer × relative density of glycerine; or, $34 \times 1 = h \times 1{\cdot}26$ (relative density of glycerine = 1·26).

$$\therefore \quad h = \frac{34}{1{\cdot}26} = 27 \text{ ft. approximately.}$$

[Note. Pressure is often expressed in atmospheres. When any liquid or gas exerts a pressure of 1013,961 dynes per sq. cm. or 14·7 lbs-wt. per sq. inch, the pressure is "one atmosphere".

Normal or Standard Atmospheric Pressure:— For comparison of pressures, a *standard pressure* is necessary. This *standard* pressure (also called *normal* pressure) is defined to be that due to a column of pure mercury 76 cms. in height, at $0°C.$, at the sea-level at 45° latitude. That is,

$$\text{normal pressure} = 76 \times 13{\cdot}596 \times 980{\cdot}6 \text{ dynes/cm}^2$$
$$= 1{\cdot}013250 \times 10^6 \text{ dynes/cm.}^2 = 1{\cdot}013 \times 10^6 \text{ dynes/cm}^2$$

[Density of mercury at $0°C = 13{\cdot}596$ gms./c.c., and the value of g at sea-level at 45° latitude = 980·6 cms./sec.^{2}.]

The normal or standard *atmospheric* pressure is a pressure equal to the above and is often used for comparison of atmospheric pressures at different places

302. Why Mercury is a convenient Liquid for Barometers?— A column of mercury only 30 inches high is able to support the pressure of the atmosphere, whereas to support the same pressure, a column of water 34 ft high, or a column of glycerine 27 ft. high, will be necessary For this reason (i.e. due to the *high specific gravity*) mercury is used for barometers as a matter of convenience Besides this, mercury *does not wet glass* and *does not evaporate rapidly*.

Very little mercury vapour collects in the Torricellian vacuum and the pressure exerted by the vapour is negligible. Moreover, mercury is a grey shining liquid and can be observed well

But the advantage in the case of lighter liquids is that a small variation in the barometric height can be observed more accurately, for a much greater variation in the liquid level is produced in their case. For this reason Glycerine is sometimes used as a barometric substance Though the vapour of this liquid has a low pressure at ordinary temperatures, it has certain objectionable features. Glycerine readily absorbs moisture from the atmosphere and so its density changes. The absorbed moisture let off into the Torricellian vacuum causes greater and greater depression of the column, as time advances

Water is not a suitable barometric liquid, for it quickly evaporates even at ordinary temperatures and causes considerable pressure on the liquid column whereby the observed column becomes appreciably shorter than truly what it should be. In some countries it cannot be used in winter when it will freeze.

Example. *The force exerted by the atmosphere on a circular plate whose diameter is 4·5 ft. is equal to 33,800 pounds. Calculate the height of the mercury barometer, if the density of mercury is 13·6 and the weight of 1 cu. ft. of water 62·5 pounds.*

Let h ft. be the height of the barometer. Then the force exerted on the plate = the weight of a column of mercury of height h standing on the plate.

The volume of this mercury column $= \pi \times \left(\frac{45}{2}\right)^2 \times h = 15{\cdot}9\ h$ cu. ft.

One cu. ft. of water weighs 62·5 lbs.: hence 15·9 h cu. ft. of water will weigh $15{\cdot}9\ h \times 62{\cdot}5 = 993{\cdot}7\ h$ lbs.

But mercury is 13·6 times heavier than water; so 15·9 h cu. ft. of mercury will weigh $993{\cdot}7\ h \times 13{\cdot}6 = 13514{\cdot}32 \times h$; and this = 33800 lbs.

or, $13514{\cdot}32 \times h = 33800$; or, $h = 2{\cdot}501$ ft.

303. Variations in the Atmosphere :—

Pressure at Different Altitudes.— As we ascend through the atmosphere with a mercury barometer, the weight of air pressing upon the exposed surface of it is reduced and consequently the height of the mercury column supported by the air becomes less and less as we ascend more and more; this is confirmed by experiments of Pascal and Perrier; on the other hand, as we descend below the sea-level, say, down the shaft of a mine, the weight of air pressing upon the surface is increased and so the mercury column is pushed higher and higher. It has been found that *for low altitudes there is a variation of 1 inch in the barometric height for a vertical rise or fall of 900 ft.; but for greater heights this is not strictly true.* Hence from variation in the readings of a barometer the altitude of a place, or the depth of a mine, can be ascertained.

It has been ascertained that about 50% of the earth's atmosphere lies within 3½ miles and about 99% within 20 miles from the surface. The remaining part, *i.e.* 1% extends over several hundred miles in a rarefied condition. At a height of about 3½ miles the pressure of the atmosphere is about 30 cms., and the pressure at a height of 20 miles is approximately 7 mm. An instrument, called the *altimeter*, is used which directly indicates the pressures and the corresponding heights at different levels in the atmosphere.

Temperature at Different Altitudes.— So far as the temperature of the atmosphere is concerned, it may be roughly taken to be divided into two regions: in the lower of which, called the "*troposphere*",

depth of about 200 ft. It has been found by the experiment that it is better to supply the divers with oxygen containing helium, instead of nitrogen, as helium dissolves much less than nitrogen in the blood and it is also got rid of more quickly and so the diver can come to the surface in much less time.

306. Balloon and Airship :—*It follows from the principle of* Archimedes that if the weight of a body is less than that of the air displaced by it, the body will be forced up, or buoyed up as it is called, and will rise in the atmosphere. The difference between the weight of the body, and that of the air displaced by it, is called the *"lifting power"* of the body. The principle is applied in a *balloon* or *airship*, which contains some gaseous substance like hydrogen, or helium which is lighter than air. The combined weight of the gas, engine, passengers, etc. must be less than the weight of the displaced air in order that balloon may rise.

At greater heights the pressure of the air is smaller and so a balloon there displaces a smaller weight of air.

An *airship*, whic' [illegible] propulsion, works o [illegible] either hydrogen or l [illegible] great disadvantage t [illegible] not, so *with helium the risk of accident is much reduced*. The advantage with *hydrogen* is that it is much *lighter* and *cheaper*.

307. Parachute :—The parachute is a device like that of an umbrella, which resists the falling of a body by putting up air resistance, *i.e.* it acts as an "air-brake" to a falling body.

308. The Lifting Power of a Balloon :— If d be the density of the air, d' the density of the gas in the balloon, V the external volume of the balloon, which is the volume of the displaced air, and V' the volume of the gas, the weight of the air displaced *i.e.* the force of buoyancy due to air $= Vd$, and the weight of the gas in the balloon $= V'd'$; the *total lifting power* $= (Vd - V'd')$. In practice V is very nearly equal to V', so the total lifting power reduces to $V(d-d')$, part of which is used to raise the balloon itself, and the remainder goes to raise its passengers and cargo. The density of hydrogen $= 0·0691 \times$ density of air. So for a balloon filled with hydrogen, *the lifting power* $= V(d - 0·0691 \times d) = 0·9309Vd$. For a balloon filled with helium for which $d' = 0·1388 \times d$, lifting power $= V(d - 0·1388d) = 0·8612 \times Vd$. Thus, it is found that though helium is twice as dense as hydrogen, the lifting power of a balloon filled with helium is almost equal (93 per cent.) to that of a similar hydrogen-filled balloon.

Examples. *(1) A spherical balloon 4 metres in diameter is filled with hydrogen gas (density $\frac{1}{13}$ of that of air). The silk envelope of the balloon weighs 250 gms*

per square metre. How much hydrogen is required to fill it and what weight can it support, the weight of a litre of air being 1·293 gms.

The volume of the balloon $=\frac{4}{3}\pi\times2^3=33·52$ cubic metres, and the surface area of balloon $=4\pi\times2^2=50·285$ sq. metres. (The wt. of 1 litre of air is 1·293 gms.).

Since the wt. of one cubic metre of air $=1·293$ kilogram.

The wt. of air displaced by the balloon $=33·52\times1·293=43·34$ kgms. and the wt. of hydrogen filling the balloon

$=\frac{1}{13}\times$wt. of the same volume of air $=\frac{1}{13}\times43·34=3·333$ kgms.

The wt. of the silk envelope is 250 gms. per sq. metre

$$=\frac{250}{1000}\text{ kgms. per sq. metre.}$$

$\therefore$ The wt. of the silk envelope of the balloon $=50·285\times\frac{250}{1000}=12·571$ kgms.

Hence the wt. of hydrogen in the balloon+its envelope $=12·571+3·333$ kgms. So the wt. which the balloon can support $=43·34-(12·571+3·333)=27·436$ kgms. This is the lifting power of the balloon.

(2) *A litre of hydrogen and a litre of air weigh about 0·09 gramme and 1·3 grammes respectively at a certain temperature (t) and pressure (p). What would be the capacity of a ballon weighing 10 kilogrammes, which just floats when filled with hydrogen having the same pressure (p) and the same temperature (t) as the air?* (*C. U. 1912*)

Let V litres be the volume of the balloon. Mass of hydrogen enclosed in the balloon $=V\times0·09$ gms. Mass of air displaced by the balloon $=V\times1·3$ gms.

When a body just floats in a fluid, the wt. of the body is equal to the wt. of the displaced fluid. Hence wt. of balloon+wt. of hydrogen in it = wt. of air displaced by the balloon;

or, $10\times1000+V\times0·09=V\times1·3$; or, $V=\frac{10000}{1·21}=8264·46$ litres (nearly).

309. Boyle's Law:— Robert Boyle (1627-1691), an Irishman, first established the exact relationship between the pressure of a confined mass of gas and its volume when they are varied at a constant temperature and the law named after him may be stated as follows:—

Temperature remaining constant, the volume of a given mass of gas varies inversely as the pressure.

Thus, if P be the pressure and V the volume of a gas,

we have, $P\propto\frac{1}{V}$; or, $P=K\frac{1}{V}$ where K is a constant whose value depends on the mass of the gas taken and its temperature.

Thus, $PV=K$.

If the pressure P be changed to P_1 at constant temperature, and the corresponding volume becomes V_1, we have, $P_1V_1=K$. But $PV=K$.

$$\therefore\ P_1V_1=PV.$$

The law can be verified graphically even without a knowledge of the atmospheric pressure. To do this, plot the excess of pressure, *i.e.* pressure above the atmospheric pressure, against $1/V$ when again, a straight line will be obtained (Fig. 187); for we have $PV=K$, a constant; $(H+X)\ V=K$, or $H+X=K/V=KY$ ... (1) where H represents the atmospheric pressure, X the excess of pressure, and Y stands for $1/V$.

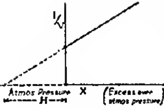


Fig 187

This is an equation of a straight line. So, if the graph of X, the excess of pressure, and Y, *i.e.* $1/V$, gives a straight line, the law is verified.

Determination of Atmospheric Pressure.—The graph just described provides a method of knowing the value of H, the atmospheric pressure. For, when $1/V$ is zero, $H+X=0$, or $H=-X$. Hence H is found.

312. Isothermal Curve:—The expansion or compression of a gas at constant temperature is said to be *isothermal* (Gk. *Isos*, equal, *thermos*, heat) expansion, or compression and the curve by which the relation between pressure and volume at constant temperature is represented is said to be an isothermal curve, or simply an *isothermal*. The curve, shown in Fig 186, obtained by a Boyle's Law experiment, is an isothermal curve.

313. Deviations from Boyle's Law:—It should be noted that for all practical purposes Boyle's Law is true for the gases like oxygen, nitrogen, air, hydrogen, etc. called the *permanent gases*. The permanent gases obey Boyle's Law under moderate pressures at ordinary temperatures. But at large pressures almost all gases deviate from the law more or less. A gas obeying Boyle's Law accurately at all pressures and temperatures is called a **perfect gas**, but ***no such gas exists really*** **(vide Chapter IV, Part II)**

314. Verification of Boyle's Law by Another Method:— Boyle's Law can be verified more simply by taking a glass tube AB about a metre long having a uniform bore of about 20 mm., closed at one end A and open at the other end B. The tube contains a mercury-index DC about 25 cms. long which encloses a column of air AD (Fig. 188)

Procedure.—Read the barometer and let P be the correct atmospheric pressure. Hold the tube vertically with the open end downwards. The atmospheric pressure in this case presses upwards on the mercury-column; so the pressure of the enclosed air is $(P-h)$, where h is the length of the mercury-column. Measure h and l_1, the length of the air-column AD

Now clamp the tube with the open end upwards. The pressure of the enclosed air now is $(P+h)$. Measure l_2, the length of the air-column now.

If a is taken to be the cross-section of the tube the volumes of the air enclosed in the two cases are al_1 and al_2. Now, by assuming Boyle's Law to be true, we have $(P-h)al_1=(P+h)al_2$,

or, $\frac{P-h}{P+h}=\frac{l_2}{l_1}$ from which P can be calculated.

The result can be *checked up* by measuring the length of the air-column when the tube is kept horizontal. The pressure in this case is P which can easily be calculated. Thus, by this method, we can approximately determine the *atmospheric pressure.*

Fig. 188

In order to have more readings for the verification of Boyle's Law by the above method, the tube can be clamped at various angles with the open end up or down. In these cases h will be the difference of the two vertical heights (h_2 and h_1) of the two ends, upper and lower of mercury-column, which can be measured by using a plumb line. The pressure of the enclosed air in these cases will be $P \pm (h_2 - h_1)$ according as the open end is up or down. All the results obtained in various positions of the tube can be tabulated, and it will be seen that the product of the pressure, $P \pm (h_2 - h_1)$, and the length of the air column and so the volume, will be constant in each case.

315. Faulty Barometer :—A barometer containing some air in the tube will always give faulty readings; the air will expand and depress the mercury-column to some extent. To *test* whether the barometer tube contains air or not, incline the tube sufficiently, or screw up the bottom of the cistern in the casing of Fortin's barometer until the whole tube will be filled with mercury, if there is no air in it. But if there be any air in the tube, it will always be left in the tube and so the tube cannot be completely filled up with mercury, however much the tube may be inclined or the bottom may be screwed up.

As the mercury rises and falls, the enclosed air obeys Boyle's Law and hence it is possible to determine the correct atmospheric pressure with such a faulty barometer by the application of Boyle's Law as follows:—

Determination of Correct Pressure.—Let h_1 be the height of the mercury-column and l_1 the length of the air-column in the tube of a faulty barometer. Now raise or depress the barometer tube in the cistern so that the air-column is about double or half of what it was. Read the new height h_2 of the mercury-column and the length l_2 of the air-column. If P be the correct atmospheric pressure, we have, by applying Boyle's Law, $(P-h_1)\times al_1=(P-h_2)\times al_2$, where a is the cross-section of the tube. From this P is determined.

strument works, while Fig. 192(*b*) illustrates the appearance of the instrument.

The instrument consists simply of a hollow-tube *BAC* open at *B* to be connected to the supply and closed at other end *C*. It is a spring tube usually made of a special quality of bronze or sometimes of brass and is of elliptical section. The pressure of the supply changes the cross-section of the tube to more and more circular form until the stress developed in the material of the tube is balanced by the pressure within, and due to this change in cross-section, the free end *C* of the tube is displaced from *B*. As a result, the pointer *P* connected to it moves over a circular scale. The instrument is previously graduated by referring to a standard gauge. Since the manometer tube is surrounded by the atmospheric air, the movement of C measures the excess or deficit of the internal pressure referred to the atmospheric pressure. To get the absolute pressure of a high pressure supply, the atmospheric pressure at the place of observation at the time of experiment is to be added to the pressure indicated by the gauge

317. Evangelista Torricelli (1608—1647):—A pupil of Galileo who succeeded him as mathematician to the Grand Duke of Tuscany He is a contemporary of Pascal Both of them lived a very short life

He was a born experimenter He showed how small beads of glass when melted could be uesd as lenses of high magnifying power He discovered a law named after him, concerning the flow of liquid from openings in a thin wall, and perhaps was the first worker in Hydrodynamics as contrasted with the science of Hydrostatics founded by Archimedes His greatest achievement, however, lies in the construction of a 'barometer'. Galileo had already measured the atmospheric pressure by means of a water-column in the tube of a deep well in Florence, though he was testing '*the power of vacuum*', an idea originating from Aristotle. Torricelli picked up the idea from him and in collaboration with Viviani tried a mercury-column in place of the water-column.

Torricelli

318. Robert Boyle :— He was the fourteenth child of Richard Boyle, the great Earl of Cork, an Irish County and was a man of means. After receiving education in a London School he went on an extensive tour throughout the continent particularly Italy where he studied Galileo's work. On return to England, he lived in a house where men of science used to meet and debate scientific topics. It was this debating club which was transformed into the Royal Society in 1662 by Charles II. After Guerike had invented the air-pump he began to study the properties of gases with it. It is he who first devised the plan of trapping some air above the mercury in the closed limb of a U-tube with the other limb kept open. The pressure of the enclosed air thus could be varied at will by setting up different heights of the mercury in the open limb. This led finally to the important law known as Boyle's Law. Edme Mariotte in Paris discovered the same law independently near about the same time and so in the continent this law is often called the Mariotte's Law. Perhaps he is the first man who made a systematic study of the elements by chemical analysis and he is considered to be one of the founders of chemical analysis. The detection of hydrogen chloride gas by precipitation with silver solution, of iron by tincture of galls; of acids by means of papers dyed with vegetable colouring matters are a few of his outstanding contributions to science. He discovered how sound is propagated through air and investigated the refractive powers of crystals.

Robert Boyle

He was a jealous supporter of Christianity and spent a huge sum of money to propagate its superiority and with that object founded the Boyle's Lecture.

Examples. (1) *What volume does a gramme of hydrogen occupy at $0°C$, when the height of the mercurial barometer is 750 mm. (1 c.c. of H weighs 0·00008958 gram at $0°C$, and 760 mm.)?*

0·00008958 gram of hydrogen at N.T.P. occupies 1 c.c.

∴ 1 gram of hydrogen at N.T.P. occupies 1/0·00008958 c.c.

If V c.c. be the volume of one gram of hydrogen at $0°C$, and 750 mm., we have by Boyle's law,

$$V\times 750=\frac{1}{0\cdot 00008958}\times 760;\ \text{or,}\ V=\frac{1}{0\cdot 00008958}\times\frac{760}{750}=11\cdot 312 \text{ litre (nearly).}$$

(2) What is the depth in water where a bubble of air would just float, when the height of the water barometer is 34 ft.? Given that the mass of 1 cubic foot of water is 62·5 lbs., and that of air is 5/4 oz.

Let h ft. be the depth at which the bubble would just float, when the density of air is d, and let d' be the density at the atmospheric pressure; then we have,

by Art 310, $\frac{34}{h+34}=\frac{d'}{d}$. But since at this depth the bubble of air just floats the density of air is just equal to the density of water; so

$$\frac{34}{h+34}=\frac{d'}{\text{density of water}}=\frac{5\cdot4}{(62\cdot5\times16)}$$

or, $h=27166$ ft.$=9055$ yds. (approx.)$=5\cdot14$ miles.

(3) At what depth in a lake will a bubble of air have one-half the volume it will have on reaching the surface? The height of barometer at the time is 76 cms. and the density of mercury 13·6. (All. 1935)

Let the volume of air-bubble at the surface be V c.c. and the depth below the surface at which the volume of the bubble is $V/2$ be x cms.

x cms. of water exerts the same pressure as $x/13\cdot6$ cm. of mercury.

Hence, the total pressure on the bubble at bottom$=76+\frac{x}{13\cdot6}$ cms.

Then, by Boyle's Law we have, $\left(76+\frac{x}{13\cdot6}\right)\times\frac{V}{2}=76V$;

or, $\frac{x}{13\cdot6}=76$; $\therefore$ $x=76\times13\cdot6=1033\cdot6$ cms.

(4) A barometer reads 30 inches and the space above mercury is 1 inch. If a quantity of air which at atmospheric pressure would occupy 1 inch of the tube is introduced, what will be the reading of the barometer? (All. 1931)

Let α by the area of cross-section of the tube, so the volume of air occupying 1 inch of the tube$=\alpha\times1$ and the pressure of the above air, before it is introduced in the tube$=30$ inches.

When air is introduced, let the mercury-column come down by x inches which, then, is the pressure of the introduced air, the volume of which is $(x+1)\times\alpha$ cu. inches. $\therefore$ By Boyle's Law, $30\times\alpha\times1=x\times(x+1)\times\alpha$.

or, $x^2+x-30=0$; or, $(x-5)(x+6)=0$; or, $x=+5$ or, -6.

According to the first value of x, the reading of the barometer will be $30-5=25$ inches, and according to the other value, the reading will be $30-(-6)=36$ inches. But as the final reading, after air is introduced, cannot be greater than the original, the second value is not admissible.

(5) A siphon barometer with a little air in its 'vacuum' indicates a pressure of only 72 centimetres, and on pouring some more mercury in the open limb until the vacuum is diminished to half its former bulk, the difference of the levels becomes 70 centimetres. What is the true height of a proper barometer? (Pat. 1922)

Let V be the volume of air in the tube and p the pressure exerted by the volume of air before mercury was poured in.

$\therefore$ The true height of the barometer$=72+p$. Then $V/2$ is the volume of this air after mercury was poured in. Let the pressure exerted by this volume of air be p_1. $\therefore$ The true height$=70+p_1$.

Then, we have, by Boyle's Law, $pV=p_1\times V/2$. $\therefore$ $p_1=2p$.

But the true height of the barometer before pouring in mercury $=72+p$, and after pouring in mercury $=70+p_1$; $\therefore$ $72+p=70+p_1=70+2p$;

$\therefore$ $p=2$. Hence the true height of the barometer $=72+2=74$ cms.

(6) A tube 6 feet in length closed at one end is half filled with mercury and is then inverted with its open end just dipping into a mercury trough. If the barometer stands at 30 inches, what will be the height of the mercury inside the tube? (C. U. 1931)

Let x ft. = height of mercury inside the tube when inverted. The initial volume of air occupies $\frac{6}{2}$ or 3 ft. length, and the initial pressure $\frac{30}{12}$ ft.; the final volume $=(6-x)$ ft. in length, and the final pressure $=\left(\frac{30}{12}-x\right)$ ft. Then, by Boyle's Law, $3\times\frac{30}{12}=(6-x)\times\left(\frac{30}{12}-x\right)$.

$\therefore$ $2x^2-17x+15=0$; or, $(x-1)(2x-15)=0$;

Hence $x=1$ ft.; or, $\frac{15}{2}$ or, $7\frac{1}{2}$ ft.

The second root is not admissible as the height cannot be $7\frac{1}{2}$ ft., *i.e.*, longer than the tube. $\therefore$ The required height $=1$ ft.

(7) The height of the mercury barometer is 30 inches at sea-level and 20 inches at the top of a mountain. Find approximately the height at the mountain, if the density of air at sea-level is 0·0013 gm. per c.c. and of mercury 13·5 gm. per c.c.

By Boyle's Law, $\frac{\text{the density of air at top of mountain}}{0\cdot0013}=\frac{20}{30}=\frac{2}{3}$;

$\therefore$ Density of air at top $=\frac{2}{3}\times0\cdot0013=0\cdot00086$

$\therefore$ Mean density $=\frac{1}{2}(0\cdot0013+0\cdot00086)=0\cdot00108$.

The difference of pressure at the two points is equal to the weight of $(30-20)$ inches of mercury standing on one square inch, i.e. of 10 cubic inches of mercury.

Now, considering the atmosphere to be homogeneous having its density equal to 0·00108, it can be found what column of this air will be equal in weight to a column of mercury 10 inches high. Hence, if h be the length of the air-column, we have

$$\frac{h}{\text{length of mercury column}}=\frac{\text{density of mercury}}{\text{density of air}}; \text{ or, } \frac{h}{10}=\frac{13\cdot5}{0\cdot00108},$$

$$\therefore\ h=\frac{13\cdot5}{0\cdot00108}\times10=125000 \text{ inches}=10416\cdot66 \text{ ft. (nearly)}.$$

(8) A bubble of air rises from the bottom of a lake and its diameter is doubled on reaching the surface. Find the depth of the lake.

Volume of a sphere $=\frac{4}{3}\pi$ (radius)$^3=\frac{1}{6}\pi$ (diameter)^3.

$$\therefore\ \frac{\text{Vol. of air-bubble at bottom}}{\text{Vol. of air-bubble at surface}}=\frac{\frac{1}{6}\pi(\text{diameter})^3}{\frac{1}{6}\pi(2\times\text{diameter})^3}=\frac{1}{8}.$$

$\therefore$ Volume at surface $=8$ times volume at bottom.

18 Describe an experiment shewing that Archimedes' principle applies to bodies immersed in a gas.

Criticise the statement 'A pound of feather weighs less than a pound of lead.' (C. U. 1944)

19 Why there is difference in the reading of a barometer at Puri and at Darjeeling? (C. U. 1947)

20. What is the effect of the pressure of the atmosphere on the weight of a body? Give reasons for your answer, and describe an experiment by which this effect can be demonstrated. (C. U. 1934)

21 As a balloon rises to greater and greater altitude, what changes are found in, (a) the atmospheric pressure, (b) the density of air, and (c) the lifting power of the balloon, by a person in it? Explain the changes (Pat. 1940)

22. The volume of a balloon is 500 cubic metres It is filled with hydrogen whose density is 0·089 gm./litre The density of the surrounding air is 1·250 gm./litre. What is the total lifting force of the gas?

[*Ans.* 580·5 kgm.]

23 A balloon, weighing 150 kgms. contains 1,000 cu m. of hydrogen and is surrounded by air of density 0·00129 Calculate the additional weight it can lift Also explain why the balloon will float in stable equilibrium at a constant altitude (Density of hydrogen = 0·00009 gm./c.c.) (Pat 1941)

[**Hints.**—Density of H per cu m $= 0·00009 \times 10^6 = 90$ gms $\therefore$ The wt of 1,000 cu m of $H = 90$ kgms So total wt $= (150+90) = 240$ kgms ; wt of $1{,}000 \times 10^6$ c.c. of air $= 1290$ kgms $\therefore$ Lifting power $= 1290 - 240 = 1050$ kgms. It will be in stable equilibrium because at a constant altitude the acceleration due to gravity, and also density of air, remain constant]

24 State Boyle's Law and show how it can be verified in the laboratory for pressures higher and lower than the atmospheric pressure.

(Dac 1941; U P B. 1944; G. U 1952; C U 1957)

25 The space above a mercury column in a barometer tube contains some air The mercury column is 23·40 inches long and the space above it is 3·05 inches long The tube is then pushed downwards into mercury so that the column is 23·14 inches long while the air space is 2·34 inches What is the true height of the barometer? (R. U 1955)

[*Ans.* 20·87 inches]

26 The height of a barometer is 75 cms of mercury and the evacuated space over mercury surface has a volume of 10 c.c One cubic centimetre of air at atmospheric pressure is introduced into the evacuated space What is the new reading of the barometer? The cross section of the tube is unity (C U, 1921, '29)

[*Ans.* 70 cms , because the other value 90 is inadmissible]

27 Find the pressure exerted by a gramme of hydrogen in a vessel of 5·55 litres capacity at 0°C., assuming that the mass of a cubic centimetre of hydrogen at 0°C and a pressure of 760 mm. of mercury is 9×10^{-5} gms

[*Ans.* 1523·5 mm.] (Dac 1930)

28 Assuming the water barometer stands at 33½ ft., find the length of a cylindrical test tube in which the water rises 1 inch if the tube is vertical and pressed mouth downward into water until the base of the tube is level with the surface of the water

[*Ans* 21 inches]

29. A column of air is enclosed in a fine tube by a thread of mercury 25 cms. long. The air-column is 5 cms. long when the tube is held vertically with its open end uppermost. On inverting the tube, the air-column measures 10 cms. Find the atmospheric pressure.

[**Hints.** $(P+25)\times 5=(P-25)\times 10$. $\therefore$ $P=75$ cms. of mercury.]

30. A narrow tube with uniform bore is closed at one end, and at the other end is a thread of mercury of known length. The tube is held vertical with the closed end (i) up, (ii) down. Show how the barometric height can be determined from the positions of the thread, assuming that Boyle's Law holds. (*Vide* Art. 298.) (Pat. 1938, '47; Gau. 1955)

31. How would you test whether the space above the mercury column in a barometer tube contains air or not? Show how a correction for the reading of a barometer containing some air above the mercury-column may be found, when no other barometer is available. (M. U. 1957)

32. A barometer whose cross-sectional area is one sq. cm. has a little air in the space above the mercury. It is found to read 77 cms. when the true height is 78 cms., and 71 cms. when the true height is 71·8 cms. Determine the volume of the air present in the tube measured under the former conditions.

(C. U. 1937; And. U. 1952)

[**Hints.**—$(78-77)\ V=(71{\cdot}8-71)\ \{V+(77-71)\times 1\}$; whence $V=24$ c.c.

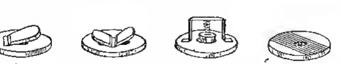
If the volume of air present is measured under normal conditions, its value (v) will be given by $\{24\times(78-77)\}=v\times 76$, whence $v=0{\cdot}31$ c.c.]

CHAPTER XIII

APPLICATION OF AIR PRESSURES: PUMPS

Air and Water-Pumps, Siphon, Diving-Bell

319. The Valves:— A **valve** is a trap door hinged in such a way that when a fluid presses on one side, it opens up a little way and



(a) (b) (c) (d)

Fig. 194—Some Different Types of Valves.

allows the fluid to pass through, but it shuts up the opening when the fluid presses on the other side. Thus a valve allows the passage

possible. Moreover, at a certain stage when the pressure of air in the receiver becomes very low, it cannot open the first valve a, after which no further evacuation is possible.]

323. Filter Pump (or Water Jet Pump) :— It is an exhaust type of air-pump ordinarily made of glass and is used when the degree of vacuum required is not lower than about 7 mm. Its special feature is that it needs no attention.

The pump is shown in Fig. 107. The side-tube B is connected with a rubber tubing to the vessel intended for evacuation. The upper end of the vertical tube A which tapers below and ends in the nozzle N is connected to the water mains, the pressure of which should remain constant. As a strong jet of water forces out of the nozzle with a very high speed, some air from around the nozzle is also entangled and carried down the tube. The draught produced thereby draws out the air from within the vessel at the same rate.

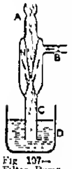


Fig. 107—
Filter Pump

324. Condensing (or Compression) Pump :— This pump is used for compressing air into a vessel usually referred to as a *receiver*. It consists of a barrel AB in which a piston P works (Fig. 108). The barrel is connected to the receiver R into which air is compressed. Both the piston and the end of the barrel contain valves b and a respectively opening towards the receiver. So, *it is like an exhaust-pump with the valves reversed.* There is a stop-cock T at the mouth of the receiver which may be closed after the required amount of compression is attained.

Action.— The piston is moved outwards (*backward stroke*) and inwards (*forward stroke*) alternately.

Backward stroke.— To start with, the piston P is at the end B of the barrel, and as it is moved up, the pressure of air in the barrel below the piston falls; the valve a is closed by the pressure of air in the receiver; the atmospheric air acting on the other side of the piston opens the piston valve b; and the barrel is filled up with air at the atmospheric pressure.

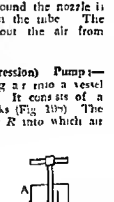


Fig. 108

Forward stroke.— To start with the piston P is near the top A of the barrel, and as it is

moved down, valve b is closed at some stage when the pressure within the barrel below it exceeds the atmospheric pressure and the compressed air in the barrel enters the receiver by forcing the valve a open. Greater compressions are required at each new forward stroke to enable the air to enter the receiver, as the pressure within it increases when the strokes are repeated.

325. The Density and Pressure of Air in the Receiver after n Nmber of Strokes:—

Let V=volume of the receiver and the connecting tube;

V_1=volume of the barrel between the higher and the lower valves;

d=density of atmospheric air; d_n=density of air in the receiver after n strokes. The mass of air originally present in the receiver$=V.d.$

At each down-stroke, a volume V_1 of a'r at atmospheric density d enters the receiver. Hence after n complete strokes mass of air in the receiver$=(V+nV_1)d$. But its volume is V.

$$\therefore \text{ Its density, } d_n=\frac{\text{mass}}{\text{volume}}=\left(\frac{V+nV_1}{V}\right)d=\left(1+n\frac{V_1}{V}\right)d \quad \dots \quad (1)$$

If the temperature remains constant, the pressure will be directly proportional to density.

If P_n be the pressure in the receiver after n strokes and P the original pressure, we have

$$\frac{P_n}{P}=\frac{d_n}{d}=\left(1+\frac{nV_1}{V}\right), \text{ from (1); i.e. } P_n=\left(1+\frac{nV_1}{V}\right) \text{ atmospheres.}$$

326. Difference between the Compression and the Exhaust-Pump:— (1) Both the pumps are provided with a valve in the piston and a valve at the end of the barrel. But the difference in their construction is that in the compression pump both these valves open towards the side of the receiver while in the exhaust-pump they open up in the opposite direction. (2) *In the compression pump* a quantity of air, whose volume is the same as that of the barrel, is forced into the receiver at each stroke, and as air from outside easily enters the barrel on every backward stroke, the density of the air which is forced into the receiver at each inward stroke, is always the same as that of the outside air and consequently the mass of air admitted per stroke is constant. But in the *exhaust-pump*, the density of air extracted from the receiver diminishes with each stroke though the volume may be the same, and hence the mass of air withdrawn per stroke diminishes as evacuation proceeds.

327. Compression Pump in different Forms:—

(a) The Bicycle Pump.—An ordinary bicycle pump (Fig. 199) is an example of the simplest kind of a compression pump. It is made of a vulcanite or metal cylinder B with a piston P inside, which

is fitted with a cup-shaped leather washer W, the rim of the cup being directed towards the bottom of the pump. During the up-stroke (left figure), the cup collapses inwards, the pressure below gradually falling more and more below atmosphere owing to the expansion of the enclosed air, and air from above passes down readily between the washer and the wall of the cylinder into the lower part of the barrel, and during the down-stroke (right figure), the increased pressure of air presses the leather washer W air-tight against the walls of the cylinder and so no inside air can pass out. As the piston is pushed down, the air pressure becomes greater and it forces its way into an air-tube made of rubber (housed in an outer jacket, called the tyre) when the increased air pressure is sufficient to open up an *inlet valve* with which the air tube is provided

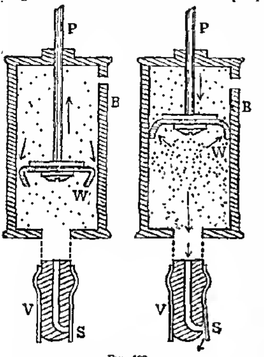


Fig 199

The connector of the pump is screwed on to this *valve* This latter consists of a narrow metal tube V lined up inside with a rubber valve having a central bore ending in a small hole at the side which is normally closed by a flapping part S acting as the *valve*. During the *up-stroke* of the piston, the pressure of the air in the air tube presses the closing flap of the rubber valve which seals up the small side hole, and so the air cannot flow back from the air tube into the pump. During the *down-stroke*, the compressed air in the pump forces its way through the small hole deflecting the closing flap and enters the air tube.

(b) Football Inflator.—This is also a compression pump similar in action to the bicycle pump. The difference in construction is that the inlet valve is different in construction and forms a part of the bottom of the barrel of the pump called the *nozzle*. The nozzle *N* [Fig. 200] is a metal tube of special design [entirely closed at the delivery end except for two slanting holes (*S, S*.) which can communicate with the rubber bladder of the football so that it can be easily introduced into the neck of the rubber bladder with which the grip should be tight. The central hole in the nozzle is convergent-divergent and a solid ball *G* can fully shut up the throat of the nozzle and is not carried past it while moving towards the barrel. During the up-stroke (left figure) the greater pressure of the air in the bladder forces the ball *G* to close the throat and so no air from the bladder can leave it. During the down-stroke, the air pressure below the leather washer *W* in the piston gradually increases and pushes the ball forward, but the latter is caught up finally by the delivery end of the nozzle, and at that position, sufficient gap for the forced air to pass through the two slanting holes is still left at the sides.

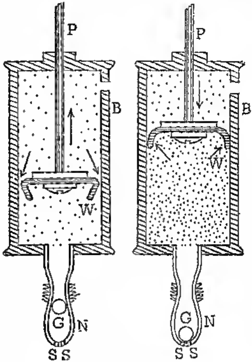


Fig. 200

328. Some uses of Compressed Air :—

The **Soda-water machine** acts as a compression pump. It forces carbon dioxide gas into a bottle containing water. The water absorbs the gas and is said to be *aerated*. This water is ordinarily called *soda-water*.

An **air-gun** may be regarded as a compression pump without any valve. At each stroke some more air is forced into barrel of the gun and becomes compressed. When suddenly released, the compressed air expands with a great force. This force is used in *air-guns*, when the released high pressure air works upon a spring which throws out the bullet at a great force.

If the air in an air-gun is released slowly, then a steady force may be obtained and this may be applied against a surface. The **'Westinghouse automatic brake'** employed in some trains works on this principle.

In the **air-cushion**, which is nothing but a hollow rubber bag having a connecting nozzle fitted with a valve or stop-cock, air is compressed into the bag by means of a condensing pump and the compressed air serves as the cusion.

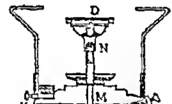


Fig 200(a)

Compressed air is used in the **air-brush** for spraying paints on smooth surfaces without leaving brush marks.

In Oil stoves air is pumped by a hand-driven compression pump into a pot which forces the oil to rise along a vertical tube [Fig 200(a)] This oil coming in contact with a hot surface is converted into vapour and burns.

Compressed air is widely used in working what are known as pneumatic tools, e g *drills* etc used in quarrying, street repairs, etc

Examples. *(1) The barrel and receiver of a condensing pump have capacities of 75 c.c. and 1000 c c respectively How many strokes will be required to raise the pressure of the air in the receiver from one to four atmospheres?* (C U 1925)

Pressure after n strokes, $P_n = \left(1+n\frac{V_1}{V}\right)$ atmospheres, where

V_1 = volume of the barrel and V = volume of the receiver

$$\therefore \quad 4 = \left(1+n\frac{75}{1000}\right), \text{ or, } 120=3n, \text{ or, } n=40$$

(2) If the pressure in a pump were reduced to $\frac{1}{2}$ of the atmospheric pressure in 4 strokes, to what would it be reduced in 6 strokes? (Pat. 1931)

Pressure P_4 after 4 strokes is given by, $P_4=\left(\frac{V}{V+V'}\right)^4 P$, where P=original pressure, V=volume of the receiver, and V'=volume of the barrel.

But $P_4=\frac{1}{3}P$; $\therefore \frac{P}{3}=\left(\frac{V}{V+V'}\right)^4 \times P$; $\therefore \left(\frac{V}{V+V'}\right)^4=\frac{1}{3}$; or, $\frac{V}{V+V'}=\frac{1}{\sqrt[4]{3}}$.

After 6 strokes, $P_6=\left(\frac{V}{V+V'}\right)^6 P=\left(\frac{1}{\sqrt[4]{3}}\right)^6 P=\frac{1}{3^{\frac{3}{2}}}P=\frac{P}{3\sqrt{3}}$.

That is, the pressure is reduced to $\frac{1}{3\sqrt{3}}$ of the original pressure.

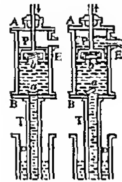
329. The Water Pumps:— These are instruments for raising water from a lower to a higher level, most of which depend on the principle that the atmospheric pressure is capable of supporting a column of water up to a height equal to the height of the water-barometer. This principle will be clear by considering the action of an ordinary syringe.

The Syringe.—It is an instrument the working of which depends on the atmospheric pressure. It is the simplest type of water pump. It consists of a hollow glass or metal cylinder ending in a nozzle and provided with a water-tight piston. When the piston is drawn up from its lowest position in the cylinder (the nozzle being dipped under a liquid), a partial vacuum is created within the cylinder below the piston. So the atmospheric pressure acting on the liquid surface outside the nozzle becomes greater than the pressure inside the cylinder and thus the liquid is pushed up into the cylinder. After sufficient liquid has been drawn into the syringe, it is removed; when owing to the greater external pressure, the liquid cannot escape through the nozzle. When the piston is pushed down, the pressure inside becomes greater and so the liquid is forced out. This is the underlying principle of all the pumps, which are described below, in which the water is said to rise by *suction*. *The drinking of liquid by drawing it through a straw tube is also a familiar example of the principle of suction.*

Pen-filler.—The ordinary pen-filler used for fountain pens, which consists of a rubber bulb fitted at one end of a piece of glass tubing drawn out to a jet, works on the same principle as above. On compressing the bulb some air from inside the tube is driven out, and when the jet is now placed in the ink and the pressure on the bulb released, ink rises up into the tube due to the external pressure on the ink surface being greater than the pressure inside the tube.

In the *self-filling* fountain pen, the filler is inside the pen. It consists of a long rubber bag which is compressed by pulling out a metallic lever in the side of the barrel of the pen. The lever presses a metal strip against the bag and this drives out some air. On reinstating the lever after immersing the nib in ink, the pressure is released and so some ink is drawn up into the pen.

330. Common or Suction Pump (Tube-well Pump) is like an ordinary syringe with an extended nozzle T to beneath the surface D of the water to be raised (Fig 201) The nozzle, or the suction pipe, is connected at the bottom of the barrel, or cylinder AB in which the piston P works. Two valves or trap doors a and b opening upwards are fitted, one at the bottom of the barrel, and the other within the piston. There is an exit spout E at the top of the barrel.



Down-stroke *Up-stroke*

Fig. 201—The Common Pump

Action.— As the piston is *raised* up during the first up-stroke, the pressure inside the barrel below the piston falls, the valve a opens due to the greater pressure of the air inside the pipe T and the valve b closes due to the atmospheric pressure (which is greater) acting from above The pressure on the surface of water in the pipe is thus less than the atmospheric pressure which acts upon the water outside the pipe. So, the water is forced up into the pipe

As the piston comes *down* during the down-stroke the valve a is closed by the weight of the water above, and the water in the barrel being compressed escapes through the valve b Further pumping will raise more water into the barrel and finally water will rush through the valve b at the down-stroke and flow out by the spout at the up-stroke.

One disadvantage of this pump is that it gives only an intermittent discharge (on up-stroke only).

331. Limitation of the Suction Pump:—It should be noted that water is raised in the tube by the atmospheric pressure, and the atmospheric pressure can support a vertical column of mercury 30 inches in length, and a column of water (30 × 13.6) inches, or 34 ft. long; so the *'head of water' above the water surface, i.e. the distance between the valve 'a' and the surface of water 'D' must not exceed the height of the water-barometer, that is to say, 34 feet.* In practice however, the height is less than 34 feet (practically about 25 ft. only), as the valves have got weight and the pump is never absolutely air-tight. This kind of pump is now being widely used in the tube-well.

Examples. (1) *What is the discharge of a pump having a diameter of 1 foot, a stroke of 2 feet, and worked at the rate of 20 strokes per minute?*

The volume of the barrel of the pump $= \pi \times (\frac{1}{2})^2 \times 2 = 1.5714$ cu. ft.

In a single acting pump, half the number of strokes per minute is only effective in discharging water Hence volume of liquid discharged per minute $= 1.5714 \times \frac{20}{2} = 15.714$ cu ft.

valve *b*. At the time of the discharge of water on the down-stroke, some water is collected into the air-chamber which compresses the inside air. On the up-stroke the compressed air expands and forces the water below it to flow up the pipe of the air-chamber and thus a continuous flow is obtained.

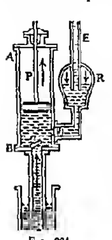


Fig. 204

[*Note*.—Applying sufficient force to the handle of the piston, water can be raised to *any height*, if the machine be strong. If the height be very great, then water can be collected by one pump into a reservoir at a certain height from which it can be raised again by a second *pump*.]

Fire-Engines.—These are used for extinguishing fire and are merely machine-driven force-pumps. With the help of an air-chamber as described above a continuous flow of water is obtained from these pumps.

In the present form of the fire-engine, the continuity of the flow of water is maintained more efficiently by means of two force pumps connected to a common air-chamber and working with alternating strokes, *i.e.* when one piston moves down, the other moves up.

In the most modern types, a continuous flow of water is supplied by means of a rotary centrifugal pump operated by petrol or electric power.

Maximum Height to which water can be raised by Suction and Force Pumps.—The **suction pump** depends on the atmospheric pressure for its working, and the height to which it can raise water is therefore limited to 34 ft. theoretically—much less in actual practice.

In a **force pump**, pressure is directly applied to the liquid by means of a piston, and the action of the pump is not therefore dependent on the atmospheric pressure. The height to which water can be raised by such a pump depends on the strength of its parts and the power applied (hand, steam, or electric). The maximum height to which it is safe to raise water in this way is, however, about 800 ft. There is no valve in the piston of a force pump.

334. The Rotary Pump:— A pump of the rotary class is valuable for use where lack of space prevents the adoption of an ordinary plunger pump. Its discharge is continuous and can be worked over a wide range of speeds. Moreover, a rotary exhaust-pump is superior to a piston-pump, for it is simpler, faster and can produce higher vacua. Its principal disadvantage is due to leakage past the rotating surfaces, which results in loss of efficiency.

The principle of a **Hyvac-rotary pump** is illustrated in Fig. 205. Such a pump can be used to reduce the pressure in a vessel to about 0·001 mm. of mercury.

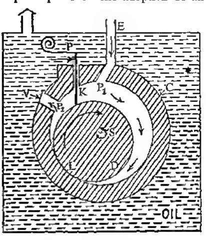


Fig. 205—The Rotary Pump.

A cylindrical drum D acts in it as rotor and it is mounted eccentrically to a shaft S which passes along the axis of a cylinder C. The shaft is rotated by an electric motor. The drum and the cylinder are machined accurately such that the surface of the drum just slides on the inner surface of the cylinder as the shaft rotates. In the figure the line of contact is shown by L at some instant of time. P_1 is the entrance port through which air from the vessel to be evacuated enters into the cylinder, and P_2, the exit port through which the air leaves the cylinder. The exit port P_2 is provided with a simple valve V opening only outwards. A scraping vane K is constantly pressed on to the drum, between the entrance and exit ports, by the action of a spring P, whatever be the position of the eccentric drum in course of its rotation. The whole arrangement is immersed in some oil of low vapour pressure contained in a box as shown in the figure. Pipe E connected to the entrance port projects out to be connected to the vessel intended for evacuation. The oil lubricates the shaft and prevents air leakage along the shaft into the high vacuum in the cylinder.

The drum D, as shown, is rotated in the clockwise direction at the rate of a few hundred revolutions per minute. At the instant shown in the figure, the volume, on the entrance side of the line of contact L, *i.e.* at the tail end of the rotor, is increasing and so the pressure diminishing thereby causes the air in the vessel to flow into it. On the other hand, the volume, on the exit side of L, i.e. at the head end of the rotor, is decreasing. This means that the air in front

of the rotor is driven out through the exit valve V. The scraper vane K prevents any air from flowing from the head end of the rotor to its tail end.

As the rotor D continues to rotate, a time comes when the line of contact L passes the exit port P_2. So the exit port then becomes exposed to the vessel to be evacuated and the atmospheric pressure closes the valve V. Soon after the line of contact passes also the entrance port P_1, when the volume of air in front begins to be swept out again as in the previous cycle.

334(a). Langmuir's Condensation Pump :—

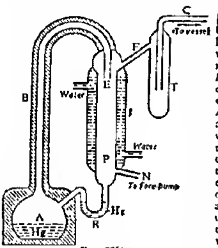


Fig. 205(a)

Mercury in the bulb A [Fig. 205(a)] is boiled by heating with a gas burner or by an electric heater. The vapour passes through B and issues out through the orifice E in vessel P which is covered by a water-jacket J. The condensed mercury returns to A through R. The bulb A and the tube B are lagged with asbestos to prevent the mercury vapour from condensing before it issues through E. The vessel to be evacuated is connected to F directly or through a liquid air-trap T (which is partly immersed in liquid air contained in a Dewar flask) wherein the mercury atoms diffusing through F may be condensed. The vessel P is connected at the bottom through the side tube N to a fore-pump which reduces the pressure in the vessel to about 1 mm of mercury before the diffusion-condensation pump begins to operate.

If there is a large concentration of mercury vapour in the vessel P above the level of the jet E, it would tend to cut down the speed of pumping because the gas molecules will then have to diffuse through mercury before coming in contact with the stream of the jet. So the level of water in the jacket J should always be sufficiently above the level of the jet E.

Such a pump can produce a vacuum of the order of 10^{-6} to 10^{-8} mm. of mercury depending on the design. Without the liquid air trap T, the mercury atoms diffuse into the vessel to be evacuated and pressure reduction will be less.

The dimension of the connecting tubes are important in determining the speed of exhaustion. The connecting tubes should be short and wide for high speed of evacuation.

Instead of mercury, organic liquids (oils) with high boiling points and low vapour pressures are now-a-days increasingly used in such pumps.

335. The Centrifugal Pump:—It is also a rotary pump with continuous discharge and can be worked over a wide range of speeds. It is suitable where a large volume of liquid is to be discharged against low heads and is widely used in irrigation. In a centrifugal pump, pressure energy is imparted to a mass of liquid, water ordinarily, by the rotation of an impeller wheel. The wheel is formed of a number of curved blades (Fig. 206) which entangles the liquid and revolves in a suitable casing. The liquid passes from a suction pipe into the centre or *eye*, as it is called, of the impeller. As the wheel is rotated, say, by an electric motor or any other device, the liquid acquires a high whirling velocity, resulting in an increase of pressure in a radial direction outwards and a tendency to outward flow due to centrifugal action. Thus the velocity is reduced and changed to pressure. If the speed of rotation is sufficiently high, the increase in pressure becomes large enough to more than balance the static head (provided it is low) against which it is to act and the flow takes place. This reduces the pressure and causes the fluid to rise in the suction pipe and enter the wheel at its centre. The flow takes the liquid into an outer shell called the *volute* chamber which leads to the *discharge outlet* of the pump.

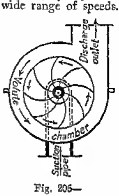


Fig. 206—
A Centrifugal Pump.

336. The Siphon:—It consists of a bent tube with one of the arms *AB* longer than the other, *CD* (Fig. 207). The tube is first filled with the liquid to be drawn off; the two ends are then temporarily closed with fingers, and the shorter leg is placed in the vessel to be emptied below the level of the liquid. On opening the two ends, the liquid begins to flow.

Let P=atmospheric pressure, d=density of the liquid and h, h'= vertical heights of D and B above the liquid surfaces on their sides.

The pressure p_2 at D urging the portion of the liquid at D to the left=$P-h\,d\,g$.

The pressure p_1 at B urging the portion of the liquid at B to the right=$P-h'dg$.

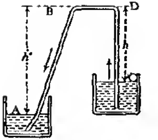


Fig. 207—The Siphon.

$\therefore$ $p_1 - p_2 = (h' - h)dg$. But $h' > h$
$\therefore$ The pressure at $D >$ the pressure at B.

Hence the water flows from D to B and the water from the vessel is raised by the atmospheric pressure to D for filling up the vacancy so caused. Thus the flow is maintained.

(a) Conditions for the working of the Siphon.—(1) In the beginning the whole tube must be completely filled with the liquid.

(2) The end A of the longer tube must be *below* the level C of the liquid in the vessel to be emptied; otherwise h' will not be greater than h and so the pressure at B will not be less than the pressure at D and the liquid will not flow.

(3) *The height 'h' must be less than the height of the corresponding liquid barometer*, otherwise the pressure of the atmosphere will not be able to raise the liquid to D. The greatest height of h, in the case of water, is 34 ft.

(4) The siphon would not work in vacuum, for the atmospheric pressure which raises the liquid is non-existent in a vacuum.

(b) Effect of making a hole in the Siphon.—When a hole is made at any point in the longer arm AB (Fig. 207) above the surface C of liquid in the vessel in which the shorter leg is placed, the siphon will cease to act, for, at the hole the pressure being atmospheric, the pressure at B will not be less than the pressure at D, a condition which must be fulfilled to enable the liquid to flow from D towards B.

If, however, a hole is made at a point in AB *below* the surface at C, the remaining portion above that point will still form a siphon and through the hole the liquid will continue to flow.

Example. *The two arms of a siphon having an internal diameter of 2 inches are respectively 12 and 8 inches in length. The shorter arm is immersed in a liquid to a depth of 2 inches. Calculate the velocity of flow of the liquid and also the amount of the liquid discharged through the siphon in one second (g = 32·2 ft./sec².)*

Ans. The flow of the liquid depends upon the height $(h' - h)$ (see Fig. 207). So we have from the law of falling bodies the velocity of flow per sec., $v = \sqrt{2g(h' - h)}$

Here h' = 12 inches = 1 ft., h, i.e. the actual height above the level of water = (8 − 2) = 6 inches = 0·5 ft.

$$\therefore \quad v = \sqrt{2 \times 32{\cdot}2(1 - 0{\cdot}5)} = 5{\cdot}67 \text{ ft. per sec}$$

The amount of liquid discharged in one sec. = velocity of flow × cross-sectional area of the tube $= 5{\cdot}67 \times \left\{ \pi \left(\frac{2}{12 \times 2} \right)^2 \right\}$ cu. ft. $= 0{\cdot}124$ cu. ft.

337. The Intermittent Siphon :— Fig. 208 represents an intermittent siphon, which is an example of the application of the principle of a siphon. The vessel is at first empty, but as any liquid is poured into it, and the level of the liquid gradually reaches the top of the bend, the liquid will begin to flow to *O*. If the supply of the liquid is discontinued, or the liquid escapes faster than it is supplied to the vessel, the flow will cease as soon as the shorter branch no longer dips in the liquid. But the flow will, however, resume when the level of the liquid reaches the bend again on the supply being restored.

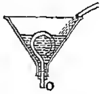


Fig. 208—The Intermittent Siphon.

Tantalus Cup.—The above principle is applied in the toy siphon known as the *Tantalus Cup*, in which the siphon is concealed inside the figure of Tantalus, placed inside a vessel. It is seen that the bend of the siphon is just below the lower lip of the figure (Fig. 209). As water is poured, the level of water in the shorter branch of the siphon rises graduall[illegible] until it reaches the top of the bend, i.e. [illegible] beneath the lips of Tantalus, when the wate[illegible] flow out keeping Tantalus thirsty for ever.


Fig. 209—Tantalus Cup.

Automatic Flushes.— The same [illegible] also applied in automatic flushes fitte[illegible] latrines etc. A siphon is fitted inside [illegible] is emptied as soon as water fills [illegible]

and this tension increases, as the bell sinks more and more and the weight of displaced water becomes less.

Taken 34 ft. as the height of the water-barometer, the pressure of air within the bell at a depth of 34 ft. will be 2 atmospheres; consequently, the volume of air is halved, and the water would rise half-way up the diving-bell. As this is obviously inconvenient for the workmen inside the bell, a constant supply of fresh air is pumped into the bell through a pipe in order to prevent water from entering the chamber and also to enable the workmen to breathe.

Examples. *(1) A bottle whose volume is 500 c.c. is sunk mouth downwards below the surface of tank containing water. How far must it be sunk for 100 c.c. of water to run up into the bottle? The height of a barometer at the surface of the tank is 760 mm. and the sp. gr. of mercury is 13.6.* (Pat. 1928)

The volume of the air inside the bottle, when 100 c.c. of water rushes in $= 500 - 100 = 400$ c.c.

If P be the pressure in cms. when the volume of the enclosed air is 400 c.c. then by Boyle's Law, $P \times 400 = 76 \times 500$; or, $P = \frac{76 \times 500}{400} = 95$ cms.

$\therefore$ The pressure exerted by water only $= 95 - 76 = 19$ cms. of mercury $= 19 \times 13.6$ cms $= 258.4$ cms. of water. ($\because$ Atmos. press. $= 76$ cms.)

$\therefore$ The bottle must be sunk below 258.4 cms. of water.

(2) Find to what depth a diving-bell must be lowered into water in order that the volume of air contained may be diminished by one quarter, the length of the bell being 3 metres, atmospheric pressure 760 mm. of mercury, and the sp. gr. of mercury 13.6. (Pat. 1943)

Length of the bell $= 3$ metres $= 300$ cms.

If P be the total pressure in cms. when the bell is lowered into water in order to diminish its volume by one quarter, we have, by Boyle's Law,

$(300 \times a) \times 76 = (\frac{3}{4} \times 300 \times a) \times P$, where a is the area of the base of the bell.

$\therefore$ $P = \frac{76 \times 4}{3}$ cms. of Hg $= \frac{76 \times 4 \times 13.6}{3}$ cms. of water

$\therefore$ The pressure exerted by water only

$$= \left\{\frac{76 \times 4 \times 13.6}{3} \quad (76 \times 13.6)\right\} = \frac{76 \times 13.6}{3} \text{ cms. of water.}$$

The volume of air inside being diminished by one quarter, the height of water inside the bell $= \frac{1}{4} \times 300 = 75$ cms., and so the length of air inside $= (\frac{3}{4} \times 300) = (3 \times 75)$ cms.

$\therefore$ The depth to which the bell is lowered, i.e. the height of water from the surface up to the top of the bell $= \left\{\frac{76 \times 13.6}{3} - (3 \times 75)\right\}$ cms. $= 119.53$ cms.

339. Otto von Guericke (1602—1686):—He was a German lawyer, Senator and Physicist. He was born at Magdeburg, descendent of a noble family. During Tilly's siege of Magdeburg (1631) he acted as "Defence or war-lord" of his native town. When Tilly was driven off, and his native town came under Swedish protection, he helped in rebuilding the bridges and fortifications of his native town the well-being of which was his constant anxiety. He was appointed its Burgomaster in 1646. Without the requisite scientific knowledge, he started experiments which he did not leave before success came to him. His ranking with great scientists is not due to his invention of the air-pump but how he conceived to make use of the same in solving outstanding problems in nature. He had a special fascination for large apparatuses for his experiments so that the uninitiated might be attracted. The discoveries of Galileo, Pascal and Torricelli generated an urge in him for producing the first vacuum and he invented the first air-pump. In the year 1654 he performed before the emperor, Ferdinand III, his famous experiment of the Magdeburg hemispheres to prove that air has weight and exerts pressure. It is said that two teams of twenty-four horses, a team on either side, were required to separate two hemispheres, when the air was pumped out from within. Boyle made use of Guericke's pump to prove the law which bears his name. Guericke made other inventions too. He discovered electrical repulsion for the first time and constructed also a frictional machine which Leibniz used and thereby produced electric sparks for the first time. From his studies in astronomy he predicted the periodic return of comets. He is said to have devised a water-barometer by which the approach of storms could be forecast. He died at the age of 84 in Hamburg.


Otto Von Guericke

Questions

1. Describe in detail an air-pump giving a diagram and explain its action. (C. U. 1923; Pat. 1925, '29, '38; P. U. 1929; U. P. B. 1930)

After four strokes the density of the air in the receiver of an air-pump is found to bear to its original density the ratio of 256 to 625. What is the ratio of the volume of the barrel to that of the receiver? (C. U. 1923)
[*Ans.* 1 : 4]

2. Describe briefly the action of the air-pump in its simplest form and explain how the degree of rarefaction produced by a given number of strokes can be approximately calculated. Can the apparatus you describe create perfect vacuum? If not, why? (Pat. 1931, '38, '41; P. U. 1931)

3 If the cylinder of an air pump is one third the size of the receiver, what fractional part of the original air will be left after 5 strokes? What will a barometer within the receiver read, the outside pressure being 76 cms

[Hints. $d_5/d=(3/4)^5$; again $P_5=(3/4)^5\times 76$] (Pat 1929)

4 Compare the pressures in the receivers of a condensing and exhausting air pump after the same number of strokes in each case and account for the fundamental difference in form of the two expressions. (Pat, 1931)

5 Describe a double barrelled air pump and explain its action. (C. U. 1938, '47, '53)

6 Can you get perfect vacuum with an air pump? If not, why not? Explain how the air pump differs in operation from a water pump. (C. U. 1953)

7. A mercury barometer is in the receiver of an air pump, and at first its height is 76 cms. After two strokes the height is 72 cms. What will it be after ten strokes? (Neglect the volume of the barometer) (Pat 1937)

[Ans. 58 cms approx.]

8 Write a short note on 'Filter pump'. (All. 1946)

9 What do you mean by a compression pump? Cite two common examples. Describe with a diagram the working of an ordinary bicycle pump and the action of the valve in the bicycle tube (C U 1952)

10. Describe in detail with a diagram a condensing pump and its mode of action (C U 1925, '24)

11. Describe with the help of a neat sketch, the working of an ordinary bicycle pump, and the action of the valve in the bicycle tube (Pat 1944)

12 Describe and explain the action of a bicycle pump What is the difference between such a pump and an ordinary exhaust pump? (Pat 1943)

13 Explain the mode of action of a football inflating pump (Pat 1923)

14 Describe a suction pump Water cannot be raised to a height much greater than 34 ft by means of such a pump State the reason for this and describe a laboratory experiment by which you prove your explanation to be correct. (C. U. 1930, '34, Dac. 1932; cf U P B 1961)

15 Describe in detail with a diagram a common pump and its mode of action Is there any limit to the depth from which it can raise water? (C U 1924; Pat 1935, Dac 1932)

16 Explain clearly the working of the usual types of Lift or Force pumps

A lift pump is used to pump oil of sp gr 0·8 from a lower into an upper tank What is the maximum possible height of the pump above the lower tank when the pressure of the atmosphere is 76 cms of mercury? Is this height practically obtained? Give reasons for your answer (Pat 1920)

[Hints. $h\times a\times 0{\cdot}8\times g=76\times a\times 13{\cdot}6\times g$. $\therefore$ $h=\dfrac{76\times 13{\cdot}6}{0{\cdot}8}=1292$ cms]

17 Explain clearly, with the aid of a neat sketch, the working of the usual types of lift pumps Is there any limit to the depth from which it can raise water?

The barrel of a suction pump is 5 in in diameter and the stroke is 8 in. How many upward strokes of the plunger will be required to lift 1 000 gallons of water if there is 12% slip? [1 cu ft. of water = 6·24 gallons] (C U 1927)
[Ans. 2003]

18. What mechanism would you suggest to lift water from a well which is deeper than 34 ft.? (A. B 1952)

19. Name different kinds of pumps for producing high vacuum. Explain the construction and working, with the help of a diagram, of any one of them.

(R. U. 1952)

20. Explain the action of a siphon.

(C. U. 1926, 37; Pat. 1921; Dac. 1926; All. 1946)

21. A siphon is used to empty a cylindrical vessel filled with mercury. The shorter limb of the siphon reaches the bottom of the vessel which is 45 inches deep, but it is found that mercury ceases to run before the vessel is empty. Explain this observation, and calculate what fraction of the volume of the vessel will remain full of mercury. The barometric height may be taken as 30 inches. (Pat. 1935; *cf.* C. U. 1926; Dac. 1930)

Ans. [$\frac{1}{3}$].

22. Explain the principle and use of the siphon, and state how the principle is used in Tantalus Cup. (C. U. 1928)

its temperature falls, except when it is not changing its state such as water changing into steam, or water changing into ice, etc.

(3) *Change of Dimension.*—A body, whether a solid, a liquid, or a gass, expands on heating and contracts on cooling.

(4) *Change of Composition (Chemical change)*.—Many substances become chemically changed when heated. Sugar, for example, when heated in a test tube, is turned into carbon, which is left at the bottom of the tube, and water vapour, which condenses at the top of the tube.

(5) *Change of physical properties*.—Many substances, when heated, become weak possibly due to some internal change in the arrangement of their molecules. Thus, iron, when heated to redness, differs materially from iron at ordinary temperatures, and ordinarily, glass when heated becomes weakened.

(6) *Electrical effect*.—(i) When by heating one of the two junctions of a thermo-couple formed of two dissimilar metals, say copper and iron, a difference of temperature is produced between the junctions, and electrical current flows round the wires. This is known as *thermo-current*. (ii) When heated, the electrical resistance of a metal increases.

5. Measurement of Temperature:—We can have an idea about the temperature of a body, i.e. the degree of its hotness, by our sense of touch. But the measurement of temperature by the sense of touch often gives unreliable and inaccurate results.

The sensation depends upon, (i) *the amount of heat transferred* to the skin of the body from the substance touched, when the temperature of the substance is higher than that of the body; or from the skin to the substance, when the temperature of the substance is lower than that of the body, and on (ii) *the conductivity of the substance*, that is, on the rate at which heat is transferred.

As this sensation is not a safe guide for the correct and numerical measurement of temperature, instruments, called **thermometers**, are devised for the purpose.

Strictly speaking, temperature is not a measurable quantity, but for various purposes we measure it in some indirect way. We utilise one or the other of the physical effects produced by heat, as enumerated in Art. 4, for measuring the temperature of a body, for example, in *mercury thermometers* the expansion of mercury inside the thermometer is used to indicate the temperature. Different types of thermometers depending on the different effects of heat have been constructed and each different type has its own merits and demerits and its own range of use.

6. Choice of Thermometric Substance :— In selecting the material for the construction of a thermometer, it is necessary to see that *(a) the substance always shows the same temperature for the same hotness; (b) the temperature changes continuously with the change of the degree of hotness; (c) the substance is convenient to use; (d) the change of the property, which is utilised for the measurement of temperature, is fairly large.* Expansion of a substance with rise of temperature, provided the former is uniform, is commonly utilised in ordinary thermometers.

Some liquids are suitable as thermometric substances, their expansions being fairly uniform and moderately large: solids expand little, whereas gases expand much more; of all liquids *mercury* has been found to be the best on account of its many advantages.

It should however, be noted that in all accurate measurements of temperature, a gas thermometer (*vide* Chapter IV) **is always referred to as the standard in preference to all other thermometers.**

7. The Hypsometer :— It is a specially constructed *constant temperature bath* in which steam is generated under the existing atmospheric pressure by heating water. The temperature of the steam is related to the pressure and has, therefore, a connection with the height of the place. So the apparatus is named a hypsometer which, in Greek, means a 'measurer of height'.

The apparatus consists of a brass vessel having an internal chamber B and an external chamber A around it closed at the bottom (Fig. 8). The internal chamber is in communication with a boiler D placed below. An open-tube manometer M, connected as shown in the figure, is used to indicate the pressure of the vapour raised from the boiler. Ordinarily, water is taken as the boiler liquid. The top of the apparatus is covered except at a central opening which is closed by a cork C. A thermometer T is inserted through a hole bored in the cork such that the bulb of the thermometer is held above the liquid level in the boiler. The hot steam rises up in the inner chamber and then passes down the outer one, as shown in the figure, to escape finally into the atmosphere through an exit tube E provided at the bottom. The liquid formed by condensation of the escaping vapour is collected in a basin and can be used again in the boiler. The outer chamber, through which the hot steam passes down, protects the steam rising in the inner chamber from condensation. The heating of the water boiler is so regulated that the liquid attains the same level in both the arms of the manometer. The

Fig 1

steam pressure then equals the atmospheric pressure and the temperature indicated by the thermometer at this stage gives the temperature of boiling of the liquid at the place of observation.

8. Construction of Mercury Thermometer:—A thick-walled glass tube of *uniform capillary bore* with a bulb *B* blown at one end is taken (Fig. 1). At *C*, near the open end, the tube is heated and drawn out so as to make a narrow neck there.

A small funnel *E* is fitted at the open end by means of a piece of rubber tubing Some pure dry mercury is put in the funnel *E*, but the mercury cannot get into the tube owing to the contained air and fineness of the bore. The bulb is heated gently to drive out some of the air in it, which on cooling, contracts in volume, and the mercury from the funnel passes down the tube into the bulb due to the atmospheric pressure acting from above, which is greater than the pressure inside *This process of alternate heating and cooling* is repeated several times till sufficient mercury enters to fill the bulb and some part of the tube The funnel is then taken away and the bulb is strongly heated until the mercury fills the whole of the tube, which is then quickly sealed at *C* by a blow-pipe flame Mercury having filled the entire tube, the tube is free from air. On cooling, the mercury contracts, and, at ordinary room temperatures, fills the *bulb* and a part of the *stem*. The rest of the tube contains only a negligible quantity of mercury vapour.

Three points are to be remembered regarding the thermometer construction :—

(1) The size of the bulb and the bore of the tube will depend upon the sensitivity of the thermometer and the number of degrees and their sub-divisions which the thermometer is to register; that is, a thermometer to read to 1/5th degree or 1/10th degree must have a longer tube with a finer bore than a thermometer reading only to 1°

(2) The quantity of liquid used should be small so that it might take as little heat as possible from the source whose temperature is being recorded otherwise it will itself lower the temperature to be recorded Thus the bulb should be small in size.

(3) The *bulb* of the thermometer should be made thin so that heat from the source may quickly pass through to warm up the liquid; this is necessary in order that the thermometer may be quick in action

Graduation—The tube being filled with mercury and sealed, should be left over for several days to cool down so that it may recover its original volume. Only after such proper *ageing* the tube may be regarded as ready for *graduation*. The first step for graduation, whatever is the scale of temperature used, is to mark on the stem the positions for the mercury thread corresponding to two definite tem-

peratures. These are called the two **fixed points of a thermometer.** These are defined and experimentally determined as follows:—

(i) The Lower Fixed Point (or Ice Point).— *It is the temperature at which pure ice melts under the normal atmospheric pressure.* Since its variation with pressure is negligibly small, the *ice-point* is determined under the ordinary atmospheric pressure and no correction is necessary. The funnel F (Fig. 2) contains powdered distilled water ice washed with distilled water. A hole is made in this ice and the bulb of the thermometer T is inserted in it and the thermometer is held vertically in it by means of a stand. The mercury column descends and after some time takes a *stationary stand*, when the position of its top is marked on the glass. This gives the lower fixed point.

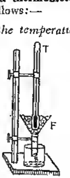


Fig. 2

(ii) The Upper Fixed Point (or Steam Point).— *It is the temperature at which pure water boils under the normal atmospheric pressure.* It is usually determined under the ordinary atmospheric pressure and a pressure correction is then made. In applying this pressure correction an empirical rule is followed, according to which the boiling point of pure water varies *directly by* 0·37°C. when the superincumbent pressure changes by *one centimetre* (in other words, the boiling point of water increases or decreases by 1°C. due to an increase or decrease of pressure by about 27 mms. of mercury) near the normal atmospheric pressure.

The thermometer T is inserted into the inner chamber of a *hypsometer* (Fig. 3), leaving the upper part projecting out above the cork C. The boiler D contains water up to a level below the bulb of the thermometer. It is heated and the steam generated from the boiling water heats up the mercury of the thermometer. The thermometer is held in the steam and not in the water, because the temperature of the water may be higher than that of the steam corresponding to the existing atmospheric pressure due to any dissolved impurity. The heating is so regulated that the pressure of the steam may always be equal to the atmospheric pressure outside, which is indicated by the equality of the mercury levels in the two arms of the manometer M. When the Hg-top in the thermometer is observed to have become stationary, it is marked.

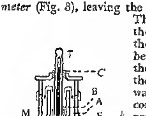


Fig. 3—Hypsometer.

After locating the positions of the two fixed points on the stem, the interval between the two points, called the **fundamental**

interval, is divided into an appropriate number of equal parts, depending on the nature of the scale of temperature desired, each part being called a degree in that scale; each degree may then be further subdivided according to requirements.

This method of marking assumes that the *bore of the tube is uniform* and that *the liquid expands uniformly*.

Should the Bore of the Tube be Uniform ?—Unless the bore is uniform, equal rise of mercury in the tube will not indicate equal rise of temperature and so the graduation shall have to be done point to point throughout the bore. Such action being tedious and costly, a tube of uniform bore is selected in commercial practice.

9. Sources of Error in a Mercury Thermometer :— (1) *Non-uniformity of the Bore*.—Each degree of a thermometer represents an equal change of temperature. When the temperature rises, the liquid column moves along the bore of the thermometer and the movement of the liquid column due to change of volume of the liquid will be uniform, only if the bore is uniform, otherwise each equal length in the different parts of the stem will not represent equal change of temperature.

(2) *Temperature of the Exposed Column*.—At the time of using a thermometer for recording a temperature, a part of the stem always remains outside the substance whose temperature is to be taken and its temperature therefore is different from that of the bulb and the rest of the stem below it. So the temperature recorded will be lower than the actual temperature, and thus it is desirable to include as much of the stem as possible inside the substance. A correction for the exposed part may then be applied (*vide* Chapter III, Art. 31).

(3) *Change of Zero*.—A thermometer placed in melting ice often indicates a reading greater than the *freezing point*. This is due to depression of the freezing point mark owing to contraction of the tube and the bulb, which takes place slowly over a long period after the marking of the fixed points. To avoid this the thermometer should be left out for a long time before the scales are marked.

10. Scales of Temperature :— There are *three* scales of temperature in use: **Centigrade, Fahrenheit** and **Reaumur**.

(i) The *Centigrade scale*,* according to some writers, was designed by Elvius of Sweden in 1710 and was reintroduced by Christen in 1743. Others associate the name of Anders Celsius†

* More recently, the name *Centigrade scale* has been replaced by *Celsius scale*, though the notation for it has been kept the same as before, name °C.

† *Anders Celsius* (1701—1744), a Swedish astronomer and Professor of Astronomy at the university of Upsala introduced a scale by taking 0° as the boiling point of water and 100° as the melting point of ice. Then at about 1742 Linne introduced the *Centigrade scale* by reversing the above with the melting point of ice at 0° and the boiling point of water at 100°. Celsius went to Lapland with a centigrade thermometer to record the temperature of the arctic region.

in this connection. The *zero* of this scale corresponds to the melting point of pure ice, and the boiling of water under the normal atmospheric pressure is taken as 100°. The interval between the two is divided into 100 equal parts.

(*ii*) The *Fahrenheit scale* was devised by **Fahrenheit**, a German philosopher (1686—1736), at about 1709. The temperature of a freezing mixture of snow and common salt (which is much below the melting point of ice) is taken as the zero of his scale. The melting point of pure ice, according to the scale, is taken as 32°, and the boiling point of water as 212°, under normal atmospheric pressure. The interval between the two is divided into 180 equal parts.

(*iii*) The *Reaumur scale* was introduced by **Reaumur** (1683—1757), a French philosopher, in 1731. In it the melting point of ice is taken as 0° and the boiling point of water, under normal atmospheric pressure, as 80°. The interval between the two is divided into 80 equal parts.

The Fahrenheit scale is generally used in Great Britain, the United States and in some English-speaking countries for household purposes. It is also used in clinical thermometers. The Centigrade (from L. *Centum*, a hundred; *gradus*, step) scale is universally used in scientific work all over the world. The Reaumur scale is used in Russia for household purposes and in some parts of the European continent.

Comparison of the three scales of temperature.—

The distance between the lower and upper fixed points of a thermometer is called the **Fundamental Interval (F.I.).**

The fundamental interval is divided into 180, 100, and 80 equal parts in the Fahrenheit, Centigrade, and Reaumur scales respectively. Fig. 4 depicts the three scales given to a mercury thermometer of which *A* and *B* are the *lower* and the *upper* fixed points respectively.

Fig. 4

The table on P. 320 gives the data about the three scales and the symbols which are used in expressing a temperature in these scales.

Scale	Symbol	Freezing Point	Boiling Point	No. of Divisions between Fixed Points
Fahrenheit ..	°F.	32°	212°	180
Centigrade ..	°C.	0°	100°	100
Reaumur .	°R	0°	80°	80

(i) We find that, $100°C. = 212° - 32° = 180°F. = 80°R.$;
or, $1°C = \frac{9}{5}$ of $1°F. = \frac{4}{5}$ of $1°R.$

(ii) Let P (Fig. 4) represent the steady position of the top of the mercury thread at some temperature and let F, C, R, be the readings of this temperature on the three scales, Fahrenheit, Centigrade, and Reaumur respectively.

Then, since AP is the same fraction of AB whatever be the scale used, we have

$$\frac{AP}{AB} = \frac{F-32}{180} = \frac{C-0}{100} = \frac{R-0}{80}\text{ , or, }\frac{F-32}{9} = \frac{C}{5} = \frac{R}{4}$$

Remember that 1 Centigrade degree is nine-fifth of a Fahrenheit degree, and 1 Fahrenheit degree is five-ninth of a Centigrade degree.

Examples. *(1) Calculate the temperature which has got the same value on both the Centigrade and the Fahrenheit scales.*

Let x be the value required. Then, $\frac{x-32}{9} = \frac{x}{5}$, or, $5x - 160 = 9x$;

or, $4x = -160$, i.e. $x = -40$. Thus $-40°C$ when converted to the Fahrenheit scale, will also be $-40°$, or, $-40°C = -40°F$

(2) The same temperature when read on a Centigrade and a Reaumur thermometer gives a difference of 1°. What is the number of degrees indicated by each thermometer?

Let x = Centigrade temperature, and y = Reaumur temperature
Then, we have, $x - y = 1$ .. (1)

Now, $x°C$, transformed into Reaumur degrees $= x \times \frac{4}{5} = y$.

$\therefore$ From (1), $(1+y)\frac{4}{5} = y$ $\therefore$ $y = 4°R$.

But $4°R = 4 \times \frac{5}{4} = 5°C$ $\therefore$ The required temperatures are $5°C$ and $4°R$

(3) Find out the temperature when the degree of the Fahrenheit thermometer will be 5 times the corresponding degree of the Centigrade thermometer.

Let x = Fahrenheit temperature, and y = Centigrade temperature

Then $x = 5y$ (1) But $x°F$. transferred into Centigrade degree $= (x-32)\frac{5}{9} = y$

$\therefore$ From (1), $(5y-32)\frac{5}{9} = y$; or, $16y = 160$. $\therefore$ $y = 10°C$.

And $10°C. = (10 \times \frac{9}{5}) + 32 = 50°F$.

Hence the required temperatures are $10°C$. and $50°F$.

(4) Two thermometers A and B are made of the same kind of glass and contain the same liquid. The bulbs of both the thermometer are spherical. The internal

diameter of the bulb of A is 7·5 mm. and the radius of cross section of the tube is 1·25 mm.; the corresponding figures for B being 6·2 mm. and 0·9 mm. Compare the length of a degree of A with that of B.

Let l_1 and l_2 be lengths corresponding to 1° rise in the temperature for A and B respectively and λ the apparent coefficient of expansion of the liquid.

Increase in volume of the liquid in the bulb of A for 1° rise $= \frac{4}{3}\pi\left(\frac{7.5}{2}\right)^3 \times \lambda \times 1$,

and this must rise in the tube, the volume being $\pi\,(1.25)^2 l_1$.

$\therefore\ \frac{4}{3}\pi\left(\frac{7.5}{2}\right)^3 \times \lambda \times 1 = \pi(1.25)^2 l_1$. Similarly, for B, $\frac{4}{3}\pi\left(\frac{6.2}{2}\right)^3 \times \lambda \times 1 = \pi(0.9)^2 l_2$

$$\therefore\ \frac{l_1}{l_2} \times \frac{(1.25)^2}{(0.9)^2} = \frac{(7.5)^3}{(6.2)^3}\ ;\ \text{whence}\ \frac{l_1}{l_2} = \frac{1.09}{1.69}.$$

11. Corrections for Thermometer Readings :— The temperature at which water boils depends upon the atmospheric pressure. It is 100°*C.* when the atmospheric pressure is normal, *i.e.* 760 mm. It increases or decreases with the increase or decrease of the atmospheric pressure. For small deviations from the normal pressure there is *a change of 0·37°C. in the boiling point of water for a change of 1 cm. in the atmospheric pressure, and so a change of about two-thirds of a degree Fahrenheit for a 10 mm. change of pressure.* The effect of the change of pressure is, however, negligible for the freezing point of water, which is lowered only by about 0·0078 of a degree Centigrade for one atmosphere increase of pressure.

So the fixed points of a thermometer can be corrected at any time by reading the height of the barometer. This will be clear from the following example:—

Atmospheric pressure = 754·96 mm.

Difference from the normal pressure = 760 − 754·96 = 5·04 mm.

There is a variation of 1°*C.* for a change of 27 mm. in the atmospheric pressure. ∴ The required correction = 5·04 ÷ 27 = 0·186°*C.*

But as the observed atmospheric pressure is less than the normal pressure, the steam point will be less than 100°*C.* Thus the true steam point = (100 − 0·186) = 99·814°*C.*

Observed steam point = 99·6°*C.* ∴ Error at steam point = 99·6 − 99·814 = −0·214°*C.*

∴ Correction at steam point = +0·214°*C.* If for

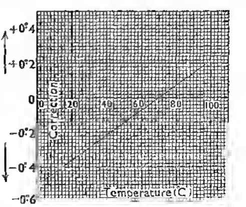


Fig. 5

the above thermometer the freezing point is 0.5° above zero, the error is +0.5°C, and the correction to be applied is −0.5°C. Thus, plotting these two points on a squared paper, the straight line (Fig 5) joining these two points will indicate the corrections at intermediate temperatures. From the graph it is evident that no correction would be required at 70°C.

Examples. (1) *The stem of a Fahrenheit thermometer has a scale upon it which is graduated in equal parts. The reading of the ice-point is 30 and that of the steam-point 300. What is the reading indicated by the thermometer (a) when placed in steam at a pressure of 73 cms of mercury and (b) in water at 50°F.*

(a) Here 300−30=270, scale divisions are equivalent to 180°F

∴ 1 scale division = (2/3)°F

The difference of pressure, (76−73)=3 cms. For 10 mm i.e. 1 cm change in pressure, the boiling point is changed by 2/3°F. For a change of 3 cms in pressure, the change in boiling point $=3\times\frac{2}{3}=2°F$ ∴ The true steam point $=212-2=210°F$ (−2 is taken because the pressure is below normal.)

Now, 2°F is equivalent to $2\div\frac{2}{3}=3$ scale-divisions of the thermometer.

Hence the reading indicated by the thermometer = 300 − 3 = 297

(b) The temperature of water is 50°F = (32° + 18°)F

∴ The reading is 18°F above the ice point, which is 30 on the scale.

Now, 18°F is equivalent to $18\div\frac{2}{3}=27$ scale divisions. ∴ The reading is 30+27=57.

(2) *If when the temperature is 0°C a mercury thermometer reads +0.5°C, while at 100°C it reads 100.8°C, find the true temperature when the thermometer reads 20°C, assuming that the bore is cylindrical and the divisions are of uniform length.* (C U 1924)

The thermometer reads 0.5°C for 0°C and 100.8°C for 100°C. So there are (100.8−0.5)=100.3 divisions between the two fixed points of this thermometer. ∴ Each division of the above thermometer $=\frac{100}{100.3}$ of a true Centigrade division. When the thermometer reads 20°C, there are (20−0.5) or 19.5 divisions above the freezing point. Hence the true temperature of the thermometer when it reads $20°C=\frac{100\times19.5}{100.3}=19.442°C$

(3) *When the fixed points of a Centigrade thermometer are verified it reads 0.5°C, at the melting point of ice and 99.2°C, at the boiling point of water at normal pressure. What is the correct temperature when it reads 15°C, and at what temperature is its reading exactly correct?* (Pat 1931)

The fundamental interval = 99.2−0.5 = 98.7 divisions. Let x be the correct temperature, then we have $\frac{15-0.5}{98.7}=\frac{x}{100}$, when the normal boiling point = 100°C, whence $x=14.7°C$.

Again, let the reading be exactly correct at $t°C$, then $\frac{t-0.5}{98.7}=\frac{t}{100}$ i.e. $100t-50=98.7t$, or, $t=38.5°C$

12. Different Forms of Thermometers :—

(1) Mercury-in-glass Thermometer.—These have been dealt with before (*vide* Arts. 8 to 11).

(2) Alcohol Thermometer.—Alcohol is sometimes used as a thermometric substance instead of mercury. Its advantages and disadvantages as a thermometric substance have been treated in Art. 14. The liquid requires to be coloured with some dye in order that the top of the column may be easily read.

(3) Water Thermometer.—Water has almost all the disadvantages of alcohol and its advantages are very few. Besides this it cannot be used as a thermometric substance due to its peculiar behaviour between 0°C., and 10°C., which has been discussed in Chapter III.

(4) Gas Thermometer.—In these thermometers gases like air, nitrogen, hydrogen, helium, etc. are used as thermometric substance. These have been dealt with in Chapter IV.

(5) Maximum and Minimum Thermometer.—It is often found necessary to know the highest or lowest temperature attained during a given period of time. The maximum temperature reached during the day and the minimum temperature during the night are recorded in meteorological stations as a routine work. Both of such information are important for meteorological as well as agricultural purposes. *A maximum thermometer* automatically registers the highest temperature and a *minimum thermometer*, the lowest temperature, during an interval.

(6) Electrical Thermometer.—There are two common forms of electrical thermometers: (*i*) *resistance thermometers*; (*ii*) *thermocouple or thermo-electric thermometers.* These have been dealt with under Current Electricity (Vol. II).

(i) Rutherford's Maximum and Minimum Thermometer.—These are two separate instruments, but are ordinarily mounted on

Fig. 6—Rutherford's Maximum and Minimum Thermometer.

the same frame (Fig. 6). The maximum thermometer is an ordinary mercurial thermometer placed in a horizontal fashion. As the tem-

perature increases, the mercury pushes forward a steel index which is left in its place to indicate the maximum temperature.

The minimum thermometer uses alcohol, instead of mercury, as the thermometric liquid. It is also fixed in a horizontal position. For recording the minimum temperature an index of glass is placed in the liquid and this allows the alcohol to expand, when the temperature rises, without moving it. But when the temperature falls and the alcohol contracts, the glass index, which is wetted by alcohol, is dragged backwards by the surface film at the end of the alcohol column.

The instrument can be reset for fresh observations by inclining the frame when the indices slide down. The steel index can be made to slide down by using a bar-magnet too.

Fig. 7—Clinical thermometer

(ii) The Clinical Thermometer (or Doctor's Thermometer).—This thermometer is used by doctors for the determination of the maximum temperature of a body. The temperature of the human body seldom varies beyond the limits, 95°F to 110°F and so in a doctor's thermometer this range is graduated in degrees Fahrenheit and each degree again is ordinarily, sub-divided into fifths. Mercury is commonly used as the thermometric substance and by using the requisite quantity of mercury in the bulb, the length of the stem is made short. Fig. 7 shows a clinical thermometer having bulb B which contains mercury and a constriction C in the bore above it. During use when the bulb is placed under the armpit or the tongue, the mercury being heated expands and is forced up into the stem S across the constriction. When the thermometer is taken out the mercury below the constriction contracts with the fall of temperature, but the mercury thread above cannot follow the mercury below because of the constriction. So the thread breaks at the constriction, the farthest end of the standing thread giving the temperature of the body. Thus the clinical thermometer acts as a maximum thermometer. To reset the instrument, the thermometer is held by the stem, bulb downwards and is given a few jerks. This forces the mercury in the stem to go back into the bulb.

To graduate the instrument it is placed in a thermostat at 95°F, and a scratch is made in the stem against the head of the mercury thread when the same is steady. Afterwards it is again placed in a thermostatic bath at 110°F, when again a scratch is made as above. The interval between these two marks are uniformly divided into 15 equal parts and each part into fifths, assuming the bore to be uniform.

As a caution, it should be remembered that a clinical thermometer must not be dipped into hot water or any other hot liquid for the determination of temperature, for the bulb would crack.

(ii) Six's Thermometer.— It is a combined form of maximum and minimum thermometer (Fig. 8).

It consists of a graduated U-tube with a bulb at each end. The tube on the left-hand side of Fig. 8 and a part of the bulb D at that end contain alcohol. The upper part of the bulb contains alcohol vapour only, and so room for expansion is left there. The bent tube contains a column of mercury which merely serves as an *index*, as its movement indicates expansion or contraction of alcohol which, is above it, and in the other tube which is completely full of alcohol. The alcohol in the right-hand tube and the bulb C constitutes the real thermometric part of the instrument.

A small steel index fitted with a spring (shown on the side of Fig. 8) is inside the tube at each end of the mercury column. Each index (P or Q) can be brought into contact with the mercury head at that end by means of a magnet from outside the tube.

When the temperature rises, the alcohol in the right-hand tube expands and so the mercury thread on the left-hand tube rises pushing the index P above it. When the temperature falls, the mercury thread comes down leaving the index in its position (as it is prevented from returning by the spring), but the mercury thread in the right-hand tube rises pushing the index Q above it, which remains there when the alcohol expands again due to rise in temperature. Thus the lower end of the index in the right-hand tube shows the minimum temperature while that in the left-hand tube shows the maximum temperature.

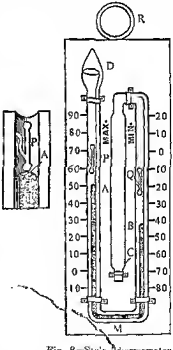


Fig. 8—Six's Thermometer.

13. Advantages of Mercury as Thermometric Substance:— Mercury remains in the liquid state over a wide range (freezing point $-39°C.$, boiling point $357°C.$) and thus can be used over this range of temperatures. The range can be extended to higher temperatures also by filling the space over the mercury with nitrogen, argon or carbon dioxide gas under pressure using a short tube. At ordinary temperatures the vapour pressure of mercury is low and

so the indications of a mercury thermometer are little affected by the pressure of the vapour. Mercury can be easily obtained pure and being a shining grey liquid its position in a glass tube can be ascertained easily. It does not wet glass and has, therefore, no tendency to stick to the walls when the temperature changes. It is a very good conductor of heat and so attains the temp. of a bath very quickly. It has high sensitivity to temperature variations, for its coefficient of expansion is large. It absorbs only negligible heat from any material with which it is placed in contact owing to its low specific heat and, therefore, the temperature which is measured by it is not altered by its use. *The most important property, however, is that its expansion is almost uniform at all part of its scale.*

14. Comparison of the Advantages and Disadvantages of Mercury and Alcohol as Thermometric Substances:—(1) Alcohol freezes at $-130°C$ while mercury at $-39°C$. The former boils at $78°C$, and the latter at $357°C$. So the range of use on the low temperature side is greater for alcohol than for mercury while the range on the high temperature side is greater for mercury than for alcohol.

(2) For a given rise of temperature alcohol expands much more than mercury. So the sensitivity to temperature variations is greater for the former than for the latter.

(3) Although the specific heat of alcohol is greater than that of mercury, a given volume of alcohol will absorb from a bath a much smaller quantity of heat than an equal volume of mercury will do in being raised through the same range of temperature, sp. gr. of alcohol being much less.

(4) Alcohol wets glass while mercury does not. So during a rise of temperature the former can move smoothly in a tube of fine bore while the latter moves in a jerky way.

(5) As alcohol wets glass it tends to stick to the wall as the temperature changes, while mercury has no such tendency.

(6) With rise of temperature alcohol does not expand uniformly but mercury does in a more satisfactory way. So an alcohol thermometer is graduated by comparison with a mercury thermometer, placing both in the same bath.

(7) Alcohol is not a good conductor of heat but mercury is. So an alcohol thermometer cannot attain the temperature of a bath so quickly as a mercury thermometer can.

(8) Alcohol is a light volatile liquid which vaporises appreciably and collects in the space above the liquid meniscus and the pressure rises. The effect is negligible in the case of mercury, a heavy liquid which does not vaporise so easily.

(9) Alcohol requires to be coloured with a dye in order to be visible while mercury is itself a shining opaque liquid.

SOME NOTEWORTHY TEMPERATURES

	Deg. °C.		Deg. °C.
Sun	6000	Mercury boils ...	357
Electric arc light ...	3400	Mercury freezes ...	—39
Iron melts	1500	Blood heat ...	37
Platinum melts ...	1760		
Iron, white hot ...	1300		
Hydrogen boils	—252 to —253	Red heat ...	500—1000
Hydrogen solidifies	—256 to —257	White heat	... above 1000
Lowest temperature obtained ......0·018° absolute.			

Questions

1. A bicycle pump gets heated when the tyre is pumped. Explain. (G. U. 1950)

2. Distinguish between temperature and quantity of heat. (C. U. 1934; Pat. 1921)

3. Briefly describe the process of constructing a mercury-in-glass thermometer. Why is it necessary to note the height of the barometer, when determining the upper fixed point of a thermometer? How would you prepare a thermometer, if you are in a deep coal mine? (Pat. 1932)

[**Hints.**—*See* Arts. 8 and 11. Note the barometric height inside the coal mine and calculate the boiling point of water which will be the upper fixed point of the thermometer.]

4. There are two thermometers of which one has the larger bulb and the other a finer bulb. Explain the advantages and disadvantages in each case. (U. U. 1941)

5. Describe the construction of a mercurial thermometer. Is it necessary that the tube should be of uniform bore throughout? Give reasons for your answer. How is it graduated? (C. U. 1926, '41, '45; *cf.* Pat. 1920, '22, '44)

6. How does a sensitive mercury-in-glass thermometer differ in construction from a less sensitive thermometer?

Describe fully the method followed to mark the scale on a mercury-in-glass thermometer. (C. U. 1952, '56).

7. What is meant by the 'Fundamental Interval' (F.I.) of the thermometer scale in a thermometer? Describe an experiment to determine it accurately.

A thermometer A has got its F.I. divided into 45 equal parts and another *B* into 100. If the lower point of *A* is marked 0 and that of *B* 60, what is the temperature by *A* when it is 110 by *B*? (Pat. 1940)

[*Ans.* 27°]

8. What is the difference between the temperature of a substance and the total heat possessed by it?

Describe the construction of a mercury-in-glass thermometer. Why is mercury preferred for use as the liquid in the thermometer?

What are the fixed points of a thermometer? What should be the marking at a point midway between these fixed points in the centigrade scale and in the Fahrenheit scale? (C. U. 1956)

[*Ans.* 50°*C.*, 122°*F.*]

9. The fundamental interval of a thermometer *A* is arbitrarily divided, into 60 equal parts and that of another thermometer *B* into 120 equal parts. If the freezing point of *A* is marked 60° and that of *B* marked 0°, what is the temperature by *A* when it is 100° by *B*? (Pat. 1954)

[*Ans.* 110°].

involved are known as **temperature stresses.** In iron structures, such as bridges, buildings, etc. and such other structures, where large temperature stresses are likely to occur, provisions must be made such that the stresses produced due to the likely change of temperature do not damage or destroy them.

An idea about the magnitude of such forces may be obtained in the laboratory by a simple experiment such as that of the *breaking bar* (Fig. 11). In such an experiment a heavy iron bar *A* provided with a screw and nut at one end and a transverse hole H_1 near the other, rests in slots on two stout iron stands fixed to a base plate of iron. A cast iron pin, say $\frac{1}{4}$ inch in diameter, is passed into the hole across the bar. The bar is then heated by means of a burner and tightly clamped by means of the screw. When the bar cools down, the cast iron pin snaps due to the tremendous force of contraction of the bar. A similar force resulting from the expansion of the bar, when the latter is heated, may be demonstrated by using the cast iron pin in the hole H_2 and screwing the clamp tightly when the bar is cold.

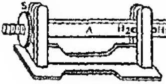


Fig. 11—The Breaking Bar

19. Linear Expansion :—As already stated, it is different for different solids but is in all cases very small. An iron rod one metre long would increase in length, when heated through $100°C$, by about 0.12 cm., and a brass rod, under similar conditions, by 0.18 cm.

Experiments show that the increase in length of bar (*i*) is proportional to the length of the bar, (*ii*) is proportional to the increase of temperature, and (*iii*) depends on the nature of the substance.

Coefficient of Linear Expansion of a Solid.—It is the ratio of the change in length to the original length of a solid at 0° per unit change of temperature.

Let l_0 be the initial length of a rod at 0° and let l_t be the length when heated through $t°$, then the expansion of the rod for a rise of temperature $t° = (l_t - l_0)$. The ratio of the change in length to the original length for $t°$ rise $= \frac{(l_t - l_0)}{l_0}$, and the ratio of the expansion to the original length at 0° for 1° rise $= \frac{(l_t - l_0)}{l_0 \times t}$.

Hence, the coefficient of linear expansion α (pronounced *"alpha"*) is given by, $\alpha = \frac{l_t - l_0}{l_0 t}$; or $l_t = l_0 \, (1 + \alpha t)$

Or, the mean coefficient of linear expansion for a given rise of temperature $= \dfrac{\textit{Increase in length}}{\textit{Original length at } 0° \times \textit{Rise in temperature}}$.

20. Does α depend on the Unit of length & Scale of Temperature ? $\alpha = \dfrac{\text{Change in length}}{\text{(Original length) (change of temp.)}}$. It is to be noted that, $\dfrac{\text{change in length}}{\text{original length}}$ is a ratio and has the same value whether length is measured in the C.G.S. or the F.P.S. unit of length. Therefore,

(a) *Coeff. of linear expansion has the same value both in cms. and inches, if the unit of temp. is the same.*

(b) Coeff. of linear exp. per degree Centigrade is 9/5 times larger than that per degree Fahrenheit, since $1°C. = 9/5°F.$ So the value of the coeff. of linear exp. depends on the scale of temperature used.

The coefficient of linear expansion of iron per °C., is 0·000012 means that 1 cm. of an iron rod raised in temperature by $1°C.$ expands by 0·000012 cm.; or,

1 yard of an iron rod raised in temperature by $1°C.$ expands of 0·000012 yard; or,

1 foot of an iron rod raised in temperature by $1°C.$ expands of 0·000012 foot etc.

21. Coefficient of Expansion at Different Temperatures :— We have seen that in defining the coefficient of linear expansion of a solid we should refer to its length at 0°, but practically it is not always convenient to measure the length at 0° and so *generally the length at the beginning of the experiment, i.e. at the temperature of the room, is taken, instead of its length at* 0°. In the case of solids, the error made by doing so is very small and can be neglected.

The length of a rod, which is initially not at 0° but at some other temperature, say $t_1°$, may be calculated thus—

Let l_0, l_1, and l_2 be the lengths at 0°, $t_1°$, and $t_2°$ respectively, where t_2 is greater than t_1;

then $l_1 = l_0(1+\alpha t_1)$; and $l_2 = l_0(1+\alpha t_2)$.

$$\therefore \quad \frac{l_2}{l_1} = \frac{(1+\alpha t_2)}{(1+\alpha t_1)} = (1+\alpha t_2)\,(1+\alpha t_1)^{-1} = (1+\alpha t_2)\,(1-\alpha t_1) = 1+\alpha(t_2-t_1),$$

neglecting terms containing higher powers of α.

$$\therefore \quad l_2 = l_1\{1+\alpha(t_2-t_1)\}\,;\ \text{or, } \alpha = \frac{l_2-l_1}{l_1(t_2-t_1)}.$$

Hence, the modified definition of the mean coefficient of linear expansion may be expressed as,

$$\textbf{Coeff. of linear expansion} = \frac{\textit{Increase in length}}{\textit{Original length} \times \textit{Rise in temperature}}.$$

22. Measurement of Linear Expansion :—

(i) Lavoisier and Laplace's Method.—To measure the coefficient of linear expansion of a metal by **Lavoisier and** Laplace's method,

23. Substances not affected by Changes of Temperature :— There are a few substances, like fused *quartz*, fused *silica*, and invar, which are very little affected by change of temperature. Vessels, made of fused *silica*, or fused *quartz*, expand or contract very little when their temperatures are changed. In the laboratory the crucibles can be made red-hot and then suddenly cooled without any risk of cracking.

Invar which is an alloy of nickel and steel, containing 36 per cent. of nickel, invented by the French metallurgist M Guillaume, shows very little change of length with change of temperature ; its coefficient of linear expansion which is 0·0000009 per °C is almost negligible. The name *invar* is derived from the word 'invariable'.

Note — It may be remembered here that glass and platinum expand or contract almost equally

24. Superficial and Cubical Expansions :— The *coefficient of superficial expansion* is the ratio of the change *in area* to the original area of a surface at 0° for unit change of temperature

If S_0 and S_t be the initial area at 0° and final area at $t°$ of a body, $t°$ the rise in temperature, then the mean coefficient of superficial expansion,

$$\beta \text{ (pronounced "beta")} = \frac{S_t - S_0}{S_0 t}, \quad \text{or,} \quad S_t = S_0(1 + \beta t) \tag{1}$$

As in Art 21 it can be shown that $\beta = \dfrac{S_2 - S_1}{S_1(t_2 - t_1)}$, where S_2 is the area at $t_2°$, and S_1 at $t_1°$

25. Relation between α and β :— Consider a square surface (Fig. 15) of a homogeneous isotropic solid, each side of which is l_0 at 0° and l_t at $t°$. The area of the surface at 0°, $S_0 = l_0^2$, and at $t°$, $S_t = l_t^2$

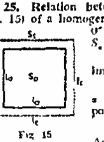


Fig 15

But $l_t = l_0(1 + \alpha t)$, where α is the coefficient of linear expansion

$S_t = \{l_0(1 + \alpha t)\}^2 = l_0^2(1 + 2\alpha t + \alpha^2 t^2)$ Since α is very small, terms containing α^2 and higher powers of α can be neglected.

$$\therefore \quad S_t = l_0^2(1 + 2\alpha t) \tag{2}$$

$$\text{Again from (1), } S_t = S_0(1 + \beta t) \tag{3}$$

$\therefore$ From (2) and (3), $1 + \beta t = 1 + 2\alpha t$ $(\because S_0 = l_0^2)$ or $\beta = 2\alpha$

That is, coefficient of area expansion = 2 × coefficient of linear expansion

Note.—The error due to neglecting $\alpha^2 t^2$ can be seen as follows —

Let us take the case of iron, where $\alpha = 0·000012$, and $\beta = 0·000024$

The part neglected is $= \alpha^2 = (0·000012)^2$

$\therefore$ Percentage error in the value for the coefficient of superficial expansion

per °C $= \dfrac{(0·000012)^2}{0·000024} \times 100 = 0·0006$ This is a negligible error

26. The Coefficient of Cubical Expansion of a Body :—It is the ratio of the change *in volume* to the original volume at 0° for unit rise of temperature.

Thus, if V_o, V_t be the volume at 0° and t° respectively and γ (pronounced *"gamma"*), the mean coefficient of cubical expansion then,

$$\gamma = \frac{V_t - V_o}{V_o \times t}; \quad \text{or,} \quad V_t = V_o(1+\gamma t).$$

As in Art. 21, it can be shown that $\gamma = \dfrac{V_2 - V_1}{V_1(t_2 - t_1)}$, for all practical purposes, where V_2 is vol. at t_2° and V_1, vol. at t_1°, expansion of all solids being small.

27. Relation between α and γ:—Consider a solid cube each side of which is l_0 at 0°, and l_t at t° (Fig. 16). Then, we have, as before, $V_0 = l_0^3$, and $V_t = l_t^3$, where $l_t = l_0(1+\alpha t)$.

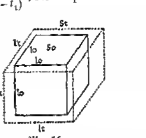


Fig. 16

$\therefore$ $V_t = \{l_0(1+\alpha t)\}^3 = l_0^3(1+3\alpha t+3\alpha^2 t^2 + \alpha^3 t^3) = l_0^3(1+3\alpha t)$ (neglecting the terms containing α^2 and α^3) $= V_0(1+3\alpha t)$.

But $V_t = V_0(1+\gamma t)$. Hence, we have $1+\gamma t = 1+3\alpha t$; whence $\gamma = 3\alpha$ approximately, *i.e. the coefficient of cubical expansion = 3 × coefficient of linear expansion.*

Examples.—(1) *A glass rod when measured with a zinc scale, both being at 20°C., appears to be one metre long. If the scale is correct at 0°C., what is the true length of the glass rod at 0°C.? The coefficient of linear expansion of glass is 8×10^{-6} and that of zinc 26×10^{-6}.* (Pat. 1920)

$\therefore$ At 0°C. each division of the zinc scale is 1 cm. and at 20°C. each division $=(1+0{\cdot}000026\times20)=1{\cdot}00052$ cms.

$\therefore$ 1 metre or 100 cms. of the zinc scale at 20°C. $=100\times1{\cdot}00052=100{\cdot}052$ true centimetres.

Hence, the correct length of the glass rod at 20°C. $=100{\cdot}052$ cms.

(The true length of the glass rod at 0°C.) $\times(1+0{\cdot}000008\times20)=100{\cdot}052$.

$\therefore$ The true length of the glass rod at 0°C. $= \dfrac{100{\cdot}052}{1+0{\cdot}000008\times20} = 100{\cdot}036$ cms.

(2) *A steel scale reads exact millimetres at 0°C. The length of a platinum wire measured by this scale is 621, when the temperature of both of them is 17°C. Find the exact length in millimetres of the platinum wire. What would be the exact length of the wire at 0°C.?*

(a) Coefficient of linear expansion of steel $=0{\cdot}000012$.

At 17°C. one scale division of the steel scale which is correct at 0°C. is not exactly 1 mm., but a little greater than 1 mm.

1 scale divisions at 17°C. would contract to 1 mm. at 0°C.

$\therefore$ 621 scale divisions at 17°C. would contract to 621 mm. at 0°C.

$\therefore$ The exact length in mm. of 621 scale divisions at 17°C.

$=621(1+0{\cdot}000012\times17)=621{\cdot}127$.

(b) Coefficient of linear expansion of platinum $=0{\cdot}000008$.

$\therefore$ Length of the platinum wire at 0°C. $\times\{1+0{\cdot}000008\times17\}=621{\cdot}042$ mm.

$\therefore$ Length of the platinum wire at 0°C. $=\dfrac{621{\cdot}127}{1{\cdot}000136}=621{\cdot}042$ mm.

So the clock will lose (86,400−86,375·8) = 24·2 seconds per day.

(7) A clock which keeps correct time at 25°C. has a pendulum rod made of brass. How many seconds will it gain per day when the temperature falls to the freezing point? (Coefficient of linear expansion of brass = 0·000019.)

(C. U. 1937)

Let l_0 = length at 0°C ; l_{25} = length at 25°C.

t_0 = period corresponding to the length l_0 ; t_{25} = period corresponding to the length l_{25}. Then, we have, $\frac{t_{25}}{t_0} = \sqrt{\frac{l_{25}}{l_0}} = \sqrt{\frac{l_0(1+0 \cdot 000019 \times 25)}{l_0}}$

$= (1+0 \cdot 000475)^{\frac{1}{2}} = (1+\frac{1}{2} \times 0 \cdot 000475)$, approx. $= 1 \cdot 0002375$.

But because the pendulum keeps correct time at 25°C., the value of t_{25} = 1 second,

$t_0 = \frac{1}{1 \cdot 0002375}$ sec.

There are 86,400 seconds in a day. So the pendulum makes 86,400 swings at 25°C. when it keeps correct time, *i.e.* when $t_{25} = 1$. ∴ When period $= \frac{1}{1 \cdot 0002375}$ sec., the number of swings $= 86,400 \div \frac{1}{1 \cdot 0002375} = 86,420 \cdot 52$.

∴ *The pendulum gains (86,420·52−86,400) = 20·52 seconds.*

28. Practical Examples of Expansion of Solids :—In many cases precautions have to be taken against expansions or contractions of metals arising from changes of temperature.

(a) Why in laying rails, a small gap is left in between ?

When railway lines are laid, a space of about a quarter of an inch is left between successive rails in order to allow for expansion when heated. But for these gaps the rail would buckle and cause train derailments.

[Similarly, allowances are to be made for expansion in mounting girders for iron bridges. The electric train lines, however, are welded together. These lines serve as electrical conductors and are continuous. As they are embedded in the ground the variation of temperature is small. The joints of gas and water pipes are made like those of a telescope in order to allow a certain amount of 'play' at the ends.]

(b) The length of metal chains used in surveying requires correction for variation of temperature. An ordinary clock fails to keep correct time owing to changes in the length of the pendulum consequent on the variations of temperature of the atmosphere. It goes slow in summer when the pendulum lengthens and fast in winter when it shortens. To keep correct time the length has to be periodically regulated.

(c) In rivetting boiler plates, red-hot rivets are used, which on cooling, contract and grip the plates tightly and make the joints steam-proof.

The same principle is adopted in fixing iron tyres on cart wheels. The tyre is at first made somewhat smaller in diameter, and then heated until it expands sufficiently to be easily put on the wooden wheel. On cooling, the tyre contracts and binds the wheel firmly.

Fire alarms are also based on this principle. One form of this consists of a compound bar of brass and iron. When hot it bends over and completes an electric bell circuit, and rings the bell.

(d) Why in drinking hot water, a thin-bottomed glass is taken ?

Thick-bottomed drinking glasses frequently crack if hot water is poured into them. Glass is a bad conductor of heat. So it fails to transmit heat quickly from the neighbouring parts to equalise the temperatures in different portions, due to which there is unequal expansion of the inner and outer layers and hence it cracks. For identical reasons the hot glass-chimney of a lantern cracks, if a drop of cold water falls on it.

For similar reasons, a tightened glass stopper sticking in a bottle may be made loose and taken out by pouring hot water round the neck of the bottle. By this the neck expands before the stopper does and so the stopper becomes loose.

(e) In sealing metallic wires into glass, why platinum is used ?

Sometimes it becomes necessary to seal metallic wires into glass. If a piece of copper is sealed through glass the joint usually fractures on cooling due to unequal contraction of copper and glass. But platinum and glass have almost the same coefficient of expansion and so platinum can be safely used for this purpose without fear of cracking.

Example.—*The distance between Allahabad and Delhi is 390 miles. Find the total space that must be left between the rails to allow for a change of temperature, from 36° F in winter to 117° F in summer.* (A., 1932)

(Coefficient of expansion of iron = 0.000012 per °C.)

$36°F = (36-32) \times \frac{5}{9} = \frac{20}{9}°C$, $117°F = (117-32) \times \frac{5}{9} = \frac{425}{9}°C$.

390 miles $= 390 \times 5280 \times 12 \times 2.54$ cms. The total space to be left = expansion of iron rails 390 miles long for $\left(\frac{425}{9} - \frac{20}{9}\right)°C$ change of temperature

$= (390 \times 5280 \times 12 \times 2.54) \times 0.000012 \times \left(\frac{425}{9} - \frac{20}{9}\right) = 0.21$ mile

[**Alternatively** —Coefficient of expansion of iron $= 0.000012 \times \frac{5}{9}$ per °F

$\therefore$ The total space to be left = total expansion $= 390 \times \left(0.000012 \times \frac{5}{9}\right) \times (117 - 36)$ mile $= 390 \times \left(0.000012 \times \frac{5}{9}\right) \times 81$ mile $= 390 \times 0.000012 \times 5 \times 9$ mile $= 0.21$ mile.]

29. The Compensated Pendulum :—In a pendulum clock the time-keeping quality depends upon its length, *i.e.* the distance from the point of suspension to the centre of gravity of the bob, because the period of oscillation of the pendulum changes with the change of length according to the relation, $t = 2\pi\sqrt{\frac{l}{g}}$.

It is evident from the above expansion that if l increases, t will become greater. In order that the rate of a clock may be uniform

the length of the pendulum must not vary with temperature. *If the length increases, the period of oscillation will increase and the clock will lose time; if the length decreases, the clock will gain time.* So generally in summer, the clock will lose, and in winter, the clock will gain time.

In order to nullify the effects of thermal expansion and contraction, compensated pendulums are constructed employing some special device whereby a constant length from the point of suspension to the centre of gravity of the bob is always maintained in spite of any variations of temperature. Such pendulums are called *compensated pendulums*.

Harrison's Grid-iron Pendulum.—This is the best form of a compensated pendulum. The principle of construction can be explained as follows :—

Let AB and CD be two parallel rods of different metals (Fig. 17), say, steel and brass, being connected by a cross-bar BC. If the point A is fixed, AB will expand downwards, while CD will expand upwards when the temperature rises. Now, if the lengths of the rods are such that the downward expansion of AB is equal to the upward expansion of CD for any rise of temperature $t°$, the distance AD will remain unaltered. So, if α, α' be the coefficients of expansion of AB and CD, and l, l' their lengths respectively, we have, $l\alpha t = l'\alpha' t$; or $l\alpha = l'\alpha'$.

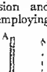


Fig. 17

$$\text{or, } \frac{l}{l'} = \frac{\alpha'}{\alpha},$$

i.e. the lengths of the rods should be inversely proportional to their coefficients of expansion. It is also evident that CD which is shorter must be constructed with *more expansive metal* than AB.

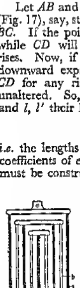


Fig. 18—Harrison's Grid-iron Pendulum.

The actual pendulum consists of a framework (Fig. 18) containing alternate rods of steel (shown in thick line), and brass (thin lines). The central steel rod C, passing through holes in the lower cross-bars of the frame, carries the bob B at its lower end. The arrangement is such that the steel rods expand *downwards*, while the brass rods expand *upwards*, and the centre of gravity of the bob is neither raised nor lowered, if the total upward expansion is equal to the total downward expansion. It should be *noticed* that in a Grid-iron pendulum all the bars, except the central one, are in pairs.

So, if there are 5 steel rods, each l_1 cm. long, and 4 brass rods, each l_2 cm. long, the effective length of the steel rods is $3l_1$, and that of the brass rods is $2l_2$, and taking the coefficient of linear expansion of brass to be 0·000019 and that of steel 0·000012,

we shall have, $\dfrac{3l_1}{2l_2} = \dfrac{0{\cdot}000019}{0{\cdot}000012} = \dfrac{19}{12}$.

In constructing good clocks and watches precautions have to be taken to counteract the effects of expansion, in order to get a correct rate of movement of the mechanism.

Note.—It is now-a-days usual to make the pendulum rod of a clock of **Invar**, an alloy of nickel and steel, the coefficient of expansion (0 0000009) of which is almost negligible.

Fig 18(a)—Mercury Pendulum

The Mercury Pendulum.—The bob of this pendulum is a framework provided with two glass cylinders containing mercury [Fig. 18(a)]. The principle of compensation is similar to that of the Grid-iron pendulum; the rod carrying the bob expands downwards, while the mercury expands upwards, and the quantity of mercury is so adjusted that the effective length of the pendulum, *i.e.* the distance between the *point of suspension* and the *centre of oscillation* remains unchanged when the temperature changes and so the rate of the clock remains unaffected.

30. Compensated Balance Wheel ;—Fig. 19 illustrates the balance wheel of a watch. The time of oscillation of the wheel depends upon the average diameter of the wheel -the smaller the diameter, the quicker the oscillation So an ordinary wheel oscillates quicker in winter than in summer owing to contraction of the wheel due to low temperature The compensation for temperature change is secured in the following way. The rim of the balance wheel is made of three segments, each segment being supported at one end *A* by a spoke joined to the centre of the wheel, the free end carrying an adjustable mass *W*. Each segment is made of two strips of dissimilar metals, the more expansible one being on the outside

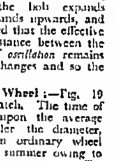


Fig 19—The Balance Wheel.

With the rise of temperature, as a spoke increases in length carrying the attached segment outward, the free end of the segment moves inwards, the outer strip of the segment expanding more than the inner one. The wheel is so constructed that the outward shift of the masses due to the increase in length of the spokes is equal to the inward shift of the masses due to the curling of the segments, when the temperature increases. The fine adjustment for this condition is made by means of the riders *W*. The average diameter of the wheel is thus kept constant, and the time period is unaffected by any increase of temperature. When temperature falls, the effects and adjustments are only opposite.

Example.—*There are 5 iron rods, each 1 metre long and 4 brass rods in a Grid-iron pendulum. What is the length of each brass rod? (The coefficient of expansion of iron is 0 000012, and that of brass 0 000019)*

The effective length of the iron rods $= 3 \times 1 = 3$ metres.

and if l metre be the length of each brass rod, its effective length $= 2l$.

$$\therefore \quad \frac{2l}{3} = \frac{0 \cdot 000012}{0 \cdot 000019} = \frac{12}{19}; \quad \text{or,} \quad l = \frac{3 \times 12}{2 \times 19} = 18/19 \text{ metre.}$$

Questions

1. A rod of iron and a zinc rod are each 2 metres long at 0°C., and both are heated equally. At 50°C., the zinc rod is found to be longer by 0·181 cm. Find the coefficient of linear expansion of iron when that of zinc is 0·0000298 per °C. (C. U. 1927)

[*Ans.* 0·0000117 per °C.]

2. The length of a copper rod at 50°C. is 200·166 cm. and at 200°C., it is 200·664 cm. Find the length at 0°C. and the coefficient of linear expansion of copper.

[*Ans.* 200 cm. ; 0·0000166 per °C.]

3. State the laws of the simple pendulum. The pendulum of a clock is made of wrought iron and the pendulum swings once per second. If the change of temperature is 25°C., find the alteration in the length of the pendulum. (Coefficient of expansion of wrought iron is 119×10^{-8}) (Pat. 1920 ; Dac. 1942)

[In this case, $t = 2$ secs. So $1 = \pi\sqrt{l/g} = \pi\sqrt{l/981}$; where $l = 99 \cdot 39$ cms.

If l be the initial length of the pendulum, the length after the temp. is increased by $25°C. = l(1 + 0 \cdot 0000119 \times 25)$. $\therefore$ The alteration in length

$= l(1 + 0 \cdot 0000119 \times 25) - l = l \times 0 \cdot 0000119 \times 25 = 99 \cdot 39 \times 0 \cdot 0000119 \times 25 = 0 \cdot 02956$ cm.]

4. Define the coefficient of cubical expansion of a solid. Does it differ when, (*a*) the lengths are measured in centimetres or feet, (*b*) the temperature is measured in Fahrenheit or Centigrade ? (C. U. 1931)

5. Define the co-eff. of linear expansion. Does it depend on (i) the unit of length, (*ii*) scale of temperature ? (Vis. U. 1952)

6. A brass scale reads correctly in mm. at 0°C. If it is used to measure a length at 33°C., the reading on the scale is 40·5 cms. What is the correct measurement of the length ?

[*Ans.* 40·525 cms.]

7. A zinc rod is measured by means of a brass scale (which is correct at 0°C.), and is found to be 1·0001 metres long at 10°C. What is the real length of the rod at 0°C. and at 10°C. ? (Pat. 1949 ; Nag. U. 1951 ; Utkal, 1951)

α (zinc) $= 0 \cdot 000029$ per °C. ; α (brass) $= 0 \cdot 000019$ per °C.

[*Ans.* (*i*) 1·000000018 metre ; (*ii*) 1·000290019 metres.]

8. A platinum wire and a strip of zinc are both measured at 0°C., and their lengths are 251 and 250 cms. respectively. At what temperature will their lengths be equal, and what will be their common length at this temperature. (The coefficient of linear expansion of zinc is 0·000026 and that of platinum = ·0000089.)

[*Ans.* 234°C. ; 251·523 cms.]

9. A brass scale measures true centimetres at 10°C. The length of a copper rod measured by the same scale is found to be 100 cms. at 20°C. Find the real length of the rod at 0°C. (The coefficient of linear expansion of copper is 0·000017 and that of brass 0·000019.)

[*Ans* 99·935 cms.]

10. How could you show that brass expands more than iron when rods of these two metals are heated through the same temperature?

11. Define co-eff. of linear expansion of a solid. How is it related to the co-eff. of cubical expansion?

If steel railroad rails are laid when the temp. is 35°*F*., how much gap must be left between each standard 30 ft. rail section and the next if the rails should just touch when the temp. rises to 120°*F*? α for steel $= 12 \times 10^{-6}$ per °C (C. U. 1956)

[*Ans* 0·27 inch.]

12. A railway line is laid at a temperature of 7°C. If each rail be 40 ft. long and firmly clamped at one end, calculate how much space should be left between the other end of the rail and the next one when the temperature rises to 31°C. (The coefficient of linear expansion for iron is 0·0000109 per °C.)

[*Ans* 0·141264 inch.]

13. What space should be allowed per mile of engine rail to avoid stress in the rails for the variations of temperature between 25°C and −5°C.

[α (iron) = 0·0000109 per °C.]

[*Ans* 1·7265 ft.]

14. Railway lines are laid with gaps to allow for expansion. If the gap between steel lines 66 ft. long is 0·5 in., at 10°C., at what temperature will the lines just touch? (α for steel $= 11 \times 10^{-6}$ per °C)

[*Ans* 67·3°C.] (C. U. 1953; G. U. 1951)

15. The diameter of an iron wheel is 3 ft. If its temperature is raised 400°C., by how many inches is the circumference of the wheel increased?

[*Ans.* 0·493 inch.]

16. A steel tyre 4 ft. in diameter is to be shrunk on to a cart wheel of which the average diameter is 1/8 inch greater than the inside diameter of the tyre. Calculate the necessary rise of temperature of the tyre in order that it may easily slip on the wheel (coff. of expansion of the tyre = 0·0000112) (Pat. 1952)

[*Ans.* 232·5°C. nearly.]

17. An iron ring of diameter 1 ft. is to be shrunk on a pulley of diameter 1·005 ft. If the temperature of the ring is 10°C., find the temperature to which it must be raised so that it will slip on the circumference of the pulley.

(α for iron = 0·000012 per °C.) (E. P. U., 1953)

[*Ans.* 426·6°C.]

18. The coefficient of linear expansion of brass is 0·000019; if the volume of a mass of brass is 1 cubic decimetre at 0°C., what will be its volume at 100°C.?

[*Ans.* 1·0057 cubic decimetre.]

19. A lump of iron has a volume of 10 cu. ft. at 100°C. Find its volume at 25°C. (α for iron = 0·000012 per °C.).

[*Ans.* 9·97 cu. ft.].

20. The volume of a lead bullet at $0°C.$ is 25 c.c. The volume increases at $98°C.$ by 0·021 *c.c.* Find the co-efficient of linear expansion of lead.

[*Ans.* $28{\cdot}6\times10^{-6}$ per $°C.$]

21. Two bars of iron and copper differ in length by 10 *cms.* as $0°C.$ What must be their lengths in order that they may differ by the same amount at all temperatures. (The coefficients of linear expansion of iron and copper are 0·000012 and 0.000018 respectively.)

[*Ans.* Iron, 30 cms. ; Copper, 20 cms.]

22. Describe any method for determining the coefficient of linear expansion of a solid. (E. P. U. 1951 ; C. U. 1942, '53 ; All. 1925 ; G. U. 1953 ; Nag. U. 1955 ; Pat. 1920 ; Dac. 1934)

23. One end of a steel rod is fixed and the other presses against an end of a lever 10·5 cms. from the fulcrum. The rod on being heated turns the lever through 2°. Find the increase in length of the rod. (Pat. 1926)

[*Ans.* 0·366 cm. nearly.]

24. One end of a steel rod of length 61 cms. is fixed and the other presses against an end of a lever 10·5 cms. from the fulcrum. The rod on being heated through $500°C.$ turns the lever through 2°. Find the co-eff. of linear expansion of the rod ($90° = \frac{\pi}{2}$ radians.)

[*Ans.* 12×10^{-6} *per* $°C.$]

25. Define the coefficients of linear and cubical expansion.

Show that the latter is three times the former.

(E. P. U. 1951 ; C. U. 1951 ; Vis. U. 1951 ; P. U. 1952 ; Pat. 1936, '49, '52, *cf.* C. U. 1953 ; G. U. 1955)

26. A brass ball whose vol. is 100 c.c. and whose mass is 820 gms. is heated from $0°C.$ to $500°C.$ If the coeff. of linear exp. of brass is 0·000018, find the difference in the density of brass at the two temps. (C. U. 1951)

[*Ans.* 0·216 gm./c.c.]

27. A grid-iron pendulum is made of 5 iron rods and 4 brass rods. Each of the brass rods is 50 *cms.* in length. Find the length of each iron rod. (C. U. 1948)

(α for iron $=12\times10^{-6}$ per $°C.$

α for brass $=18\times10^{-6}$ per $°C.$)

[*Ans.* 50 cms.]

28. Describe the effect of varying temperature on the rate of a clock or watch. Explain how chronometers are constructed so as to keep accurate time in spite of changes of temperature ? (C. U. 1925)

29. Why should the time of oscillation of a clock pendulum change with rise of temperature ? What arrangement is made to make the clock give correct time both in warm and cold weather ? Given that the coefficient of linear expansion of brass is 0·000019 and that of steel 0·000011 ; what must be the relative lengths of the bars of the metals used in the Grid-iron pendulum ?

[*Ans.* 11 : 19] (Pat. 1936 ; G. U. 1949)

30. Write explanatory notes on compensated clock-pendulums and watch balance wheels ; give diagrams. (Utkal, 1954)

CHAPTER III

EXPANSION OF LIQUIDS

31. Dilatation or Expansion of Liquids :—Liquids must always be kept in vessels, and since the liquids have no definite shape of their own, and always take the shape of the containing vessels, the thermal expansion or contraction in the case of liquids is always cubical and linear or area expansion has no meaning for them.

Real and Apparent Expansions.—In any experiment on the thermal expansion of liquids, the liquid has to be placed in a vessel of some sort, and the heat applied will also, in most cases, make the vessel expand. As a result, the liquid expansion which we observe, called the *apparent expansion of the liquid*, is less than its real expansion. The expansion of the vessel partly makes the expansion of the liquid and makes the latter appear less than what it really is

In Fig. 20 temperatures are represented along the abscissa and volumes along the ordinate. Consider a glass vessel containing a volume OB of a given liquid at $0°C$. Suppose its temperature is raised to $t°C$, represented by OA. Let the straight line BE represent the expansion curve of the vessel, assuming the expansion to take place uniformly as the temperature rises so that at $t°C$ the volume is AE. Again, let the straight line BF represent the expansion curve of the liquid so that its volume at $t°C$ is AF, assuming the liquid expansion to be greater and also uniform over the temperature range considered. Let the horizontal line through B meet the vertical line AF at D, so that DF gives the real expansion of the liquid for $t°C$. rise in temperature, while DE gives the expansion of the vessel for the same rise. Therefore, the observed expansion, i.e. the apparent expansion, will be given by EF only. Since $DF = DE + EF$, the real expansion DF of the liquid is equal to the apparent expansion EF of the liquid plus the expansion DE of the vessel.

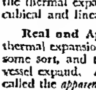


Fig. 20

Or, apparent expansion = real expansion − expansion of vessel.

N.B. If the liquid is more expansible than the material of the vessel, there will be, on the whole, an apparent expansion of the liquid. In the reverse case, the liquid will apparently contract. If the two expand equally, the volume of the liquid will appear to remain constant. Because the liquids, in general, expand more than the solids, there is ordinarily an apparent expansion, when a liquid is heated in a vessel.

Note also that a hollow vessel expands as if it were solid, having the same volume, because if the hollow of the vessel were also solid, after expansion it would fit in with the outer vessel.

Coefficient of Expansion (or Dilatation).—(*i*) *The coefficient of apparent expansion of a liquid is the ratio of the apparent increase in volume produced by a rise of temperature of* 1° *to the volume of the liquid at* 0° ; *or, symbolically,* $\gamma_a = \dfrac{\textit{apparent increase in volume}}{V_0 \times t}$.

(*ii*) *The coefficient of real (or absolute) expansion of a liquid is the ratio of the real increase in volume produced by a rise of temperature of* 1° *to the volume of the liquid at* 0° ; *or, symbolically,*

$$\gamma_r = \frac{\textit{real increase in volume}}{V_0 \times t}.$$

The above two coefficients are little affected if the increase in volume is referred to the original volume at any temperature instead of to the volume at 0°, for the expansions of all liquids are small. So, as pointed out in the cases of linear and superficial expansions, the mean coefficient of liquid expansion, real or apparent as the case may be, may also be expressed as,

$$\textit{Coeff. of expansion} = \frac{\textit{increase in volume}}{\textit{original volume} \times \textit{rise in temp.}}.$$

Relation between γ_r and γ_a.—If a volume V_0 of liquid be heated through $t°$, its real expansion $= V_0 \gamma_r t$, apparent expansion $= V_0 . \gamma_a t$, and the expansion of the vessel $= V_0 \gamma . t$, where $\gamma =$ coeff. of cubical expansion of the material of the vessel. So because *real expansion = apparent expansion + expansion of the vessel,* $V_0 . \gamma_r . t = V_0 . \gamma_a t + V_0 \gamma . t$;

$$\text{or, } \gamma_r = \gamma_a + \gamma.$$

32. Variation of Density with Temperature :—We know that density $= \dfrac{\text{mass}}{\text{volume}}$. Let m gm. of a substance (say, a liquid) occupy V c.c. at 0°*C*., then its density at this temperature, $d_0 = m/V_0$ gms./c.c. ....(1). The volume occupied by the same mass at $t°C$. will be V_t, when the density, $d_t = m/V_t$ gms./c.c. ....(2)

But $V_t = V_0(1 + \gamma_r t)$..(3), where γ_r is the coefficient of real cubical expansion of the liquid.

From (1) and (3), $\dfrac{d_0}{d_t} = \dfrac{V_t}{V_0} = \dfrac{V_0(1+\gamma_r t)}{V_0} = (1 + \gamma_r t)$;

$$\text{or, } d_0 = d_t(1 + \gamma_r t) \qquad (4)$$

$$\text{or, } d_t = d_0(1 + \gamma_r t)^{-1} \text{; or, } d_t = d_0(1 - \gamma_r t), \text{ approximately} \qquad (5)$$

$$\therefore \quad \gamma_r = \frac{d_0 - d_t}{d_0 t}.$$

[**Note.** Compare equations (3) and (5).]

Examples.—*(1) The density of mercury is 13.59 at 0°C. What will be the volume of 30 kilograms of mercury at 100°C., coefficient of expansion of mercury being 1/5550.*

Let d_{100} = density of mercury at 100°C., d_0 = density of mercury at 0°C.

We have, $$d_0 = d_{100}(1+\gamma_r t)$$

or $$d_{100} = \frac{d_0}{1+\gamma_r t} = \frac{13.59}{1+(\frac{1}{5550}\times 100)} = \frac{13.59\times 5550}{5650},$$

So, the volume of mercury $= \dfrac{30\times 1000}{d_{100}} = \dfrac{30\times 1000}{\frac{13.59\times 5550}{5650}} = 2247.27$ c.c.

(2) A glass hydrometer reads specific gravity 0.920 in a liquid at 45°C. What would be reading at 15°C.? Coefficient of cubical expansion of the liquid = 0.000525 and that of glass = 0.000024

Let V_{45}, V [illegible]

Again the mass of V_{15} c.c. of liquid at 15°C. $= V_{15}\times d_{15}$.

$\therefore\quad V_{15}\times d_{15} = (V_{45}\times 0.99928)\times d_{45}\times 1.01575$

$\therefore\quad d_{15} = \dfrac{V_{45}\times 0.99928\times d_{45}\times 1.01575}{V_{15}} = 0.9345 \qquad (\because\ d_{45} = 0.920)$

(3) A cylinder of iron 20 inches long floats vertically in mercury, both being at the temperature 0°C. If the common temperature rises to 100°C., how much will the cylinder sink? (Sp. gr. of iron at 0°C = 7.6, sp. gr. of mercury at 0°C = 13.6, cubical expansion of mercury between 0°C and 100°C = 0.018153, linear expansion of iron between 0°C and 100°C = 0.001182) (Pat. 1918)

Let l_0 and l_{100} be the lengths of the cylinder immersed in mercury and A_0, A_{100} be the areas of the cylinder at 0°C and 100°C respectively.

The density of iron at 0°C = (7.6 × 62.5) lbs. per cu. ft. = d_0 say, and that of mercury at 0°C = (13.6 × 62.5) lbs. per cu. ft. = ρ_0, say, and let their corresponding densities at 100°C be d_{100} and ρ_{100}, then from eq. 5, Art. 32,

$d_{100} = d_0(1-3\times 0.001182)$ and $\rho_{100} = \rho_0(1-0.01815)$.

By the law of floatation we have $(20\times A_0)\times d_0 = (l_0\times A_0)\times\rho_0$ (1)

and $\{20(1+0.001182)A_{100}\}\times d_{100} = (l_{100}\times A_{100})\times\rho_{100}$ (2)

From (1) we have, $l_0 = \dfrac{20\times d_0}{\rho_0} = \dfrac{20\times(7.6\times 62.5)}{13.6\times 62.5} = 11.176''$

and from (2), $20(1+0.001182)\times d_0(1-3\times 0.001182) = l_{100}\times\rho_0(1-0.018153)$,

or, $20(1+0.001182)\times(7.6\times 62.5)(1-0.003546)$

$= l_{100}\times(13.6\times 62.5)(1-0.018153)$, whence $l_{100} = 11.355''$

So the extra length of the cylinder which will sink in mercury when the temperature rises to 100°C = (11.355 − 11.176) = 0.179".

33. Determination of the Coefficient of Apparent Expansion of a Liquid :—

(i) The Weight-thermometer Method.—

The following method in which a weight-thermometer is used is a convenient laboratory method for determining the coefficient of apparent expansion of a liquid. The common form of such a

thermometer consists of a glass-bulb (Fig. 21) having a bent capillary stem drawn out of a narrow nozzle.

A glass tube of suitable size and material is taken. It is, at first, carefully cleaned and then dried. By blowing, a bulb having a capillary stem of the type shown in Fig. 21 is then made. The wt.-thermometer so constructed is then carefully weighed empty (w gms.). It is then completely filled with the given liquid by dipping the nozzle inside the liquid and alternately heating and cooling the bulb. With the nozzle still inside the liquid the rest of the bulb is kept immersed for sometime in water in a tub at the room temperature. After the contents have attained the steady temperature (say t_1°C.) of the water which is recorded by an ordinary mercury thermometer inserted in the water, the bulb is taken out, wiped dry, and weighed again (w_1 gms.). The bulb is again put under water in the tub with the nozzle now projecting outside. The water is kept well-stirred and gradually heated until a suitable steady temperature, (say t_2°C.) is attained as indicated by the inserted thermometer. The contents of the weight-thermometer now have attained the raised temperature of the bath. As the temperature is raised, the liquid inside the weight-thermometer expands and some of it is continuously forced out until it reaches a steady temperature. The thermometer is now removed from the bath, allowed to cool and finally brought to the room temperature by dipping it inside water as was done previously. It is then removed from the bath, wipped dry, and weighed again (w_2 gms.). The residual liquid in the bulb, however, contracts to a smaller volume due to cooling.

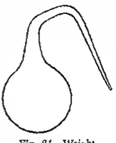


Fig. 21—Weight-Thermometer.

Calculation—

Mass of the liquid filling the thermometer at t_1°C.

$$=w_1-w=m_1 \text{ gms. (say).}$$

Again, mass of the liquid filling the thermometer at t_2°C.

$$=w_2-w=m_2 \text{ gms. (say).}$$

Neglecting the expansion of the *weight-thermometer* itself, it is evident that the volume occupied by m_1 gms. of the liquid at t_1°C. is the same as that occupied by m_2 gms. of the liquid at t_2°C. Now the volume of m_1 gms. of the liquid at t_1°C. is equal to m_1/ρ c.c. where ρ=density of the liquid at t_1°C. in gms./c.c. So this is also the volume occupied by m_2 gms. of the liquid at t_2°C. But the volume of m_2 gms. of the liquid at t_1°C. is m_2/ρ c.c. So we find that a mass of m_2 gms. of the liquid, when heated from t_1°C. to t_2°C., *apparently*

expands through $(m_1/\rho - m_2/\rho)$. In other words, the coeff. of apparent expansion of the liquid, $\gamma_a = \dfrac{m_1/\rho - m_2/\rho}{\dfrac{m_2}{\rho} \times (t_2 - t_1)} = \dfrac{m_1 - m_2}{m_2(t_2 - t_1)}$

$$= \frac{\text{mass of liquid expelled on heating}}{\text{mass remaining} \times \text{rise of temp.}}.$$

Since the coefficient is obtained in the expt. from different *weights*, the method is known as the weight-thermometer method. The method is not suitable for volatile liquids.

Absolute Expansion.—The coefficient of *absolute expansion* of the liquid can also be calculated in the following way from the above data :—

Let $t_2 - t_1 = t$. Then $V_2 = V_1(1 + \gamma . t)$, where γ is the coefficient of cubical expansion of glass, and $d_1 = d_2(1 + \gamma_r . t)$ (*vide* Art. 32), where γ_r is the coefficient of absolute expansion of the liquid.

$\therefore$ From Eq. 1, $\dfrac{m_1}{m_2} = \dfrac{V_1 d_1}{V_2 d_2} = \dfrac{V_1 d_2(1 + \gamma_r t)}{V_1 d_2(1 + \gamma t)} = \dfrac{1 + \gamma_r t}{1 + \gamma t}$;

or, $\quad m_2 + m_2 \gamma_r t = m_1 + m_1 \gamma t$,

or, $\quad m_2 \gamma_r = \dfrac{m_1 - m_2}{t} + m_1 \gamma$, or, $\gamma_r = \dfrac{m_1 - m_2}{m_2 t} + \dfrac{m_1}{m_2} \gamma$.

If only the apparent expansion is required, γ should be neglected and the coefficient of apparent expansion becomes,

$$\gamma_a = \frac{m_1 - m_2}{m_2 \times t}.$$

Notes.—(1) Because in the above experiment *weights* (and not volumes) are taken for the determination of the coefficient of expansion, it should not be thought that the coefficient of expansion is equal to the increase per unit mass of the liquid for 1° rise of temperature.

(2) The above instrument is called a *Weight-thermometer*, because by knowing the coefficient of apparent expansion of a liquid and by finding the weight of liquid expelled at the higher temperature we can determine an *unknown temperature*.

Examples.—*(1) The mass of mercury overflowed from a weight-thermometer is 5.4 gms. when heated from ice to steam point. The thermometer is placed in an oil bath at 20°C. On heating the bath, 8.64 gms. of mercury flow out. Determine the temperature of the bath.*

The mass of mercury overflowed for $(100 - 0)$°C $= 5.4$ gms.

$\therefore$ The mass overflowed for 1°C $= 5.4 \div 100 = 0.054$ gm.

So for the overflow of 8.64 gms. of mercury, the rise of temperature of oil bath $= \dfrac{8.64}{0.054} = 160$°C.

Hence the actual temperature of the bath $= 160 + 20 = 180$°C.

(2) A weight-thermometer weighs 40 gms. when empty, and 490 gms. when filled with mercury at 0°C. On heating it to 100°C., 6·85 gms. of mercury escape. Calculate the coefficient of linear expansion of glass, the coefficient of real expansion of mercury being 0·000182.

Mass of mercury in the thermometer at 0°C.$=490-40=450$ gms.
The mass of mercury left in the thermometer at 100°C.

$$=450-6{\cdot}85=443{\cdot}15 \text{ gms.}$$

∴ The coefficient of apparent expansion of mercury

$$=\frac{6{\cdot}85}{443{\cdot}15(100-0)}=0{\cdot}000155.$$

Hence, the coefficient of cubical expansion of glass = coefficient of real expansion of mercury − coefficient of apparent expansion of mercury.

$$=0{\cdot}000182-0{\cdot}000155=0{\cdot}000027.$$

∴ The coefficient of linear expansion of glass $=0{\cdot}000027\div3=0{\cdot}000009$.

(3) If the coefficient of apparent expansion of mercury in glass be $\frac{1}{6500}$, what mass of mercury will overflow from a weight thermometer which contains 400 gms. of mercury at 0°C., when the temperature is raised to 90°C ? (C. U. 1930)

We have, $\gamma_a=\dfrac{m_0-m_t}{m_t(t-t_0)}$; or, $\frac{1}{6500}=\dfrac{400-m_t}{m_t(90-0)}$;

whence $m_t=\dfrac{2600000}{6590}=394{\cdot}53$.

∴ The mass of mercury expelled $=m_0-m_t=400-394{\cdot}53=5{\cdot}47$ gms.

(ii) Dilatometer or Volume Thermometer Method.—A dilatometer (Fig. 22) consists of a glass bulb with a graduated stem of small bore leading from it. It is used as follows: Weigh the dilatometer empty. Let this be w_1 gms. Introduce mercury in the tube to fill the bulb and a part of the stem up to the zero mark A. Weigh again, and let this weight be w_2. Put in more mercury to fill, say, up to B, the length AB being l cms. Weigh again. Let this third weight be w_3 gms. Then the weight of mercury occupying l cms. of the stem $=(w_3-w_2)$ gms. =say, m_1 gms., and the weight of mercury in the bulb and stem up to the zero mark $=(w_2-w_1)$ gms. $=m_2$ gms., say.

Fig. 22—The Dilatometer.

∴ m_2 gms. of mercury would occupy $\left(\dfrac{m_2}{m_1}\times l\right)$ cms. of the stem, and the volume of the bulb up to the zero mark of the stem $=\dfrac{m_2}{m_1}\times l\times a$ (if a sq. cm. = area of cross-section of the bore of the stem).

The bulb and part of the stem of the dilatometer is then put in a water bath, the temperature t_1 of which is measured, and the length l_1 of the mercury height, at temperature t_1, is read accurately. Increase the temperature of the water bath up to t_2°C., and read the level of mercury which is, say, at C now, the length AC being l_2 cms.

Then the volume expansion of (l_2-l_1) cms. of mercury column for (t_2-t_1)°C.

$=(l_2-l_1)\times a$ c.c., and the original volume $=\left\{\left(\frac{m_2}{m_1}\times l\times a\right)+l_1 a\right\}$ c.c.

∴ Mean coefficient of expansion between t_1°C. and t_2°C.

$$=\frac{\textit{increase in volume}}{\textit{Original volume} \times \textit{rise in temperature}}$$

$$=\frac{(l_2-l_1)\times a}{\left(\frac{m_2}{m_1}\times l\times a+l_1 a\right)\times(t_2-t_1)}=\frac{(l_2-l_1)}{\left(\frac{m_2}{m_1}\times l+l_1\right)(t_2-t_1)}.$$

Note.—The calculation will be easier if the density of the liquid is supplied (*vide* example 2 below).

Examples.—(1) *A long glass tube of uniform capillary bore contains a thread of mercury which at 0°C. is one metre long. At 100°C. it is 16.5 mm. longer. If the average coefficient of volume expansion of mercury is 0.000182, what is the coefficient of expansion of glass?* (C. U. 1910)

Coefficient of expansion of mercury $=\frac{\text{Increase in volume}}{\text{Original volume} \times \text{rise in temp}}$

$=\frac{16.5 \text{ cm.} \times \text{area of the cross-section}}{100 \text{ cm.} \times \text{area of cross-section} \times 100}=0.000165$

Coefficient of cubical expansion of glass = coefficient of absolute expansion of mercury − coefficient of apparent expansion of mercury (*vide* Art. 31)

$=0.000182-0.000165=0.000017$

Coefficient of linear expansion of glass $=\frac{0.000017}{3}=0.0000056.$

(2) *A glass bulb with an accurately graduated stem of uniform bore weighs 30 gms. when empty, 356 gms. when filled with mercury up to the 16th division, and 356.15 gms. when filled up to 110th division. Find the mean coefficient of apparent expansion of the liquid which fills the bulb and the stem up to the zero of the graduations at 0°C., and up to the 80th division at 10°C. (The density of mercury is 13.6.)*

The capacity of the bulb and 16 divisions of the stem $=\frac{356-30}{13.6}=\frac{326}{13.6}$ c.c.

and the internal volume of each division $=\frac{356.15-356}{13.6\times(110-16)}=\frac{0.15}{13.6\times94}$ c.c.

Hence the capacity of the bulb with the part of the stem below the zero mark $=\frac{326}{13.6}-\frac{0.15\times16}{13.6\times94}=\frac{15320.8}{13.6\times47}$ c.c. Thus the initial volume of the liquid $=\frac{15320.8}{13.6\times47}$ c.c., and the total apparent increase of volume for 10°C. $=\frac{80\times0.15}{13.6\times94}$ c.c.

Hence the coefficient of apparent expansion of the liquid

$$=\left(\frac{80\times0.15}{13.6\times94}\Big/\frac{15320.8}{13.6\times47}\right)\div10=0.00003915$$

(3) *The coefficient of absolute expansion of mercury is 0·00018; the coefficient of linear expansion of glass is 0·000008. Mercury is placed in a graduated tube and occupies 100 divisions of the tube. Through how many degrees the temperature of the tube must be raised to cause the mercury to occupy 101 divisions?*

Let t be the number of degrees; then the length of the mercury column for $t°$ rise of temperature $=100(1+0·00018t)$.

This becomes equal to 101 divisions of the tube after expansion. $\therefore$ 1 division of the tube $=\dfrac{100(1+0·00018t)}{101}$. But 1 division of the tube becomes $(1+0·000008t)$ divisions at $t°$.

$$\therefore \quad \frac{100(1+0·00018t)}{101}=1+0·000008t;$$

$$\text{whence } t=\frac{1}{0·018-0·000808}=\frac{1}{0·017192}=58·2°C.$$

34. Exposed Stem Correction for a Thermometer:—The correction for the exposed portion of the stem of a thermometer will be best understood by the following example:—

A mercurial thermometer is placed with its bulb and lower part of the stem in a liquid and indicates a temperature t°C. The upper portion of the stem containing 'n' division of mercury column is in air at θ°C. Find the true temperature of the liquid.

The true temperature $T°$ of the liquid is that which the thermometer would indicate, if completely immersed in the liquid. Then n divisions of the mercury column, now at $\theta°C$, would be at $T°C$., and at that temperature would occupy $n\{1+\gamma(T-\theta)\}$ divisions, where γ is the coefficient of expansion of mercury in glass.

The corrected length of the exposed portion would be greater than the actual length by $\quad n\{1+\gamma(T-\theta)\}-n=n(T-\theta)\gamma$.

Hence, the true temperature of the liquid, $T=t+n(T-\theta)\gamma$.

Example.—*The bulb of a mercurial thermometer and the stem up to the zero mark are immersed in hot water at 100°C., while the remainder of the stem is in the air at 20°C. What will be the reading of the thermometer?*

Using the formula given already, we have $T=100$, $n=t$, $\theta=20$, $\gamma=·000155$

$\therefore \quad 100=t+t\times(100-20)\times0·000155=1·0124t$ or, $t=98·77°C.$

35. Coefficient of Absolute Expansion—(a) Dulong and Petit's Method:—In 1816 Dulong and Petit developed a method of determining the coefficient of real expansion of a liquid, *i.e.* in which the expansion of the containing vessel has no effect on the observations from which the expansion is to be calculated.

The method consists in balancing the *pressure* of one column of mercury at a certain temperature against another column of the said liquid at a different temperature. Since pressure is measured by the force per unit area, it is independent of the cross-section of the liquid column, *i.e.* the method is independent of the expansion of the tubes

containing the liquid. So the method gives the coefficient of real or absolute expansion of the liquid. The liquid taken by them was mercury.

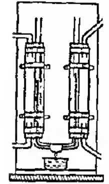


Fig 23—Dulong and Petit's Apparatus

The apparatus consists of a U-tube filled with the liquid (Fig. 23). One limb of the U-tube is kept cool by packing one of the jackets with melting ice, while the temperature of the other is increased and maintained by passing steam though the jacket. A piece of blotting paper constantly soaked with water is placed on the horizontal portion in order to prevent a flow of the liquid from one limb to another. Thus two different temperatures are maintained in the liquid in the two limbs.

Let h_t and h_0 be the heights of the two liquid columns at t°C. and 0°C. respectively.

Let d_0 be the density of the cold liquid of the column, and d_t be that of the hot column. Then the pressure exerted on the horizontal portion of the tube by the cold column $=h_0d_0g+P$, and that by the hot column $=h_td_tg+P$, where P = atmospheric pressure But, since the two liquid columns are in equilibrium,

we have $h_0d_0g=h_td_tg$, or $\frac{d_0}{d_t}=\frac{h_t}{h_0}$ But $d_0=d_t\ (1+\gamma_r t)$, where γ_r is the coefficient of real expansion of the liquid.

$$\therefore \quad 1+\gamma_r t=\frac{h_t}{h_0}\ ;\ \text{or,}\ \gamma_r=\frac{h_t-h_0}{h_0 t} \qquad (1)$$

Laboratory Experiment.—The above experiment can be done in a laboratory by circulating water at the room temperature through the left-hand jacket, instead of melting ice. The formula (1) should then be slightly changed as follows

Let h_1 and h_2 be the heights of the cold and hot columns, and t_1, t_2 their temperatures. If d_1, d_2 be the densities of the cold and hot columns respectively, we have, $h_1d_1g=h_2d_2g$,

$$\text{or,}\quad h_1\frac{d_0}{1+\gamma_r t_1}=h_2\frac{d_0}{1+\gamma_r t_2}.\quad [\because\ d_0=d_1(1+\gamma_r t_1)]\ ;$$

$$\text{or,}\quad h_2(1+\gamma_r t_1)=h_1(1+\gamma_r t_2)\ ; \qquad \text{or,}\quad h_2+h_2\gamma_r t_1=h_1+h_1\gamma_r t_2;$$

$$\text{or,}\quad \gamma_r=\frac{h_2-h_1}{h_1t_2-h_2t_1}.$$

Sources of error.—

(1) For liquids having small coefficients of expansion, such as mercury for which the value is 0·00018, the difference in height between the two columns will be small, and with the apparatus described above it will not be an easy task to measure it very accurately.

A cathetometer telescope may, however, be used, instead of a metre scale, for greater accuracy in this respect.

(2) Some parts of both the columns are always outside the jacket; temperatures of these exposed parts are not known definitely; nor are they taken into consideration in the calculation.

(3) The blotting paper moistened with water placed on the horizontal part of the tube is used to prevent convection currents but it fails to do so completely, and so the hot and the cold liquids will mix up to some extent.

(4) Temperatures of the mercury heads in the two limbs were different. This introduces a difference in level due to inequality of surface tension.

[**Note.**—The above method is independent of the cross-sections of the two columns and so the diameters of the two limbs may be different without in any way interfering with the result.]

In the modified actual arrangement used by Dulong and Petit, the upper ends of the two vertical limbs were again bent at right angles towards each other and were placed side by side for convenience of reading. A plane mirror placed behind these two tubes was used to avoid parallax. In Dulong and Petit's apparatus, the top of the hot column of mercury had to project above the bath in order to be visible and hence did not attain the correct temperature. The two mercury surfaces also had different curvatures, because the surface tension of mercury is much less when hot than when cold, and this difference in curvature was difficult to allow for. Regnault subsequently removed the above-mentioned defects in a highly improved apparatus.]

(b) Indirect Method.—Knowing the coefficient of absolute expansion of mercury by Dulong and Petit's method and the coefficient of apparent expansion of a liquid and also that of mercury by the weight-thermometer or any other method, the coefficient of cubical expansion of the material of the weight-thermometer can be obtained and also the coefficient of absolute expansion of the liquid as shown below.

Suppose the *co-effs.* of apparent expansion of mercury and glycerine are determined by the same weight-thermometer.

Let γ_r^m = Coeff. of real expansion of mercury.
γ_a^m = ,, ,, apparent ,, ,, ,,
γ_r^g = ,, ,, real ,, ,, glycerine.
γ_a^g = ,, ,, apparent ,, ,, ,,
γ = ,, ,, cubical ,, ,, container.

We know, $\gamma_r^m = \gamma_a^m + \gamma$ .. .. (1), and
$\gamma_r^g = \gamma_a^g + \gamma$ .. .. (2).

From (1) and (2), $\gamma_r^g = \gamma_a^g + (\gamma_r^m - \gamma_a^m)$. Knowing γ_r^m and experimentally determining γ_a^m and γ_a^g by the same weight-thermometer, γ_r^g can be indirectly thus determined. γ for the container can be calculated either from (1) or (2).

(c) Regnault's Method.—Regnault's apparatus consists of two vertical iron tubes AB and CD (Fig. 24) joined at the top by a horizontal cross-tube AD which has a top hole L. Suppose one of the tubes say, AB, is placed in a water bath at the room temperature t_1 and the other tube CD is immersed in a hot bath whose temperature can be maintained constant at any desired temperature t_2. For uniformity of temperature, stirring arrangements are provided in both the baths. The horizontal cross-tube BC which connects AB and CD at the bottom is interrupted in the middle at E and G where two vertical glass tubes EF and GJ are joined and connected with each other. The inter-connected tubes EF and GJ are connected through a side tube P to an air-reservoir whose pressure can be modified by an air pump. The tubes EF and GJ are placed inside a common water bath at the room temperature. Mercury is poured into AB and CD and cold air is forced in through the pipe P from an air-reservoir by means of a pump whereby the level of mercury in AB and CD becomes equal, any excess mercury flowing out through the opening L. The pressure on the top of mercury in the columns EF and GJ is the same and equal to the pressure of air in the reservoir. Regnault measured the temperature of the hot column CD by immersing into the hot bath the bulb of an air thermometer and that of the cold column AB by means of a mercury thermometer.

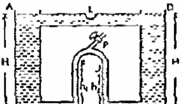


Fig. 24—Regnault's Apparatus

Theory.—Suppose ρ_1 and ρ_2 are the densities of mercury at temperatures t_1 and t_2 respectively. Now the pressure on the top of the mercury column $FE = (H-h_1)\rho_1 g$ and that on the top of the column $GJ = H\rho_2 g - h_2\rho_1 g$, and these pressures are equal.

$$\therefore \quad (H-h_1)\rho_1 g = H\rho_2 g - h_2\rho_1 g\,,$$

$$\text{or,} \quad (H-h_1+h_2)\rho_1 = H\rho_2\,;$$

$$\text{or,} \quad \frac{\rho_2}{\rho_1} = \frac{H-h_1+h_2}{H} = \frac{H-(h_1-h_2)}{H} \qquad (1)$$

But $\rho_2 = \rho_1\{1-\gamma_r(t_2-t_1)\}$, where γ_r = Coefficient of absolute expansion of mercury. $\quad \rho_2/\rho_1 = 1-\gamma_r(t_2-t_1)$ (2)

$$\therefore \quad 1-\gamma_r(t_2-t_1) = \frac{H-(h_1-h_2)}{H}, \text{ from (1) and (2)}$$

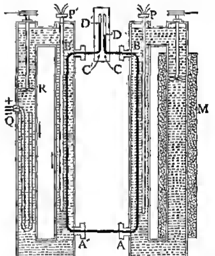
$$\text{That is, } \gamma_r = \frac{h_1-h_2}{H(t_2-t_1)}.$$

Advantages over Dulong and Petit's Method.—In Dulong and Petit's apparatus the temperatures at the different parts of the hot or cold column were uncertain, for the baths in which they were

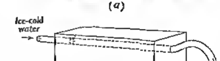
placed could not be stirred. Regnault placed them inbaths which could be constantly stirred, and moreover the hot column could be given any desired constant temperature. In Dulong and Petit's method, the heads of the mercury in the two comparing columns being at different temperatures, the effects of surface tension on them were unequal resulting in an error introduced in the observed difference in heights. To remedy this defect, Regnault brought the heads of the two columns close together and placed them at a constant temperature in the same bath.

Moreover, Regnault's determination of the temperature of the hot column was more accurate, as it was done with an air thermometer.

35(a). Callendar and Moss's method of determining the coefficient of real expansion of mercury :—A simple description of the method is as follows : AB and $A'B'$ are two vertical tubes each about two metres long, bent twice at right angles having portions BC and $B'C'$ horizontal, and portion AA' narrowed to smaller diameter in order to reduce circulation of mercury from one vertical tube to the other [Fig. 24$A(a)$]. The tube system contained mercury. AB is surrounded by a water jacket which is cooled to 0°C. by means of ice packed in a jacket M around it, the water in the jacket being kept in forced circulation with the help of a mechanically driven paddle. $A'B'$ is surrounded by an oil bath, the oil being electrically heated by means of an wire loop Q immersed in it and the oil also kept in forced circulation caused by a second paddle R. P and P' are pt.-resistance thermometers, the bulb of each of which extends through almost the whole length of the bath. They indicate respectively the mean temperatures of the cold and



(a)



(b)

Fig. 24A

the hot bath accurately. The portions CD and $C'D'$ of the two tubes were also at 0°C. (arrangement not shown in the figure). The portion BC and $B'C'$ of the tubes are made strictly horizontal where they project out from the baths. In the actual apparatus of Callendar and Moss there were six pairs of hot and cold columns placed in series, the successive columns being alternately hot and cold. In order that heat may not pass along the horizontal tubes each of the two vertical array of tubes is silver-soldered to a massive brass block kept at 0 C. by means of ice-cold water flowing continuously through a tube passed through it [Fig. 24A(b)]. The assumption is that the temperature is 0°C. at all points except in the hot bath

Callendar and Moss measured the longer heights with a carefully calibrated steel tape and difference $D'D$ between the mercury tops in the hot and cold columns of mercury with a cathetometer

Theory.—Let H_t and H_0 be the lengths of $A'B'$ and AB at temperatures t°C. and 0°C. Suppose h_0 and h'_0 to be the lengths CD and $C'D'$ when both of these columns are at 0°C Then the pressures at A and A' will be $P+gh_0\rho_0+gH_0\rho_0$ and $P+gh'_0\rho_0+gH_t\rho_t$, if ρ_0 and ρ_t are the densities of mercury at 0°C. and t°C. and P=atmospheric pressure. They being equal, we have,

$\rho_0(H_0+h_0)=H_t\,\rho_t+\rho_0 h'_0=H_t\times\dfrac{\rho_0}{1+\gamma_r t}+\rho_0 h'_0$, where γ_r= co-eff. of absolute or real expansion of mercury

$\therefore\ \ \gamma_r=\dfrac{H_t-H_0+h'_0-h_0}{[H_0-(h_0-h'_0)]\times t}$. The quantity (h'_0-h_0) is the difference in levels DD'

The mean value of γ_r between 0°C and 100°C, as determined by Callendar was $1\ 82\times10^{-4}$ per °C. and it increased as the temperature increased.

36. Apparent Loss in Weight of a Solid dipped in a Liquid at Different Temperatures :—A solid of volume V c.c and known weight is weighed in the liquid at 0°C Let the apparent loss in weight be W_0. It is then weighed again in the liquid raised to temperature t°C, and let the apparent loss in weight be W_t.

Let d_0, d_t=densities of the liquid at 0°C. and t°C respectively ; γ=mean coefficient of cubical expansion of the solid between 0°C. and t°C. ; g=acceleration due to gravity.

We have, according to Archimedes' principle, weight of the displaced liquid at 0°C.$=W_0=V\times d_0\times g$ ... ... (1) where V is the volume of the solid at 0°C. and so the volume of the liquid displaced at 0°C.

When the temperature increases to $t°C.$, the volume of the solid becomes $=V(1+\gamma t)$, which is also the volume of the liquid displaced at $t°C.$ $\therefore$ The weight of the displaced liquid at $t°C.$,

$$W_t=\{V(1+\gamma t)\}d_t\times g \qquad \ldots \qquad \ldots \qquad (2)$$

From (1) and (2), $\frac{W_0}{W_t}=\frac{Vd_0g}{V(1+\gamma t)d_tg}=\frac{d_0}{d_t(1+\gamma t)}=\frac{d_0}{d_0(1-\delta t)\ (1+\gamma t)}$

$$=\frac{1}{1-\delta t+\gamma t-\delta\gamma t^2} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (3)$$

where δ = mean coeff. of expansion of the liquid between $0°C.$ and $t°C.$

So the loss in weight W_t, at a higher temperature, is less than W_0, the loss at the lower temperature, since $\delta>\gamma$. Therefore, *the weight of the solid in the liquid will increase with rise of temperature of the liquid.*

Coefficient of Expansion : (*Hydrostatic Method*).—

Knowing the value of γ, we can also apply this method in determining the coefficient of expansion of the liquid.

We have, from (3), $\frac{W_0}{W_t}=\frac{1+\delta t}{1+\gamma t}$;

whence $\delta=\frac{W_0-W_t}{W_t.t}+\frac{W_0}{W_t}\gamma \qquad \ldots \qquad \ldots \qquad \ldots \qquad (4)$

Example.—*A piece of glass weighs 47 grams in air, 31·53 grams in water at 4°C. and 31·75 grams in water at 60°C. Find the mean coefficient of cubical expansion of water between 4°C. and 60°C., taking that of glass as 0·000024.* (*C. U. 1922*)

Wt. of displaced water at $4°C.=47-31·53=15·47$ gms.

$\therefore$ Volume of displaced water $=15·47$ c.c. and this = volume of glass at $4°C.$

Again, the volume of glass at $60°C.=15·47\{1+0·000024(60-4)\}$
$=15·49$ c.c. = volume of displaced water at $60°C.$

Wt. of displaced water at $60°C.=47-31·75=15·25$ gms.
$\therefore$ Density of water at $60°C.=15·25\div15·49$.

Now, if d = density of water at $4°C.$, d' = density of water at $60°C.$;
γ = coefficient of cubical expansion of water,

we have $d'=d\{1-\gamma(60-4)\}$; or, $\frac{15·25}{15·49}=d\{1-\gamma(60-4)\}$;

whence $\gamma=·000276$, since, $d=1$.

[N.B. The value of the coefficient of expansion can also be determined by using Eq. (4), Art. 36 (*Hydrostatic Method*).]

the central part, therefore, gradually falls and comes to 0°*C.*, when ice begins to be formed. Small crystals of ice tend to rise to the surface, melt and cool the water in the upper part, causing a rapid fall of temperature there. So, now the upper part of the water comes to 0°*C.*, as indicated by the thermometer t_1 ; all this time the water at the bottom remains at 4°*C.* Crystals of ice formed in the central part float up to the surface being lighter than the water there.

Densest liquid occupies the lowest position and as the lower thermometer indicates a constant temperature of 4°*C.*, it is concluded that *water attains its maximum density at 4°C.* ; otherwise, it may be stated that *water at 4°C. expands whether it is heated or cooled.*

The reading of the two thermometers, entered in a graph, will be as represented by Fig. 27(*b*).

41. Practical Importance of Hope's Experiment :—The fact that water has a maximum density at 4°*C.* and expands both at higher and lower temperatures, has a great practical importance in nature. If the density continued to increase until 0°*C.* was reached, ponds in cold countries would freeze solid from top to bottom in severe frosts, and ultimately the whole of a pond would be a mass of ice, and that would destroy the aquatic animal life. But that does not actually take place and what really happens can be explained as follows :—

Let us consider a pond where the air above the water surface is below 0°*C.* (Fig. 28). The water on the surface, on cooling, becomes denser than that below and gradually sinks downwards. This proceeds until the water temperature falls to 4°*C.* As the surface water cools below this, it becomes less dense than the water below, which is at 4°*C.*, and is the densest. It therefore remains at the top, though cooling more and more, and finally freezes into ice. As ice it also remains at the top, for ice is lighter than water. The layer of ice formed acts as a thermal barrier and does not allow much heat to pass from the water below to the colder atmosphere above, for ice is a poor conductor of heat. Extremely slowly the thickness of the layer of ice develops, the rate of heat transfer from water being very small. The temperature of the deeper layers of the water in the pond remains nearly at 4°*C.* and falls gradually to 0°*C.* upwards till the layer of ice is reached. The aquatic life in the water is thus preserved.

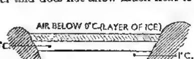


Fig. 28—Frozen Water Surface in a Pond.

42. Correction of Barometric Reading:—The pressure exerted by a column of zero-degree-cold pure mercury (density $=13{\cdot}596$ gms/c.c.), 76 cms. in height, at the sea-level at 45° latitude (where $g=980{\cdot}6$ cms/sec^2) is called the **Standard pressure.** If a comparison is to be made, the observed barometric height at a place should be so transformed as to correspond to the above standard conditions. But before the observed height is transformed to standard conditions, it has to be corrected, because the scale with which the height is measured may be at a different temperature from that at which it is graduated.

Temperature Correction for Scale.—

Suppose the scale is graduated at 0°C. At higher temperatures, each division of the scale will extend in length. So the observed height, say h_t, at a temperature t°C as measured by such an expanded scale will be smaller than its real value. Let h_0 be the correct height, had the scale been maintained at 0°C. So, $h_0=h_t(1+\alpha t)$, where α=coeff. of linear expansion of the material of the scale.

Transformation of Corrected Observed Height to Standard Conditions.—

(*a*) *Transformation to zero-degree-cold mercury*—

The corrected height h_0 is a column of mercury at t°C. To transform it to zero-degree-cold mercury with which the height will be, say H, we have

$H\,d_0=h_0 d_t$, where d_0 and d_t are the densities of mercury at 0°C. and t°C., i.e. $H=h_0\cdot\dfrac{d_t}{d_0}=h_0\,\dfrac{d_0(1-\gamma t)}{d_0}$, where γ=coeff. of cubical expansion of mercury.

$\therefore\quad H=h_0(1-\gamma t)=h_t(1+\alpha t)\,(1-\gamma t)$, after applying the temperature correction for the scale

$=h_t\{1-(\gamma-\alpha)t\}$, approximately

(*b*) *Transformation to the sea-level at 45° latitude.*—

The value of g at a place depends on the latitude of the place and its elevation above the sea-level. If g=accel. due to gravity at the place of observation, and g_a, that at the sea-level at 45° latitude ($g=980{\cdot}6$ cms. sec^{-2}), and if the corrected height H measured by zero-degree-cold mercury, on transformation to sea-level at 45° latitude becomes H_a, then

$$H_a\rho_0 g_a=H\rho_0 g$$

$$\text{or,}\quad H_a=H.\frac{g}{g_a}=h_t\{1-(\gamma-\alpha)t\}\times\frac{g}{g_a}.$$

Note.—γ for mercury=0·000182 per 1°C., α for brass=0·000018 per 1°C.; α or glass=0·000008 per 1°C.

Hence for a barometer with **brass scale,** we have as follows :

$$\text{True height} = \text{observed height} \times (1 - 0{\cdot}000164\, t)\ \frac{g}{980{\cdot}6}\ ;$$

and for a barometer with **glass scale :**

$$\text{True height} = \text{observed height} \times (1 - 0{\cdot}000174\, t)\ \frac{g}{980{\cdot}6}\ .$$

Examples.—(1) *The glass scale of a barometer reads exact millimetres at 0°C. The height of the barometer is noted as 763 divisions at 18°C. Find the true height of the barometer at 18°C. (The coefficient of linear expansion of glass=0·000008 ; coefficient of absolute expansion of mercury=0·000180).*

From Art. 42, we have true height $H = h\{1-(\gamma-\alpha)t\}$

$$= 763\{1-(0{\cdot}000180-0{\cdot}000008)18\} = 760{\cdot}637 \text{ mm.}$$

(2) *A barometer provided with a brass scale, which is correct at 50°F., reads 754 mm. at 40°F.; what will be the true height at 32°F. ?* (*cf. Utkal, 1951*)

The coefficient of linear expansion of brass is 0·000018 per 1°C., so the value for 1°F., will be ($\frac{5}{9}\times 0{\cdot}000018$)=0·00001 and similarly, the coefficient of cubical expansion of mercury for 1°F.=0·0001.

Let t_1 be the lower temperature at which the height should be corrected, t_2 the observed temperature, and t_3 the temperature at which the graduations are correct. (It should be noted that here the barometer is corrected at a higher temperature.)

We have,
$$h_{t_1} = \frac{h_{t_2}[1+\alpha\{(t_2-t_1)-(t_3-t_1)\}]}{\{1+\gamma_r(t_2-t_1)\}}$$

$$h_{32} = \frac{h_{40}[1+\alpha\{(40-32)-(50-32)\}]}{1+\gamma_r(40-32)}$$

$$= \frac{754\{1+0{\cdot}00001(-10)\}}{1+(0{\cdot}0001\times 8)} = 753{\cdot}32 \text{ mm.}$$

(3) *The brass scale of a barometer was correctly graduated at 15°C. At what temperature the observed reading will require no temperature correction ?*

Let t be the required temperature, then $h_0 = \dfrac{h_t\{1+0{\cdot}000019(t-15)\}}{(1+0{\cdot}000181\,t)}$

(Coeff. of linear expansion of brass=0·000019). Here, we have $h_t = h_0$.

$\therefore\ 1+0{\cdot}000181\,t = 1+0{\cdot}000019(t-15)$; or, $t = -1{\cdot}76°C.$

43. Henry Victor Regnault (1810—1878) :—A French scientist who began his life as an assistant in a pharmaceutical shop. He had to work hard at the day time. Instead of taking leisure at night he used to devote his time to private studies on elementary Chemistry and Medicine. His poverty could not separate him from his studies.

In 1832 he started for Paris where he somehow got admitted to the Ecole Polytechnic. From this institute he passed out with distinction in 1836 and accepted an appointment as a Professor at Lyons. Though he began his scientific career as an Organic Chemist, by 1840 his name became widely known as a physicist too and he was offered the Professorship of Natural Philosophy at his own *Alma Mater*, the Ecole Polytechnic. His principal contributions to science belong to the domain of Physics.

His name will endure for ever for his systematic researches on liquids and gases, e.g. on the absolute expansion of mercury, density of water vapour, specific heats of gases, vapour pressures, humidity of air and velocity of sound. Regnault's table of vapour pressure of water is an achievement of great practical importance. He designed a number of apparatuses for various types of laboratory measurements, such as those for the absolute expansion of mercury, constant pressure expansion of air, specific heat of gases at constant pressure, specific heat of solids, dewpoint, etc. which all bear his name and are universally used all over the world.

44. Thomas Charles Hope (1766—1844) :—A brilliant Edinburgh graduate who first acted as Professor of Chemistry at the [illegible], was [illegible]. He [illegible]s and obtains a maximum value at 4°C. is a result of his researches and is a fact of outstanding practical importance.

Questions

1. Distinguish between real and apparent expansions in the case of liquid. Establish a relation between them and the expansion of the material of a vessel. (C. U. 1926, '30 ; Pat. 1927, '28, '30, '41 ; *cf.* All. 1944 ; G. U. 1949)

2. When hot water is thrown on the bulb of a thermometer, the mercury column first falls and then rises. Why is this ?

3. The readings of two thermometers containing different liquids agree at the freezing point and boiling point of water respectively, but differ at other points of the scale. What inferences do you draw from this ?

4. The coefficient of expansion of mercury is $\frac{1}{5550}$. If the bulb of a mercurial thermometer is 1 c.c. and the section of the bore of the tube 0·001 sq. cm., find the position of mercury at 100°C., if it just fills the bulb at 0°C. (Neglect the expansion of glass) (C. U. 1916)

[*Ans.* 18 cms. nearly.]

5. Describe how to measure the absolute expansion of a liquid with the [illegible] liquid at 15°C., [illegible] nsion of glass is [illegible]

6. Describe with theory an accurate method for determining the apparent coefficient of cubical expansion of a liquid. How can the coefficient of real expansion be obtained from it ? (Utkal, 1951)

7. The density of mercury at 20°C. is 13·546, and its coefficient of cubical expansion is 0·000182. Find the mass of 500 c.c. of mercury at 80°C. Also find the volume of 500 gms. of mercury at this temperature.

[*Ans.* 6699 gms. ; 37·3 c.c.]

8. The density of mercury is 13·6 gm./c.c. at 0°C. and at 100°C., it is 13·35 gms./c.c. Calculate the coefficient of absolute expansion of mercury. (Utkal, 1949)

[*Ans.* $1·84 \times 10^{-4}$/°C.]

9. The density of water at 20°C. is 0·998 gm./c.c. and at 40°C. it is 0·992. Find the coefficient of cubical expansion of water between the two temperatures.

[*Ans.* 0·0003/°C.]

10. Two scratches on a glass rod 10 cms. apart are found to increase their distance by 0·08 mm., when the rod is heated from 0°C. to 100°C. How many c.c. of too much boiling water will a measuring flask of the same glass hold up to a scratch on the neck which gave correctly one litre at 0°C. ?

[*Ans.* 1002·4 c.c.]

11. The coefficient of linear expansion of glass is 8×10^{-6} and the coefficient of cubical expansion of mercury is $1·8 \times 10^{-4}$/°C. What volume of mercury must be placed in a specific gravity bottle in order that the volume of the bottle not occupied by mercury shall be the same at all temperatures ?

[*Ans.* $\frac{2}{15}$ of the vol. of the bottle.]

12. The apparent expansion of a liquid when measured in a glass vessel is 0·001029, and it is 0·001003 when measured in a copper vessel. If the coefficient of linear expansion of copper is 0·0000166, find that of glass.

[*Ans.* 0·0000079.]

13. A weight-thermometer contains 700 gms. of mercury at 100°C. What is its internal volume at that temperature ? (Density of the mercury = 13·6 ; coefficient of expansion = 0·000182).

[*Ans.* 52·4 c.c.]

14. Calculate the coefficient of apparent expansion of mercury from the following data :—

A mercury thermometer wholly immersed in boiling water reads 100°C. When the stem is withdrawn so that graduations from 0° upwards are at an average temperature of 10°, the reading is 98·6°. (C. U. 1940)

[*Ans.* 0·000157/°C.]

15. A wt.-thermometer containing 100 gms. of mercury at 0°C. is surrounded by liquid in a bath when 4 gms. of mercury flow out. What is the temperature of the bath if the apparent coefficient of expansion of mercury is 0·00018 ? (East Punjab U. 1952)

[*Ans.* 231·5°C.]

16. A glass wt.-thermometer has a mass of 6·34 gm. when empty, and 153·81 gm. when filled with mercury at 0°C. If 2·08 gms. are expelled when it is heated to 100°C., find the coefficient of relative expansion of mercury in glass. (R. U. 1952)

[*Ans.* 0·000143 per °C.]

These three variables are commonly called the *factors of state* of a gas. They are found to be such that if any one of them is kept constant, the other two, when they vary, follow a definite law, known as a gas law. This gives us the following three *gas laws* :—

(1) the relation between pressure P and volume V, when *temperature* (t) *is constant* ; this relation is given by the *Boyle's Law* ;

(2) the relation between volume and temperature, when *pressure is constant* ; this relation is given by the *Charles' Law* (Art. 46) ;

(3) the relation between pressure and temperature, when *volume is constant* ; this relation is given by the *Pressure Law* (Art. 50).

For a *given mass of a gas*, all the three variables as stated above are not independent ; when any two of them are given, the third becomes automatically fixed up, as will be seen afterwards.

The first of the above three relations, which the Boyle's law embodies, has already been treated in full in Art. 309, Part I *et seq.*

46. Expansion of Gases at Constant Pressure :—

Charles' Law.*—*The law states that the pressure remaining constant, the volume of a given mass of any gas increases (or decreases) by the constant fraction* $\frac{1}{273}$ *of its volume at 0°C. for each degree centigrade increase (or decrease) of temperature.*

This constant fraction is, therefore, the coefficient of expansion of a gas at constant pressure and may be simply called the *volume coefficient* of a gas and ordinarily denoted by γ_p. Thus if V_0 and V_t be the volumes of a given mass of any gas at 0°C. and t°C. respectively then according to Charles' law,

$$V_t = V_0(1+\gamma_p t) = V_0\left(1+\frac{t}{273}\right) = \frac{V_0}{273}(273+t) = \frac{V_0}{273}\,T, \text{ where}$$

T is the absolute temperature (*vide* Art. 51) corresponding to t°C.,

$$\text{or} \qquad V_t \propto T$$

This gives us another form of the Charles' law which may be stated as, "*the volume of a given mass of any gas, at constant pressure, varies directly as its absolute temperature.*" Evidently, the graph between the temperature and the volume of a given mass of any gas will be a straight line ; such a graph has been shown in Fig. 32.

Working Formula of Charles' Law in Fahrenheit Scale.—A Fahrenheit degree is $\frac{5}{9}$ of a Centigrade degree ; so the value of the coefficient of expansion of a gas at constant pressure, which is $\frac{1}{273}$ per °C. becomes equal to $\frac{5}{9}\times\frac{1}{273}$ or $\frac{1}{491}$ per °F. approximately. Therefore

* The law is also sometimes called Gay-Lussac's Law , for, though Charles first found out this relationship for a gas he did not publish his work. In 1802, Gay-Lussac proved the same Law independently ; he saw Charles' manuscripts afterwards and found that Charles had discovered the law fifteen years earlier.

according to the Fahrenheit scale, our formula for the Charles' law will be,

$$V_t = V_0\{1 + \tfrac{1}{491}(t - 32)\}.$$

N.B. The value of γ_p is a constant. It is equal to $\frac{1}{273}$ or 0·00366 per °*C.* and is *approximately the same for all gases.* It is not different for different gases as in the case of solids and liquids.

Thus 1 c.c. of a gas at 0°*C.* becomes $(1+\frac{1}{273})$ c.c. at 1°*C.*; $(1+\frac{5}{273})$ c.c. at 5°*C.*; $(1+\frac{50}{273})$ c.c. at 50°*C.*; and so on.

Again, 273 c.c. of a gas at 0°*C.* become 273 $(1+\frac{1}{273})$ c.c., *i.e.* 274 c.c. at 1°*C.*; $273(1+\frac{100}{273})$ c.c. *i.e.* 373 c.c. at 100°*C.*; 273 $(1+\frac{110}{273})$ c.c., *i.e.* 383 c.c. at 110°*C.*

47. The Importance of measuring the Expansion of a Gas with respect to its Volume at 0°C. :—In determining the coefficient of expansion of a gas, the initial volume of a gas must always be taken at 0°*C.* instead of taking it at any other temperature which can be allowed in the case of solids and, to some extent, of liquids, as the expansion of a gas for a small change of temperature is very large in comparison with the very small expansion of a solid or that of a liquid; or, in other words, the coefficient of expansion of a gas is *not a very small fraction* as in the case of solids or liquids.

For the above reason we did not so much insist on specifying any lower temperature in the formula relating to expansion of solids and liquids. But, *in calculating the expansion of gases, we should always mind the words* $\frac{1}{273}$ *"of its volume at* 0°*C.*", and we shall get wrong results if we take the original volume at any other temperature, say 10°*C.*, or 20°*C.*, as in the case of solids and liquids.

Suppose we have 373 c.c. of a gas at 100°*C.*, and we want to find its volume at 110°*C.* By directly applying the formula

$V_{110} = V_{100}(1+\frac{10}{273})$ we get,

$$V_{110} = 373(1+\tfrac{10}{273}) = 373 + 13{\cdot}67 = 386{\cdot}67 \text{ c.c.}$$

But this cannot be, for a volume of 373 c.c. at 100°*C.* will become 383 c.c. at 110°*C.*, as seen before. This shows the importance of the words "of its volume at 0°*C.*".

That the above point is not so important in the case of solids will be shown thus:

Suppose we have a rod of iron which is 100 cms. long at 0°*C.*, then at 100°*C.*, it will become $100(1+0{\cdot}000012\times100)$ or 100·120 cms. At 110°*C.* it will become $100(1+0{\cdot}000012\times110)$ or 100·132 cms.

Again by applying the formula directly, as in the above case, $l_{110} = l_{100}(1+0{\cdot}000012\times10) = 100{\cdot}12(1+0{\cdot}00012) = 100{\cdot}1320144$.

The difference in the two results which is 0·0000144, can easily be neglected for our purposes, and this clearly shows the importance of always considering the volume at 0°*C.* while calculating the expansion of gases.

48. Important Points of Difference :— Though the relation, $V_t = V_0(1+\gamma_p \times t)$ as given by Charles' law is similar to that in the case of thermal expansions of solids and liquids the following points of difference should be marked :—

(i) Unlike in solids and liquids a change in pressure considerably affects the volume of a gas and so in finding the volume coefficient of a gas, steps must be taken to keep the pressure constant while the temperature is changed.

(ii) The coefficient of expansion for gases ($\frac{1}{273}$) is quite large compared to those for solids and liquids

(iii) The value of the coefficient of expansion for gases is a *constant* and is approximately the same for all gases and not different for different gases. For solids and liquids, it is different for different substances and for the same substance the value changes, in many cases irregularly, at different parts of the temperature scale.

(iv) The volume 0°C. and not at any original temperature (which is permissible in the case of solids roughly also in the case of liquids) is to be taken for gases in applying the law of thermal xpansion

49. Determination of the Coefficient of Expansion of a Gas at Constant Pressure :—

(1) Constant Pressure Air Thermometer Method.— Take a piece of capillary glass tube T of uniform bore and about 50 cms. long (Fig. 29). Pass a stream of hot air through the tube for some time, and when the tube has been dried, seal off one end of it by a blow-pipe flame. The tube is then gently heated with the open end dipped in mercury. On allowing the tube to cool, the air contracts and a small pellet m of mercury is driven inside and this serves as an index. The tube T is now held horizontally in a wide glass tube G which is stoppered at both the ends The tube G acts as a bath and is provided with inlet tube A and outlet tube B. A thermometer P, also introduced horizontally, gives the temperature within the bath G. Pass

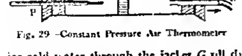


Fig. 29 —Constant Pressure Air Thermometer

ice-cold water through the jacket G till the thermometer P indicates a constant temperature 0°C and the pellet m assumes a steady position. After waiting for sometime, measure the distance of the [illegible]

constant, as shown by a steady position of the pellet. The distance

of the lower end of the pellet is again noted. Let the temperature now indicated by the thermometer P be $t°C$. As the open end of the air thermometer is exposed to the atmosphere all throughout, the pressure of the enclosed air is constant and equal to that of the atmosphere.

As the tube is of uniform bore, the volume of the enclosed air is proportional to the length of the enclosed column. Let α be the area of cross-section of the bore, and l_0 and l be the lengths occupied by the air column at $0°C$., and $t°C$. respectively ; So $l_0\alpha$ is the volume of air at $0°C$., and $l\alpha$ the volume at $t°C$., neglecting the expansion of glass, which is small compared to that of air.

$$\text{The volume coefficient, } \gamma_p = \frac{l\alpha - l_0\alpha}{l_0\alpha \times t} = \frac{l - l_0}{l_0 \times t} \cdots \qquad \cdots \qquad (1)$$

The air in the tube can be replaced by any other gas, and it *will be found that the value of γ_p in every case will be the same,* viz. $\frac{1}{273}$ approximately.

(2) Regnault's Method.—Regnault's apparatus is also an air thermometer. In it the air is enclosed in the bulb A (Fig. 30) fo one limb of a U-tube and kept dry by strong sulphuric acid poured through the other limb B. The limb having the air bulb A is graduated and directly gives the volume of the enclosed air. The limb B is open to the atmosphere. The U-tube has a short cross-tube attached to its bend and this serves as an outlet. This outlet tube is provided with a stop-cock S by opening which any excess acid in the U-tube can be dropped out. The U-tube is placed in water contained in an outer jacket and the quantity of water is so taken that the air bulb is completely immersed in it but the open limb B projects out. This outer jacket is a thick glass cylinder whose bottom is closed by means of a stout rubber cork. A copper pipe enters through this cork into the water in the jacket and leaves the water bath again through the rubber cork. When steam is passed through this pipe, the water around gets heated. By regulating the supply of steam the temperature of the water bath can be kept constant at a desired value. For uniformity of temperature throughout the water, the latter may be stirred by means of a stirrer (not shown in the figure). A thermometer T is suspended in the water and records the temperature of the bath. When it is steady, it is also the temperature of the air enclosed in the

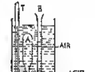


Fig. 30—Regnault's Apparatus (Constant Pressure Air Thermometer).

bulb A. Before taking readings, sufficient time should be given to the enclosed gas to attain the temperature of the water. Now sulphuric acid is poured into B or run out by opening the stop-cock S until its levels are the same in both the limbs. The air in A is then at atmospheric pressure and its temperature is noted and the volume is read from the graduations. Steam is passed through the copper pipe and the water is kept constantly stirred. The temperature-rise causes the air in the bulb to expand and force down the liquid which rises in the other limb. The temperature is kept constant for some time by regulating the steam, during which the levels of the acid are adjusted to be the same in both the limbs either by dropping out some acid by opening the stop-cock or adding more acid into B, as required; volume and temperature are read as before. The heating is continued and readings are taken at various higher temperatures until the water boils.

If V_0, V_1 and V_2 be the volume of the air respectively at 0°C., t_1°C, and t_2°C.,

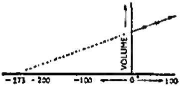


Fig. 31

we have, $V_1 = V_0(1+\gamma_p t_1)$, and $V_2 = V_0(1+\gamma_p t_2)$; or $\frac{V_2}{V_1} = \frac{1+\gamma_p t_2}{1+\gamma_p t_1}$ whence γ_p can be known as V_1, V_2, t_1 and t_2 are known.

Determination of γ_p from graph and the verification of Charles' Law.—If the temperature is plotted on the x-axis and the volume on the y-axis, a straight line graph is obtained (Fig. 31) indicating that the expansion of a gas is uniform, or the pressure of a gas remaining constant, the volume increases directly with the temperature. On producing the graph backwards it will cut the x-axis at about −273°C., which means that the volume of the air (theoretically) becomes zero at −273°C. The volumes of air V_0 at 0°C., and V_t at any convenient temperature t can be read from the graph from which γ_p may be calculated from the relation $V_t = V_0(1+\gamma_p t)$.

The result obtained for γ_p for air is about 0.00367 per °C., i.e. approximately 1/273 per °C. This verifies Charles' law.

50. Increase of pressure of a gas at Constant Volume :—

The Pressure Law.—The relation between *pressure* and *temperature* of a gas at constant volume is called the pressure law or *constant volume law*.

The law states that volume remaining constant, the pressure of a gas increases (or decreases) by a constant fraction $(\frac{1}{273})$ of its pressure at 0°C. for each degree centigrade increase (or decrease) of temperature.

This constant fraction is called the **pressure coefficient** (γ_p) of a gas and is evidently, equal to the volume coefficient of the gas (*vide* also Art. 51). Mathematically, if P_t and P_0 are the pressures of a gas at $t°C.$ and $0°C.$ respectively, then at constant volume,

$$P_t = P_0(1+\gamma_p t) = P\left(1+\frac{t}{273}\right)$$

$= \frac{P_0 T}{273}$; or, $P \propto T$, where T=absolute temperature corresponding to $t°C.$ The graphical relation between the pressure and temperature of a gas will, therefore, be a straight line as shown in Fig. 33.

N.B.—The Pressure Law is also often referred to as Charles' law, for, as is evident from above, the pressure *of a gas varies with temperature* at constant volume according to the same law as the volume varies with temperature at constant pressure.

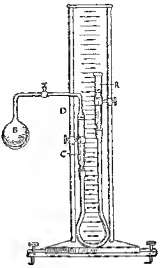


Fig. 32—Joly's Constant Volume Air Thermometer.

50(a). Determination of the Pressure Cofficient of a Gas :—

By Joly's Apparatus.—The relation between pressure and temperature of a gas at constant volume can be studied by Joly's apparatus (Fig. 32), which almost resembles the Boyle's Law Tube (*vide* Art. 311, Part I) with the addition of a glass bulb B provided with a stop-cock in place of the straight closed tube. The bulb B and the connecting tube up to the surface of mercury in the tube C contain dry air.

Expt.—Open the stop-cock and raise or lower the open tube R till the mercury in the tube C reaches some point D marked on the stem, the point being selected as near the top of the tube C as possible. Now close the stop-cock. At this stage the pressure of the air above the mercury in both the tubes is atmospheric, which, suppose, is H cms. of mercury. Next take a bath of water, say a large brass or copper basin provided with a stirrer containing water placed on an adjustable vertical stand, which may be heated from below by means of a burner. Gradually adjust the height of the bath until the whole of the bulb B is completely immersed in the water. A thermometer vertically inserted in the bath gives the temperature. Heat the bath and regulate the temperature at some constant value, say $t_2°C.$, by applying the burner or withdrawing it for some time as required. For

uniformity of temperature, the water should be stirred well. Due to temperature-rise the air in the bulb will expand and force down the mercury in the tube C. Raise the tube R till the mercury head touches the fixed mark D again in the other tube. Let the difference in levels of the mercury in the tubes C and R be h_1. Then the pressure P_1 of the air, now at temperature t_1 is $(H+h_1)$ cms. of mercury. Similarly, change the temperature to t_2°C. and note the difference in levels h_2 now, when the pressure P_2 will be $(H+h_2)$ cms. of mercury. Mark that the volume of the air enclosed in the bulb B up to the fixed mark D is kept constant in this experiment neglecting the expansion of the glass bulb B; for, at each observation, the level of mercury is brought back to the same fixed mark D by adjusting the limb R of the tube. If P_0 be the pressure at 0°C., we know from Charles' law,

$$P_1 = P_0(1+\gamma_v t_1), \text{ and } P_2 = P_0(1+\gamma_v t_2) \quad \ldots \quad \ldots \quad (2)$$

That is, $$\frac{P_1}{P_2} = \frac{1+\gamma_v t_1}{1+\gamma_v t_2} = \frac{H+h_1}{H+h_2}.$$

As t_1, t_2, h_1 and h_2 are all known and H may be determined by means of a barometer, γ_v, the pressure coefficient can be determined. The air in the connecting tube attached to the bulb B is not at the same temperature as that of B, when the temperature of the bulb is raised. This may be called the error due to *exposed column* for which a correction is needed. Strictly speaking, the air is not heated all along under constant volume as the bulb expands, however small the expansion may be, with the temperature of the bath.

Determination of γ_v from graph and the verification of the Law of Pressures.—If the temperature is increased gradually in steps keeping the volume constant and the corresponding pressures are determined a graph may be plotted with temperatures on the x-axis and pressures on the y-axis. On drawing the graph on a smaller scale and producing it backwards, it will be a *straight line* (Fig. 33) cutting the x-axis at about -273°C; that is, at zero pressure the temperature is theoretically -273°C.

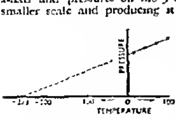


Fig. 33

The straight line indicates that the pressure increases uniformly with the temperature when the volume of the gas remains constant.

Reading from the graph, the value of P_0 at 0°C. and P_t at any convenient temperature t, γ_v can be calculated, and in this experiment, water at the temperature of the laboratory can be used instead of ice-cold water.

The result obtained for γ_v for air is about 0·00367 per °C i.e. $\frac{1}{273}$ per °C. approximately, and the same value is also obtained for other

gases which obey Boyle's law. This verifies the **Law of Pressures,** which is another form of the Charles' law.

51. Relation between γ_p and γ_v :—For any gas obeying the law of Boyle and Charles, it may be shown that $\gamma_p=\gamma_v$. If the temperature of any mass of the gas be increased from 0° to $t°$ while the pressure remains constant, we have, $V_t=V_0(1+\gamma_p t)$ .... (1). Now increasing the pressure from P_0 to P_t while the temperature remains at $t°$, until the volume is V_0, we have, by Boyle's law, $P_0V_t=P_tV_0$ .... (2). From (1) and (2), $P_0(1+\gamma_p t)=Pt$ .... (3). If, however, the temperature of the gas had been increased from 0° to $t°$ while the volume remained constant, then $P_t=P_0(1+\gamma_v t)$ .... (4).

Hence from (3) and (4), we have, $\gamma_p=\gamma_v$, or *the volume coefficient of a gas is equal to its pressure coefficient.*

52. Joseph Louis Gay-Lussac (1778—1850) :—He was born at Limousin in France. During the French Revolution his father, a Judge, was imprisoned and so Joseph's schooling began late. He passed from the Paris Polytechnic and developed a great passion for Chemistry. He began researches under Berthelot and here he discovered in 1802 the law of thermal expansion of gases independently though he did not know then that Charles had found the same fifteen years earlier. The theory of variation of temperature with altitude is due to him, and he personally climbed heights as great as 23,000 ft. in order to study the variation of magnetic field and temperature. In 1809 he became Professor of Chemistry at the Paris Polytechnic. He discovered Iodine, Cyanogen, and Prussic acid.

53. The Gas Thermometer :—Like liquids, gases also can be used as thermometric substances. In practice the gases, such as air, hydrogen, nitrogen, helium, etc. *i.e.* the gases which behave nearly as perfect gases (*vide* Art. 64) are used as thermometric substances and the thermometer (either constant pressure or constant volume) is named according to the gas used, *e.g.* the air thermometer, the hydrogen thermometer, etc. The reason for using only one or the other type of these gases only is that these gases obey Charles' law or the law of pressures quite accurately over a wide range of temperatures while other gases do not.

(1) Methods of Measurement of Temperature by a Gas Thermometer.—To find the temperature of a given bath with a constant volume gas thermometer, find P_0, the pressure of the enclosed gas at 0°C. and P_t, the pressure at the unknown temperature t of the bath. Then, we have,

$$\gamma_v=\frac{P_t-P_0}{P_0 t}\ ;\ \text{or,}\ t=\frac{P_t-P_0}{P_0\gamma_v}=273\,\frac{P_t-P_0}{P_0}.$$

The volume of P_0 can also be determined from the graph as shown in Fig. 33.

(II) Graphical Method.—Plot two points corresponding to P_0 and P_{100} at 0 C. and 100 C. respectively and join them by a straight line as in Fig. 33. Now find the pressure P_t, of the same volume of the enclosed gas, corresponding to the unknown temperature t of the bath, and from the graph read the value of t corresponding to P_t

A constant pressure gas thermometer can also be used in either of the two ways described above for the measurement of an unknown temperature. The only difference in its case will be to find V_t and V_0 in method (I) instead of P_t and P_0, and P_0, V_{100} and V_t in method (II) instead of P_0, P_{100} and P_t.

Standard Thermometer.—Though both the constant pressure and the constant volume gas thermometers are equally accurate for measurement of temperature, *the constant volume hydrogen thermometer* has been internationally accepted *as a standard thermometer*. Other thermometers, such as the different types of liquid-in-glass thermometers, the electrical thermometers like the resistance or thermocouple thermometers, the radiation thermometers, etc. should be standardised by comparison with such a thermometer. A gas thermometer, after corrections are applied for the deviations of the gas from perfect gas conditions (which are known now-a-days for all thermometric gases), furnishes a scale of temperatures which has been shown by Kelvin to be perfect from the theoretical standpoint. This is the reason, besides the numerous practical advantages listed below, why a gas thermometer is preferred to all other thermometers in all standard measurements

Advantages and Disadvantages of a Gas Thermometer.—

(a) Advantages.—A gas is light and can be obtained in pure condition. It remains gaseous and therefore can be used as a thermometric substance for a much wider range of temperatures than is possible in the case of other types of thermometers By using helium gas a temperature up to very near the absolute zero (Art. 54) can be determined The maximum temperature for which a gas thermometer can be used is determined by the temperature at which the bulb of the thermometer fuses and the permeability of the bulb to the gas used. A hydrogen thermometer may be used from −200°C. to 500 C. above which hydr[illegible]n cannot be used as it attacks the materials of the containing [illegible] (glass or porcelain). For temperatures above 500°C. hydrog[illegible] replaced by nitrogen, and for low temperatures below −200 [illegible] [illegible]m replaces hydrogen A platinum-rhodium bulb using nitrog[illegible] [illegible]een used up to 1600 C. The rate of expansion of a gas is [illegible] [illegible] and regular over the whole range of the scale. A gas th[illegible] is very sensitive too, for the thermal expansion of gases [illegible]rge. For the same reason the expansion of the envelop[illegible] does not affect the observations seriously as in [illegible]

[illegible] tively [illegible] olume [illegible] nished

by different gases are identical. So all gas thermometers read alike at all parts of the scale.

(b) Disadvantages.—A gas thermometer cannot be used for clinical or calorimetric purposes, for it is not a direct-reading thermometer. Moreover, being unwieldy in size it is inconvenient for domestic use. In case of a constant volume gas thermometer a barometer is needed for the knowledge of the pressure of the gas. Again, no permanent scale can be fixed with a gas thermometer, since the atmospheric pressure changes.

Examples. (*1*) *Find the temperature of the boiling point of a salt solution from the following readings obtaind with a constant pressure air thermometer. Position of mercury at* $0°C.=7{\cdot}2$, *and at* $100°C.=16{\cdot}8$; *position when the thermometer is in boiling solution*$=17{\cdot}3$.

Let V_t=volume of air at the unknown temperature $t°C.$, then $\gamma_p=\dfrac{V_t-V_0}{V_0 t}$.

But $\gamma_p=\dfrac{V_{100}-V_0}{V_0\times 100}$. $\therefore$ $\dfrac{V_t-V_0}{t}=\dfrac{V_{100}-V_0}{100}$.

$\therefore$ $t=\dfrac{V_t-V_0}{V_{100}-V_0}\times 100=\dfrac{17{\cdot}3-7{\cdot}2}{16{\cdot}8-7{\cdot}2}\times 100=105{\cdot}2°C.$

(*2*) *The pressure of air in the bulb of a constant volume air thermometer is* $7{\cdot}3$ *cms. of mercury at* $0°C.$, $100{\cdot}3$ *cms. at* $100°C.$, $77{\cdot}8$ *cms. at room temperature. Calculate the temperature of the room.*

As in Ex. 1, $t=\dfrac{P_t-P_0}{P_{100}-P_0}\times 100=\dfrac{77{\cdot}8-73}{100{\cdot}3-73}\times 100=17{\cdot}6°C.$

53(A). Constant Volume Hydrogen Thermometer :—A constant volume hydrogen thermometer, originally devised by Harkar and Chappius, consists of two distinct parts, a bulb A (Fig. 33A) containing hydrogen gas and a manometer $QGFG_1$ which measures the pressure of the gas at any unknown temperature and the gas bulb A (to be immersed in the bath whose temperature is to be measured), made of an alloy of platinum and iridium, one metre long having a capacity of 1 litre. The gas bulb A is connected to the manometer tube FG_1 just below a steel partition S by a capillary platinum tube B (1 metre long). The lower surface of the partition is provided with a platinum pointer P upto the tip of which the mercury level in the lower compartment of FG_1 is to be raised in order to maintain the constancy of volume of the gas in the bulb A. The upper compartment of FG_1 is directly connected to the other manometric tube GQ through a narrow cross-tube L. The lower mercury column is communicated to the same tube GQ through M. The end of the inverted manometer tube IQ (otherwise referred to as the barometer) dips in mercury in G. The barometer tube is so bent at Q that the mercury column I and that in FG_1 lie in the same vertical line. This enables one to read the levels of the mercury top in these tubes and hence the gas pressure with the same vertical setting of a cathetometer. The mercury in the manometer tubes communicates

to zero. This temperature is the lowest possible temperature on the gas scale and this temperature ($-273°C$) is $273°C$. lower than $0°C$.

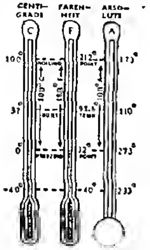


Fig 34 — Absolute Scale

The scale of temperature in which temperatures are measured from $-273°C$. as zero degree and in which other divisions are numbered starting from this temperature is known as the Absolute Scale or the Kelvin Scale (*vide* Fig. 34). It is so named, because the zero of this scale is really or absolutely the lowest temperature *we can imagine* and a temperature lower than this is impossible.

The zeroes of other scales are only arbitrary, for temperatures below $0°C$., $0°F$., or $0°R$., exist actually. In the absolute or Kelvin scale each degree above its zero is equal to a degree Centigrade or a degree Fahrenheit, or a degree Reaumur according as the scale desired is a Centigrade, a Fahrenheit, or a Reaumur absolute scale.

N.B. The above result namely that the volume as well as the pressure of gas reduces to zero at $-273°C$., is only theoretically, true and is physically impossible, as all known gases liquefy and then become solid before this temperature is reached. The result is true for a perfect gas (Art 61) only. As a matter of fact, air starts liquefying at about $-184°C$ hydrogen gas uniformly contracts in volume up to $-269°C$. By the evaporation of liquid helium a temperature as low as $-272°C$ has been reached, but *the absolute zero has never yet been reached*. At the absolute zero temperature, according to the kinetic theory (Art 60), all molecular motion must cease.

(b) Absolute Scale Value on the Fahrenheit System.— Remember that we have so long considered the Centigrade scale according to which absolute zero$=-273°C$. But if the temperature is measured on the Fahrenheit system, the absolute zero becomes equal to 491·4 Fahrenheit degrees below the freezing point ($32°F$.), because $273°$ on the Centigrade scale$=273\times\frac{9}{5}=491{\cdot}4$ on the Fahrenheit scale.

So absolute zero$=32-491{\cdot}4=-459{\cdot}4°F$. It is usual in Engineering practice to take this value as $-460°F$. (approximately).

Relations between Absolute Scale Values and other Scale Values.

Centigrade System.—Absolute value=Centigrade Scale value $+273$.

Fahrenheit System.—Absolute value=Fahrenheit Scale value $+460$.

Reaumur System.—Absolute value=Reaumur Scale value $+218{\cdot}4$.

55. Charles' Law in terms of Absolute Temperature :—

(*i*) According to Charles' law,

we get, $V=V_0\left(1+\frac{t}{273}\right)$, when pressure is constant; similarly, $V'=V_0\left(1+\frac{t'}{273}\right)$, when pressure is constant; here t and t' are in centigrade temperatures.

$\therefore \frac{V}{V'}=\frac{273+t}{273+t'}=\frac{T}{T'}$, where T and T' denote the absolute temperatures corresponding to the Centigrade temperatures t and t'.

Hence $\frac{V}{T}=\frac{V'}{T'}$, *=a constant, when P is constant.*

Or, $V \propto T$, when P is constant.

In other words, *the volume of a given mass of any gas is directly proportional to the absolute temperature when the pressure remains constant.*

(*ii*) Similarly, from the Law of Pressures, we get, $\frac{P}{P'}=\frac{T}{T'}$, when V is constant ;

or, $\frac{P}{T}=$ *a constant, when V is constant.*

Or, $P \propto T$, *when V is constant.*

In other words, *the pressure of a given mass of any gas is directly proportional to the absolute temperature when the volume remains constant.*

56. Meaning of N.T.P. :—This expression stands for '*normal temperature and pressure*'.

(*a*) *Normal Temperature.*—It is the temperature of melting ice when the pressure is one atmosphere. In the centigrade scale it is $0°C.$, or $273°A.$ In the Fahrenheit scale it is $32°F.$, or $492°A.$

(*b*) *Normal Pressure.*—It is the pressure exerted at the base by a vertical column of zero-degree-cold pure mercury, 76 cms. in height placed on the sea-level at 45° latitude. At the above conditions, density of mercury$=13{\cdot}596$ gms./c.c., and acceleration due to gravity, $g=980{\cdot}6$ cms./sec.^{2}

(iii) Value of the gas constant K for 1 gm. of air.—*One litre of air weighs 1·293 gm. at N.T.P. Find the value of K considering 1 gm. of air.*

The volume of 1·293 gm. of air at N.T.P. = 1000 c.c.

$\therefore$ " " 1 gm. " " " $= \frac{1000}{1 \cdot 293}$ c.c.

The normal pressure of the atmosphere $= 1 \cdot 013 \times 10^6$ dynes per sq. cm.

We have, $PV = P_0 V_0 = KT$. $\therefore \quad 1 \cdot 013 \times 10^6 \times \frac{1000}{1 \cdot 293} = K \times 273$.

So, $K = 2 \cdot 87 \times 10^6$ ergs/°C. for 1 gm. of air

59. Change of Density of a Gas:—It is often useful to know the changes of density instead of the changes of volume. If D, D' represent the original and final densities of a mass M of gas, and V, V' be the corresponding volumes, then,

$M = V \times D = V' \times D'$, or, $V = M/D$, and $V' = M/D'$.

So, the equation, $\frac{PV}{T} = \frac{P'V'}{T'}$, becomes $\frac{PM}{DT} = \frac{P'M}{D'T'}$.

$$\therefore \quad \frac{P}{DT} = \frac{P'}{D'T'} = \text{a constant} \qquad \ldots \qquad \ldots \qquad (1)$$

or, $\frac{P}{D} = \frac{P'}{D'}$, when $T = T'$

Hence, *the density of a gas at constant temperature varies directly as the pressure.*

Again, from (1), $DT = D'T'$, when $P = P'$.

Hence, the density of a gas at a constant pressure varies inversely as the absolute temperature.

60. The Kinetic Theory of Gases:—The simple gas laws, namely Boyle's Law, Charles' Law, Avogadro's Law*, etc. are generally obeyed by all gases. So it is reasonable to suppose that they all possess a *common* and simple structure. During his investigations on the structure of gases, **Bernoulli,** assuming a simple common structure of the molecules for all gases, first suggested that the pressure of a gas could be explained, if the molecules were endowed with considerable velocity. Starting from this kinetic concept, he actually deduced Boyle's law but his theory did not develop farther until Joule carried out in 1848 his famous experiment on the equivalence of *mechanical work* and *heat.* However, the credit of giving the Kinetic concept a concrete form lies with Clausius who formulated in 1857 the following basic postulates for the Kinetic theory of gases :—

*__Avogadro's Law.__—Equal volumes of all gases under the same conditions of pressure and temperature contain the same number of molecules.

(1) *"The molecules of a given mono-atomic gas are identical solid spheres which move in straight lines until they collide with one another or with the walls of the containing vessel.*

(2) *The time occupied in collision is negligible ; the collision is perfectly elastic and there are no forces of attraction or repulsion between the molecules themselves.*

(3) *The molecules are negligible in size compared with the size of the container."*

Clausius introduced also the idea of the mean free path of a gas molecule ; this is a very important concept in the study of molecular motion in a given boundary—*"The mean free path is defined as the averge distance traversed by a molecule between two successive collisions."*

61. Interpretation of Various Physical Quantities relating to a Gas by the Kinetic Theory :—

(1) Temperature.—A mass of gas means a vast assemblage of molecules in a given boundary, the container. The molecules are never at rest but are at random motion with very high velocities directed in the most haphazard manner. As a matter of fact, all manners of velocities are probable [*vide* Brownian Motion, Art. 62(2)]. The energy possessed by the molecules is all *Kinetic* and arises by virtue of their being in continuous motion (the molecules have no potential energy, since they neither attract nor repel each other). *The Kinetic energy manifests itself as the temperature of the gas.* This is what is called the Kinetic interpretation of temperature. When the motion becomes more rapid, the temperature increases and *vice versa.* Consistently the temperature will fall to nothing, when all molecular motion ceases. *This temperature will, therefore, be the absolute zero temperature and no temperature can be lower than this and this is a direct deduction from the Kinetic theory of gases.*

(2) Pressure.—The molecules, in course of their motion, make collisions against each other as also against the walls of the container from which they rebound back into the interior of the gas, without loss of energy, exerting a force on the walls. Blows or hits incessantly given to the walls constitute a continuous force tending to push out the walls just as the water particles rushing out from a hose tend to push off an obstacle against which they strike. The force and so the pressure (which is force per unit area) is uniform and steady for the hits are incessant and are directed against the walls of the container equally in all directions in all probability.

(3) Root Mean Square Velocity (R.M.S. Velocity) of a Gas.—The pressure of a gas can be deduced mathematically by the application of Newton's second law of motion. For, when a molecule proceeding with a certain velocity hits a wall of the container, it rebounds, and therefore undergoes a change of momentum without

loss of energy, since according to the Kinetic theory the collision is perfectly elastic. The rate of change of momentum is proportional to the force exerted on the wall. Proceeding in this way, an expression for the pressure can be deduced. For a monoatomic gas this relation is given by $pv = \frac{1}{3}mnC^2 = \frac{1}{3}MC^2$, where p = pressure, v = volume of the gas, M = mass of the gas = mass (m) of a molecule × number (n) of molecules present in the volume and C^2 = the mean value of the squares of the velocities of the individual molecules of the gas at any instant at the given temperature. The quantity C is called the *root mean square velocity* for a gas. It is proportional to the square root of the absolute temp. of the gas, for

$$C^2 = \frac{3pv}{M} = \frac{3RT}{M}, \text{ or } C = \sqrt{\frac{3RT}{M}}; \text{ i.e. } C \propto \sqrt{T},$$

R and M being constants for a given quantity of the gas.

Again $pv = \frac{1}{3}m.n.C^2 = \frac{2}{3}.\frac{1}{2}m.nC^2 = \frac{2}{3} \times K.E.$

That is, *pressure of a monoatomic gas is numerically equal to $\frac{2}{3}$ of the K.E. of the molecules per unit volume of the gas.*

(4) Distribution of Molecules.—The molecules of a gas are all alike. For a monoatomic gas, each molecule is a perfectly elastic sphere having a fixed mass. Its volume, however, is so small that it is treated as a mere *mass-point*. That is, it has a mass as well as a position (which varies from instant to instant) but no dimensions. Even a small portion of a gas contains an inconceivably large number of molecules which is of the order of 27×10^{18} molecules per c.c.* Considering the large number of molecules in a small space and the enormous velocity each molecule possesses and also the fact that the time taken during collision is negligibly small, the distribution of molecules in the volume is, in all probability, uniform in spite of the fact that by collision the directions of motion of the molecules are changing every moment, for such changes of direction of motion will occur in all directions and quite a large number of times at each instant.

62. Evidence of Molecular Motion :—

(1) Diffusion.—The phenomenon of diffusion (*vide* Art. 221, Part I) provides an evidence in support of the molecular motion in fluids. If a jar, containing a light gas like hydrogen, is inverted over another containing, say, carbon-dioxide, a heavier gas, a uniform mixture is formed after a while. This happens in spite of gravity under which the heavier gas should remain in the lower jar and the lighter one in the upper jar. A similar case happens when a strong potash permanganate solution is kept at the bottom of a cylinder and water

* The number of molecules contained in a gramme-molecule (molecular weight expressed in grammes) is a constant for any gas, according to Avogadro's hypothesis and is called the Avogadro number (N). The accepted value for N is $6{\cdot}062 \times 10^{23}$.

is slowly and carefully added from above without causing any agitation. The coloured permanganate solution gradually works up and spreads throughout the whole mass.

Such process of self-mixing of one fluid into another, sometimes even in opposition to gravity, known as *diffusion*, are possible only because the molecules of any fluid are in perpetual motion in all possible manners and this concept of molecular motion is the basis of the Kinetic theory.

(2) Brownian Motion.—A direct evidence, based on *visual* experience, first experimentally demonstrated by Dr. Brown, has established beyond all doubts, the reality of molecular motion in fluids.

In 1827 Robert Brown, an English Botanist, while observing suspensions of powdered gamboge in water (which are inanimate particles) under a highly powerful microscope found the particles moving about in the wildest fashion. Each particle, viewed under the microscope, appears like a tiny star of light in rapid and incessant motion in the most haphazard fashion. Each particle rises, sinks and rises again, or moves to a side this way or that way and so on. The motions are spontaneous and incessant. The motions are more *vigorous in a less sticky liquid or when the temperature is increased.* They are just perceptible in glycerine while most quick in gases. Such chaotic molecular motion in a fluid is called *Brownian motion.*

Gas	At N. T. P.		Avogadro number (N)
	Density (gms./c.c.)	R.M.S. velocity (cms./sec.)	
Hydrogen	$8{\cdot}9\times10^{-5}$	$18{\cdot}38\times10^{4}$	
Oxygen	$14{\cdot}3\times10^{-4}$	$4{\cdot}61\times10^{4}$	
Nitrogen	$12{\cdot}5\times10^{-4}$	$4{\cdot}93\times10^{4}$	$6{\cdot}062\times10^{23}$
Air	$12{\cdot}9\times10^{-4}$	$4{\cdot}85\times10^{4}$	
Carbon-dioxide	$19{\cdot}8\times10^{-4}$	$3{\cdot}93\times10^{4}$	

63. Explanation from the Kinetic Theory :—

Boyle's Law.—If a gas is compressed at constant temperature to half its original volume, the number of molecules per cubic centimetre is doubled, *i.e.* the density of the gas is doubled, and so the number of molecules striking against a wall per unit area per second, *i.e.* the rate of striking against the wall per unit area is doubled. So though the *K.E.* per molecule remains constant (the temperature remaining the same) still the pressure is doubled due to the rate of striking being doubled. Thus the product of pressure and volume remains constant at constant temperature. This is Boyle's law.

Pressure-Temperature Law.—When a gas is heated at constant volume, the heat energy given increases the $K.E.$ of the molecules which is, at all stages of heating, proportional to the absolute temperature T according to the Kinetic theory.

$$\text{Now (pressure} \times \text{volume)} \propto K.E.$$
$$\propto T.$$

$\therefore$ At constant volume, pressure $P \propto T$.

Change of State.—During the *change of state* of a substance from the solid to the liquid or from the liquid to the gaseous state, the temperature does not rise. How to account for the latent heat then? The heat supplied in the form of latent heat is utilised in further separating the molecules from one another against their forces of attraction without increasing their velocity.

64. What is a Perfect Gas? Is there any Gas which is Perfect?

A gas is said to be perfect or ideal, if the assumptions of the Kinetic theory of gases (*vide* Art. 60) strictly apply to its case. This is the same thing as to say that such a gas should strictly obey the equation of state, $PV = RT$, for this equation can be deduced under those assumptions. The equation combines in itself the *Boyle's law*, *Charles' law* and the *Pressure law*. A gas which strictly obeys the above laws, therefore, can be called a perfect gas. Such a gas should also follow *Joule's law** which is only a consequence of the Kinetic theory of gases. Such a gas cannot have any viscosity and should remain gaseous down to the absolute zero.

As a matter of fact, no real gas exists which strictly can be called a perfect gas Some of the gases like hydrogen, oxygen, nitrogen, air, etc. which were formerly described by Faraday as *permanent gases*, have been found to obey the above gas laws approximately under definite conditions only. For instance, under moderate pressures and at temperatures which are not low, they conform to Boyle's law and under those limited conditions they may be regarded as perfect gases and not under other conditions. For ordinary purposes, however, these gases are always referred to as perfect gases

65. Isothermal and Adiabatic Changes;—

Isothermal Changes.—Physical changes, *e.g* changes in pressure, volume, etc. brought about in a substance at constant temperature, are called *isothermal changes*. Thus the changes in pressure and volume of a gas in a Boyle's law [illegible] -rmal compression or expansion [illegible] etc. known as permanent gas [illegible] the pressure (P) vs. volume (V) graph of such a gas at a constant tempera-

* **Joule's Law.**—It states that there should be no fall of temperature of a gas when it expands into vacuum, if it is a perfect gas

ture, which is called an *Isothermal Curve* or simply an *isothermal* of the gas at that temperature, is found to be a *rectangular hyperbola* within moderate ranges of temperature and pressure (Fig. 186, Part I). That is, PV=a constant at a constant temperature.

Next, let us see what conditions are to be fulfilled for isothermal changes to be produced in a gas. Consider a gas kept in a cylinder closed by a movable piston. If the gas is compressed by pushing the piston inwards, heat will be generated equivalent to the work done on the gas; whereas, if the compressed gas is allowed to expand pushing the piston outwards, the gas will be cooled, *i.e.* heat will be used up, corresponding to the work done by the gas against external pressure. So, to maintain the temperature constant, heat is to be taken out from the gas, in the case of compression, at the rate at which it is produced, and supplied to the gas, in the case of expansion, at a rate equivalent to the work done by the gas. If the cylinder is made of the best possible conductor of heat, the rejection or absorption of heat by the gas becomes easy, if it is placed in contact with a medium of large thermal capacity. So in practice a metallic cylinder is used and the same is placed in a current of air or water, for constancy of temperature, when isothermal changes in pressure and volume take place within the gas contained in the cylinder; if the changes take place slowly, the substance gets sufficient time either to gain or lose heat, as the case may be, and the temperature remains unaltered. *So slow changes are often referred to as isothermal changes.*

Adiabatic Changes.—A physical change in a substance is said to be *adiabatic* when the substance is acted on in such a way that it neither gives out heat to, nor takes heat from, any body external to it. That is, in an adiabatic change physical changes take place without loss or gain of heat *as heat*. So in adiabatic changes the working substance requires to be kept in perfect thermal isolation from external bodies by covering the container with perfectly *non-conducting* materials; such processes are often called the processes of *lagging* in technical language; moreover, if the physical changes are sudden or rapid, chances of exchange of heat will be further reduced. That is why, *sudden changes* are often regarded as *adiabatic.*

In *adiabatic compression*, a gas is rapidly *heated up*, because the heat produced due to the work done on the gas remains lodged within the gas itself; while in the case of an *adiabatic expansion*, the gas becomes rapidly *cooled down*, for the energy equivalent to the work done by the gas is drawn from the gas itself. The relations between pressure P, volume V, and temperature T in adiabatic changes in the case of a perfect gas are as follows :—

The relation between pressure and volume is $PV^{\gamma}=K_1$, a constant. The relation between volume and temperature is $VT^{\gamma-1}=K_2$, a con-

(4) *The mass of one litre of air at 0°C. is 1·293 gms. when the pressure is $1·013 \times 10^6$ dynes per sq. cm. Find the value of K in the equation PV=KT.*

The vol. of 1·293 gms. of air is 1000 c.c. So vol. of 1 gm. is 1000/1·293 c.c.

So, we have, $1·013 \times 10^6 \times \frac{1000}{1·293} = K \times 273$ [$\because$ 0°C.=273°C. Absolute.] ;

or, $K = \frac{1·013 \times 10^6 \times 1000}{273 \times 1·293} = 2·87 \times 10^6$ ergs. per degree centigrade per gm.

[*vide* Art. 58(*b*)]

(5) *Determine the height of the barometer when a milligram of air at 30°C. occupies a volume of 20 c.c. in a tube over a trough of mercury, the mercury standing 730 mm. higher inside the tube than in the trough. (1 c.c. of dry air at N.T.P. weighs 0·001293 gm.).*

The wt. of 20 c.c. of air at 30°C.=1 mgm.=0·001 gm. and so,

wt. of 1 c.c.=0·001/20 gm.

If P be the pressure of the enclosed air,

$$\frac{P}{\frac{0·001}{20} \times (273+30)} = \frac{760}{0·001293 \times 273} \text{ ; whence } P=32·6 \text{ mm.}$$

$\therefore$ The height of the barometer=730+32·6=762·6 mm.

(6) *When the temperatures of the air is 32°C. and the barometer stands at 755 mm. the apparent mass of a piece of silver when counterpoised by brass weights in a delicate balance is found to be 25 gms. What is the actual mass ? The density of silver is 10·5 and that of brass 8·4, both at 32°C.*

Let m gm. be the true mass of the silver, then its volume is m/10·5 c.c. which is also the volume of air displaced by it.

This volume reduced to N.T.P. becomes= $\frac{m}{10·5} \times \frac{273}{(273+32)} \times \frac{755}{760}$.

But the mass of 1 c.c. of dry air at N.T.P.=0·001293 gm.

$\therefore$ The mass of the above volume of air

$$= 0·001293 \times \frac{m}{10·5} \times \frac{273}{(273+32)} \times \frac{755}{760} = 0·0011497 \times \frac{m}{10·4}.$$

Hence the apparent mass of silver in air

$$= m - \frac{0·0011497\,m}{10·5} = m\left(1 - \frac{0·0011497}{10·5}\right) \text{gm.} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

The volume of brass weights is 25/8·4 c.c. and the mass of air displaced by the weights=0·0011497×25/8·4 gm.

$\therefore$ The apparent mass of the brass weights in air

$$= 25 \left\{1 - \frac{0·0011497}{8·4}\right\} \text{ gm.} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

Since the apparent weights of silver and brass are in equilibrium, we have from

(1) and (2), $m \left\{1 - \frac{0·0011497}{10·5}\right\} = 25 \left\{1 - \frac{0·0011497}{8·4}\right\}$;

or, m=24·9934 gm.

(7) *A litre of air at 0°C and under atmospheric pressure weighs 1·2 gms. Find the mass of the air required to produce at −18°C. a pressure of 3 atmospheres in a volume of 75 c.c.* (*Pat. 1924*)

Let P be the atmospheric pressure. Then the pressure on the mass of the air=3P, and the absolute temperature T=273−18=255°.

Then from the formula $\frac{PV}{T}=\frac{P'V'}{T'}$, we have $\frac{P\times V}{273}=\frac{3P\times 75}{255}$, where V is the volume of the mass of air at $0°C$, and at atmospheric pressure P, whence $V=240.88$ c.c.

But the mass of 1 litre or 1000 c.c. of air at $0°C$. and atmospheric pressure is 1.2 gm. ∴ The mass of 240.88 c.c. of air $=\frac{240.88\times 1.2}{1000}=0.289$ gm.

Questions

1. State concisely the relations between the volume, pressure and temperature of a gas. (C. U. 1912, '50 , C U. 1950)

Describe an experiment to prove the relation between pressure and temperature when the volume is kept constant. (C. U. 1912)

2. [illegible] of mercury = 13.6.

[*Ans.* £171 5s]

3. A flask which contains 250 c.c. of air at atmospheric pressure is heated to 100°C., and then corked up. If it is afterwards immersed mouth downwards in a vessel of water at 10°C, and the cork removed, what volume of water will enter the flask, if the final pressure is atmospheric?

[*Ans.* 60.4 c.c.]

4. A flask containing dry air is corked up at 20°C, the pressure being one atmosphere. Calculate the temperature at which the cork would be blown out if this occurs when the pressure inside the flask is 1.7 atmospheres. (R. U. 1955)

[*Ans.* 225.1°C]

5. Describe an experiment to find the coefficient of expansion of a gas at constant pressure. (C. U. 1929, '35, '40, '51 , Pat 1952)

6. Describe the constant volume air thermometer and explain how you will use it to find the melting point of wax.

(All. 1927 , *cf.* 1920, Pat. 1936 , R. U. 1944, '46)

7. The pressure in a constant volume air thermometer is 770 mm. at 15°C. What will it be at 20°C ?

[*Ans.* 783 mm.]

8. Prove that for a perfect gas the volume and pressure coefficients are equal. (Nag U. 1953 ; Rajputana, 1919 , Del 1952 , U. P. B. 1947)

9. Explain how the thermal expansion of air can be utilised as a convenient means of measuring temperature. (All. 1918, '24)

10. What is the temperature of a gas whose pressure is 136.63 cm. of Hg. if its pressure at 0°C is 100 cms. of Hg. the volume remaining constant. Given the pressure co-efficient of the gas = 0.003663. (Vis. U. 1951)

[*Ans.* 100°C.]

11. A uniform vertical glass tube open at the top and closed at the bottom contains air and a pellet of mercury 3.0 cm. long. The lower end of the pellet is 30.5 cm. above the bottom of the tube when the tube is at a temperature of 5°C. [illegible] will the pellet rise, if the tube is heated to 100°C.?

(Utkal, 1952, '54)

[illegible] 40.83 cm.]

12. What is meant by 'absolute temperature'? Find the value of the absolute zero on the Fahrenheit scale. (Pat. 1928; G. U. 1951; C. U. 1938, 49)
[*Ans.* $-459 \cdot 4°F.$]

13. Why is it necessary to take account of the pressure of a gas in determining its coefficient of cubical expansion?

200 c.c. of air at 15°C. is raised to 65°C. Find the new volume, the pressure remaining unchanged. (C. U. 1915)

[*Ans.* 234·7 c.c.]

14. A gas at 13°C. has its temperature raised so that its volume is doubled, the pressure remaining constant. What is the final temperature? (Dac. 1933)

[*Ans.* 299°C.]

15. Find the percentage increase of pressure in the tyres of a bicycle taken out of the shade (59°F.) into the sun (95°F.) disregarding the expansion of the rubber.

[*Ans.* 7%]

16. At what temperature would the volume of a gas initially at 0°C., be doubled, if the pressure at the same time increases from that of 700 to 800 millimetres of mercury? (C. U. 1916)

[*Ans.* $t=351°C.$]

17. What volume does a gram of carbonic acid gas occupy at a temperature of 77°C., and half the standard pressure? (1 c.c. of carbonic acid weighs 0·0019 gram. at 0°C., and standard pressure.) (C. U. 1912; *cf.* 1918, '33)

[*Ans.* 1349 c.c. nearly.]

18. At 22°C. and pressure of 74 cm. the volume of a given mass of gas was found to be 54·02 c.c. On cooling to 0°C., the volume became 49·3 c.c., the pressure having risen to 75 cm. Find the coefficient of expansion of the gas.

[*Ans.* 0·00368/°C.]

19. State how the volume of a gas changes when its temperature and pressure both change. (Dac. 1921, '33)

20. Air is collected in the closed arm of a Boyle's tube and the volume found to be 32 c.c. the temperature being 17°C., and the height of the barometer 753 mm. while the mercury stands at 3·5 cms. higher in the closed arm than in the open one. What would be the volume of the air at 0°C., and 760 mm. pressure?

[*Ans.* 29·7 c.c.]

21. A quantity of gass collected over mercury in a graduated tube is found to occupy 25 c.c. at 27°C. The level of the mercury inside stands 15 cm. higher than the level outside while the barometer stands at 75 cm. Find the volume that the mass of the gas would occupy at a pressure of 74·5 cm. of mercury and at a temperature of 32°C.

[*Ans.* 20·5 c.c. approx.]

22. Establish the relation $PV=RT$ for a gas.

(Mysore, 1952; East Punjab, 1953; C. U. 1950; U. P. B. 1951; M. B. B. 1952)

Given that one litre of hydrogen at N.T.P. weighs 0·0896 gm., calculate the value of R for a gramme of the gas. (C. U. 1938; R. U. 1959)

Write down the value of the gas constant. (G. U. 1950; R. U. 1945)

[*Ans.* $4 \cdot 15 \times 10^7$ ergs/°C.; $8 \cdot 3 \times 10^7$ ergs/°C.]

23. Assuming the perfect gas equation to hold for carbon dioxide, calculate its gas constant R, given that 22·4 litres of CO_2 weighs 44 gms. at N.T.P.

[For 1 gm. of CO_2] (Rajputana, 1945; U. P. B. 1947)

[*Ans.* $1 \cdot 88 \times 10^6$ ergs/°C.]

24. The mass of 1 c.c. of hydrogen at 0°C. and 760 mm. pressure is 0·0000896 gm. per c.c. What will be its mass per c.c. at 20°C. and 750 mm.?
[Ans. 0·0000835 gm./c.c.]

25. A litre of hydrogen at N.T.P. weighs 0·9 gm. What is the weight of a litre of this gas at 27°C. and 75 cm. pressure? (East Punjab, 1952)
[Ans. 0·8 gm.]

26. Compare the density of air at 10°C. and 750 mm. pressure with its density at 15°C. and 760 mm. pressure.
[Ans. 54 : 53·77]

27. On a certain day the barometer reads 76 cm. and temperature is 10°C. On being taken to the bottom of a mine shaft, where the temperature is 27°C., the barometer reading increases by 4 cm. Find the ratio of the density of the air at the bottom of the shaft to that of air on the ground level.
[Ans. 0·993 : 1]

28. A flask is filled with 5 gms. of gas at 12°C. and then heated to 50°C. Owing to the escape of some of the gas, the pressure in the flask is the same at the beginning and end of the experiment. Find what weight of the gas has escaped.
[Ans. 0·67 gm.]

29. Write notes on the molecular motion in gases. (C. U. 1919)

30. How do you account for the pressure of a gas in a closed space and on what factors does it depend? (Pat. 1932)

31. Differentiate between Isothermal and Adiabatic changes with the help of simple illustrations.

Obtain a relation between the isothermal and adiabatic elasticities of a perfect gas. (R. U. 1947)

(Vide Art. 24, Part III)

CHAPTER V

CALORIMETRY

66. Quantity of Heat :— If we take 10 gms. of water and raise the temperature from 10°C. to 20°C., then the quantity of heat required for this purpose will raise the temperature of 1 gm. of water through 100°C., or 100 gms. of water through 1°C.

From this we find that the quantity of heat required to raise the temperature of a substance through a given range depends on (1) its *mass*, (2) *range of temperature*, i.e. on the number of degrees through which it is heated, and, we shall see later on, it also depends on (3) the *nature* of the substance.

67. Calorimetry and Calorimeters :— Calorimetry means the science of measurement of quantity of heat. It has come from the word *Calorie* which is a popular unit for quantitative measurement of that. The vessels in which the measurement of quantities of heat

is carried out are called *Calorimeters.* These vessels are generally made of copper. Vessels of different sizes and shapes and made of special materials are also available. Every calorimeter is provided with a *stirrer* made of the same material. The stirrer is generally taken in the form of a wire ending in a loop which is placed in the liquid used in the calorimeter and moved up and down.

68. Units of Heat :—

(a) Calorie.—It is the *C.G.S.* unit for heat and is the amount of heat required to raise the temperature of one gramme of pure water through 1°*C.* This unit is called a *calorie* or *gram-degree Centigrade Unit.* This amount is a quantity which can be added, subtracted, multiplied or divided, just like any scalar quantity.

[It is experimentally found that the quantity of heat required to raise the temperature of 1 gm. of water through 1°*C.* varies at different parts of the temperature scale, though the variation is small. So, the size of the calorie, according to the above definition, is liable to vary. This has given rise to various definitions of a calorie, each of which seeks to fix up the size of the calorie in its own way. We must at least take note of two of these *calories.* One of them is known as the 15°*C.* calorie. It is equal to the heat required to raise 1 gm. of water from 14·5°*C.* to 15·5°*C.* In accurate calculations, the scientists prefer this unit to the other unit which is called the **mean calorie.** The *mean calorie* is equal to the heat required to raise 1 gm. of water from 0°*C.* to 100°*C.* divided by 100. For ordinary calculations it is this *mean calorie* which is commonly used. Unless great accuracy is required, one need not distinguish between the different values of *calorie* over the range 0°*C.* to 100°*C.*]

(b) The **British Thermal Unit (B.Th.U.,** more recently, **B.t.u.)** or *Pound-Degree Fahrenheit Unit is the amount of heat required to raise the temperature of 1 pound of water through 1°F.* It is also expressed as *B.Th.U.*

1 Therm. $\doteq$ 100,000 B.Th.U.

(c) The Centigrade Heat Unit (C.H.U.) is the amount of heat required to raise the temperature of one pound of water through 1°*C.* It is a mixed unit and is largely used in Engineering.

69. Relations between the Units of Heat :—

1 lb. of water = 453·6 gms. of water ; and $1°F. = \frac{5}{9}C.°$

$\therefore$ 1 B.Th.U. $= 453{\cdot}6 \times \frac{5}{9} = 252$ calories.

Thus to convert from calories to B.Th.U., multiply the calories by 1/252 ; and to convert from B.Th.U. to calories, multiply the B.Th.U.'s by 252.

Again 1 Centigrade degree is $\frac{9}{5}$ of a Fahrenheit degree ; so the **Pound-Degree Centigrade Unit** $= \frac{9}{5}$ or 1·8 **B.Th.U.** ; and since 1 pound = 453·6 gms., we have,

One Pound-Degree Centigrade Unit (C.H.U.) $= 252 \times \frac{9}{5} = 453{\cdot}6$ calories.

70. Principle of Measurement of Heat :—Take two beakers of the same size. Into one of them put 50 c.c. of water (mass = 50 gms.) at 40°*C*., and in the other 50 c.c. of ice-cold water. Now quickly mix the contents of the two beakers. It will be found that the final temperature of the mixture is midway between 40°*C*. and 0°*C*. e.g. 20°*C*.

Again, if 100 gms. of water at 60°*C*. is mixed with 100 gms. of water at 20°*C*., the resulting temperature of the mixture will be 40°*C*.

In this experiment we assume that (*a*) the quantity of heat gained or lost by one gramme of water taken at any temperature for a change of 1°*C*. is constant, *i.e.* it is the same whether the temperature changes from, say, 30° to 31°, 80° to 81° or 55° to 56° ; (*b*) the exchange of heat takes place between the two quantities of water without any loss or gain of heat from any other causes

In other words, the heat lost by 50 gms. of warm water, is equal to the heat gained by 50 gms. of cold water, or again the heat lost by 100 gms. of water in cooling through 20°*C*. (from 60° to 40°) has raised the temperature of 100 gms of water through 20° (from 20° to 40°). This is the main principle of the measurement of heat, *i.e.*

heat lost = heat gained.

$50(40-t)=50(t-0)$, or, $2000=100t$; or, $t=20°C$

[Note.—If two masses m_1 and m_2 are added, the resultant mass, $m=m_1+m_2$ and if two quantities of heat Q_1 and Q_2 are added, the resultant quantity, $Q=Q_1+Q_2$, but *temperatures do not follow the addition law*, viz., if two bodies at temperatures θ_1 and θ_2 are mixed up, the resultant temperature θ of the mixture is not equal to $\theta_1+\theta_2$.]

71. Specific Heat :—We have seen that by mixing 100 grams of water at 60°*C*. with 100 grams of water at 20°*C*., the resulting temperature of the mixture becomes 40°*C*. But if 100 grams of water at 60°*C*. are mixed with 100 grams of turpentine at 20°*C*., the resulting temperature of the mixture will be about 48°*C*. Thus, the heat given out by the water in cooling through 13°C. is sufficient to raise the temperature of turpentine through 28°*C*. If any other liquid is taken, the result will be different. Again, if equal masses of different metals are heated to the same temperature, and then each of them is separately dropped into a beaker containing water at the room temperature, the mass of water in each beaker being the same, it will be found that

absorb different amounts of heat when heated through the same range of temperature.

Expt.—Place a number of balls of different metals, say lead, tin, brass, copper, iron, and of the same mass, say m gms., in a vessel of boiling water. After a few minutes remove the balls and place them on a thick slab of paraffin. The balls will melt the paraffin, but not to the same amount (Fig. 36). The ball which absorbed the greatest heat will of course, sink farthest into the paraffin.

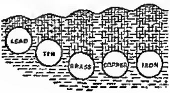


Fig. 36

Since the mass m and the rise of temperature t are the same in each case, there is some *specific property* of the substances on which the quantity of heat taken up by each of them depended. The *specific heat* of a substance refers to this *specific property* of the substance.

The heat H required to raise the temperature of m gms. of water through $t°C.=mt$ calories.

The heat required to raise m grams of mercury through the same range of temperature ($t°C.$) is much less than mt calories.

If H' denotes this amount of heat, we have, $H' \propto mt$; or $H'=s \times mt$, where s depends upon the specific property of mercury.

72. Definition of Specific Heat :—Specific heat is defined in different books in either of the following two ways. The modern view is for accepting the second definition.

(*i*) The specific heat *of a substance is given by the ratio of the quantity of heat required to raise any mass of the substance through any range of temperature to the quantity of heat required to raise an equal mass of water through the same range of temperature.*

(*ii*) The **specific heat** *of a substance is the quantity of heat required to raise the temperature of unit mass of it through one degree.*

Note.—

(*i*) According to the first definition, *the specific heat is a mere number involving no unit for it, i.e. in both the C.G.S. and F.P.S. units the value of the specific heat of a substance is the same.* Thus if m be the mass of a substance and s its specific heat,

$$s=\frac{\text{amount of heat reqd. to raise } m \text{ gms. of substance through } t°C.}{\text{amount of heat reqd. to raise } m \text{ gms. of water through } t°C.}$$

Similarly, in British units

$$s=\frac{\text{amount of heat reqd. to raise } m \text{ lbs. of substance through } t°F.}{\text{amount of heat reqd. to raise } m \text{ lbs. of water through } t°F.}$$

But the amount of heat required to raise m grams of water through $t°C.$ is mt calories.

Therefore, the amount of heat required to raise m grams of a substance through $t°C. = m \times s \times t$ calories.

Similarly, the amount of heat required to raise m pounds of a substance through $t°F. = m \times s \times t$ B.Th.U.'s.

Thus, the amount of heat required to raise the temperature of a body = **Mass × Sp. heat × Rise of temperature (Calories, or B.Th.U.'s).**

Example.—

(i) The specific heat of iron is 0·11. This means that 0·11 calorie will raise the temperature of 1 gm. of iron through $1°C.$, or that 0·11 B.Th.U. will raise the temperature of 1 lb. of iron through $1°F.$ or that 0·11 pound-degree centigrade unit of heat will raise the temperature of 1 lb. of iron through $1°C.$ Similarly, 1 gm. of iron cooling through $1°C.$ will give out 0·11 calorie of heat.

(ii) According to the second definition, the specific heat is not a number, but a quantity of heat, *i.e.* it is expressible in some unit. In the C.G.S. system, its unit is cals. per gm. per $°C.$, whereas in the F.P.S. unit, its unit is B.Th.U.'s per lb. per $°F.$ Thus, the quantity of heat required to raise the temperature of body = mass × sp. heat × rise of temperature **(Calories, or B.Th.U.'s).**

73. Thermal Capacity :—The thermal capacity of a body is the quantity of heat required to raise the temperature of the body through 1°.

If m be the mass of the body and s its specific heat, the thermal capacity of the body $= ms$ units of heat. In the C.G.S. system where m is in gms. and temp. is in $°C.$, the thermal capacity $= ms$ calories.

The specific heat of a substance gives the thermal capacity of a body per unit mass.

Example. *The densities of two substances are as 2 : 3, and their specific heats are 0·12 and 0·09 respectively. Compare their thermal capacities per unit volume.*

(C. U. 1929, '31)

Let the densities of the two substances be $2x$ and $3x$ respectively. Therefore, the mass per unit volume of the first substance is $2x$ gms., and that of the other is $3x$ gms. Hence the thermal capacity per unit volume of the first substance $= 2x \times 0·12$, and that of the second substance $= 3x \times 0·09$.

$$\therefore \frac{\text{Thermal capacity of the first substance}}{\text{Thermal capacity of the second substance}} = \frac{2x \times 0·12}{3x \times 0·09} = \frac{8}{9}.$$

74. Water Equivalent :—*The water equivalent of a body* is the mass of water which will be heated through 1° by the amount of heat required to raise the temperature of the body through 1°.

If m gms. be the mass of a body and s its specific heat, the amount of heat required to raise the temperature of the body throuh $1°C. = ms$

calories. This amount of heat will raise ms grams of water through $1°C$.

$\therefore$ Water equivalent of the body $= ms$ grams.

So thermal capacity of a body is numerically equal to its water equivalent.

75. Determination of the Water Equivalent of a Calorimeter :—Dry the calorimeter and weigh it along with a stirrer of the same material. Fill the calorimeter to about one-third with cold water, note its temperature and weigh it again, and thus get the weight of water taken. To this add quickly about an equal quantity of hot water after correctly noting its temperature. The temperature of this water should not be very high, otherwise the loss of heat due to radiation etc. (which has not been considered in the following calculation) shall have to be accounted for. Now stir the mixture and note the final temperature. When cold, weigh the calorimeter again to get the weight of water added.

Let mass of cold water $= m$ gms. ; mass of hot water $= m'$ gms. ; temperature of cold water $= t_1°C$. ; temperature of hot water $= t_2°C$. ; common temperature of the mixture $= t°C$. ; water equivalent of the calorimeter and stirrer $= W$ gms.

Heat lost by m' gms. of hot water in cooling through $(t_2-t)°C$. $= m'(t_2-t)$ calories. Heat gained by m gms. of water in rising through $(t-t_1)°C. = m(t-t_1)$ calories.

Heat gained by calorimeter and stirrer in rising through $(t-t_1)°C$. $= W(t-t_1)$ calories. Now, we have,

total heat lost = total heat gained,

i.e. $m'(t_2-t) = W(t-t_1) + m(t-t_1)$

$$\therefore \quad W = \frac{m'(t_2-t)}{(t-t_1)} - m.$$

Errors and Precautions.—Heat may be lost by the hot water when being poured into the calorimeter, and moreover, the hot mixture will lose some heat through radiation. Due to both the accounts, the final temperature will be too small. Again, unless the temperature of the mixture is small, the loss of water by evaporation will be appreciable.

The loss of heat by radiation from the mixture may be eliminated by adopting **Rumford's Method of Compensation.** In this method, the initial temperature of the water is taken as many degrees below that of the atmosphere (by addition of ice-cold water) as the final temperature of the water after mixture will be above that of the atmosphere. So, the heat lost by radiation from the calorimeter after mixture will be exactly compensated for by the gain of an equal quantity of heat by the calorimeter and its contents before mixture.

room temperature as the final temperature would be above (Rumford's Method of Compensation, Art. 75) the temperature of the room.

So, the loss of heat by the calorimeter during the second half of the expt. is compensated for, by an equal gain in the first half.

The outer and inner surfaces of the calorimeter are very often polished by which the loss of heat by radiation is reduced to some extent.

(2) Some heat is lost in transferring the hot solid from the steam-heater to the calorimeter; so an arrangement is made for dropping the hot solid directly into the calorimeter by bringing it under the steam-heater.

Some heat is also lost in heating the thermometer.

(3) The water equivalent of the calorimeter and stirrer should be taken into account in calculating the amount of the heat gained.

(4) The thermometer should be very sensitive, say graduated to $\frac{1}{10}$th or $\frac{1}{5}$th of a degree centigrade.

(5) *The change of temperature of the water in the calorimeter should be observed very accurately*, as the accuracy of the result depends more on the accuracy with which the change of temperature of the water in the calorimeter is noted, and not so much on the accuracy in weighing.

(6) The thermometer used in the steam-heater should be corrected for the boiling point.

Examples. *(1) A piece of lead at 99 C. is placed in a calorimeter containing 200 gms of water at 15 C. The temperature after stirring is 21 C. The calorimeter weighs 40 gms and is made of a material of specific heat 0·01. Calculate the thermal capacity of the piece of lead.*

Let C be the thermal capacity of the piece of lead.

Heat lost by the lead piece $= C(99-21)$ cal.

Heat gained by calorimeter and water $= 40\times 0{\cdot}01\times(21-15)+200(21-15)$ cal.

Heat lost = heat gained. Therefore,

$C(99-21)=(40\times 0{\cdot}01+200)(21-15)=200{\cdot}4\times 6$, whence $C=15{\cdot}4$ calories.

(2) An alloy consists of 60% copper and 40% nickel. A piece of the alloy weighing 50 gms. is dropped into a calorimeter whose water equivalent is 5 gms. The calorimeter contains 55 gms of water at 10 C. If the final temperature is 20 C., calculate the original temperature of the alloy. [Sp. ht. of copper = 0·095, sp. ht. of nickel = 0·11]

The mass of copper in the alloy $=\frac{60}{100}\times 50=30$ gms., and

the mass of nickel in the alloy $=\frac{40}{100}\times 50=20$ gms.

Let t°C be the original temperature of the alloy, then heat lost by copper $=30\times 0{\cdot}095\times(t-20)$ cal., heat lost by nickel $=20\times 0{\cdot}11\times(t-20)$ cal., heat gained by water $=55\times(20-10)$ cal.

Since, heat lost = heat gained,

$(t-20)\{(30\times 0{\cdot}095)+(20\times 0{\cdot}11)\}=(20-10)(55+5)$, whence $t=138{\cdot}8$ C.

(3) *Equal volumes of mercury and glass have the same capacity for heat. Calculate the specific heat of a piece of glass of specific gravity 2·5, if the specific heat of mercury is 0·0333 and the specific gravity, 13·6.* (*Pat. 1922*)

Let the volume of the piece of glass $= V$ c.c., then its mass $= V \times 2{\cdot}5$ gms. and the mass of V c.c. of mercury $= V \times 13{\cdot}6$ gms.

Capacity for heat of V c.c. of glass $(H_1) = V \times 2{\cdot}5 \times s$ (where s is the sp. ht. of glass). Capacity for heat of V c.c. of mercury $(H_2) = V \times 13{\cdot}6 \times 0{\cdot}0333$.

We have, $\quad H_1 = H_2$.

$$\therefore \quad V \times 2{\cdot}5 \times s = V \times 13{\cdot}6 \times 0{\cdot}0333\ ; \quad \therefore \quad s = \frac{13{\cdot}6 \times 0{\cdot}0333}{2{\cdot}5} = 0{\cdot}181.$$

77. Measurement of High Temperature by Calorimetric Method :—In principle the method is the same as the method of mixtures as explained in Art. 76.

A solid of known mass and sp. heat, preferably a good conductor of heat such as a metal, *whose melting point (vide Art. 95), is much greater than the temperature under measurement*, is placed in contact with the source of high temperature. After an interval of time when the solid has attained the constant temperature of the bath, it is taken out and immediately dropped into a calorimeter containing sufficient water to cover the solid, and the rise of temperature of the water is determined with a sensitive thermometer.

Let the mass of water taken	$= m$
,, ,, ,, ,, solid ,,	$= w$
Water eq. of calorimeter and stirrer	$= W$
Specific heat of the solid	$= s$
Initial and final temperatures of water	$= t_1, t_2$
Unknown temperature of the bath	$= t.$

We have $w.s(t - t_2) = (m.1 + W)(t_2 - t_1)$, whence t can be calculated.

Example. *In order to determine the temperature of a furnace, a platinum ball weighing 80 gms. is introduced into it. When it acquired the temperature of the furnace, it is transferred quickly to a vessel of water at 15°C. The temperature rises to 20°C. If the weight of water together with the water equivalent of the calorimeter be 400 gms., what is the temperature of the furnace ? (Specific heat of platinum = 0·0365.)*

Let t°C. be the temperature of the furnace. The heat lost by the platinum ball in falling from t°C. to 20°C. $= 80 \times 0{\cdot}0365 \times (t - 20)$ cal.
and heat gained by calorimeter and water $= 400(20 - 15)$ cal.

$\therefore \quad 80 \times 0{\cdot}0365 \times (t - 20) = 400(20 - 15)$; whence $t = 705$°C. (nearly).

78. Heating (or Calorific) Values of Fuels :—"The heating or calorific value of a sample of coal is 12,000 B.Th.U. per pound"—simply means that the heat given by the complete combustion of one pound of coal of that particular sample is 12,000 B.Th.U. The heating value of any other fuel—solid, liquid, or gas—can be similarly expressed.

For accurate determinations of calorific values of fuels, special fuel calorimeters such gas the Bomb calorimeter, Bunsen's gas calorimeter, Junker gas calorimeter, etc. have been devised.

Now, mix the third liquid and let T_1 be the final temperature which is greater than T but less than t_3, then we have $ms_1(T_1-T)+ms_2(T_1-T)=ms_3\times(t_3-T_1)$.

$$\text{or, } T_1(s_1+s_2+s_3)=s_3t_3+T(s_1+s_2)=s_3t_3+\frac{s_1t_1+s_2t_2}{(s_1+s_2)}\times(s_1+s_2) \qquad \text{from (1)}$$

$$=s_3t_3+s_1t_1+s_2t_2\text{ ; whence } T_1=\frac{(s_1t_1+s_2t_2+s_3t_3)}{s_1+s_2+s_3}.$$

(3) The specific gravity of a certain liquid is 0.8, that of another liquid is 0.5. It is found that the heat capacity of 3 litres of the first is the same as that of 2 litres of the second. Compare their specific heats. *(Pat. 1926)*

Volume of the first liquid $=3000$ c.c. ; mass of the first liquid $=3000\times0.8=2400$ gms. Volume of the second liquid $=2000$ c.c. ; mass of the second liquid $=2000\times0.5=1000$ gms.

Heat capacity of the first liquid, $H_1=2400\times s_1$ (where $s_1=$sp. ht. of the first liquid) ; heat capacity of the second liquid, $H_2=1000\times s_2$ (where $s_2=$sp. ht. of the second liquid).

We have, $H_1=H_2$ $\quad 2400\times s_1=1000\times s_2$. $\quad \therefore \dfrac{s_1}{s_2}=\dfrac{1000}{2400}=\dfrac{5}{12}$.

(4) A mixture of 5 kgms. of two liquids A and B is heated to 40°C and then mixed with 6 kgms. of water at 7.67°C. The resultant temperature is 10°C. If the specific heat of A is 0.1212, that of B is 0.0746, find the amount of A and B in the mixture.

Let x be the amount of A, and y, the amount of B, then $x+y=5$ kgms. (1)

Heat lost by x kgms. of $A=x\times1000\times0.1212\times(40-10)$ calories.

Heat lost of y kgms. of $B=y\times1000\times0.0746\times(40-10)$ calories.

Total heat lost by the mixture $=(3.636x+2.238y)\times1000$ calories.

Heat gained by water $=6\times1000\times(10-7.67)=13980$ calories.

Hence, $3636x+2238y=13980$. But $x=5-y$ from (1)

$3636(5-y)+2238y=13980$, from which $y=3.0043$ kgms.

$\therefore x=5-3.0043=1.9957$ kgms.

80. Specific Heat of Gases :—When heat is applied to a gas, the rise of temperature may be accompanied by an increase of pressure, or volume, or both. It may, however, be so arranged that while temperature rises either the pressure or the volume remains constant. In the case of a constant pressure air thermometer (Art. 49), the pressure is kept constant while the volume increases with the rise of temperature. In case of a constant volume thermometer, the volume is kept constant while the pressure increases with the rise of temperature (Art. 50). Therefore when the mass of a gas and the amount of heat taken to raise temperature through a certain range are known, the specific heat of the gas can be calculated at constant pressure, or at constant volume, as the case may be.

The Specific heat of a gas at constant volume (C_v) is the amount of heat required to raise the temperature of unit mass of the gas through 1°, the volume being kept constant.

The Specific heat of a gas at constant pressure (C_p) is the amount of heat required to raise the temperature of unit mass of a gas through 1°, the pressure being kept constant.

81. C_p is greater than C_v :— Suppose 1 gm. of a gas is taken which is to be heated through 1°*C*. A definite quantity of heat will be required for the purpose when the gas is heated only but not allowed to expand, *i.e.* when the *volume* is kept *constant* and the pressure increases. Again, if the gas be heated and allowed to expand at constant pressure, *i.e.* when the *pressure* is kept *constant* and the volume increases, heat is necessary not only to raise the temperature of the gas, but also for the reason that the expanding gas does *some work against the external pressure* while in the first case no such work is done. Thus, at constant pressure, in addition to the heat required to raise the temperature through 1°*C*. at constant volume, some additional heat must be necessary to supply the energy for the work done during expansion against the external pressure. Hence, *the specific heat of a gas at constant pressure (C_p) is greater than the specific heat at constant volume (C_v).* It is found that *the ratio of the specific heat of a gas at constant pressure to that at constant volume, which is ordinarily designated by γ (i.e. $\gamma = C_p/C_v$) is equal to 1·41 in case of di-atomic gases, like oxygen, hydrogen, nitrogen, air, etc., 1·67 for mono-atomic gases, while it is equal to 1·33 for tri-atomic gases.*

N.B. For solids and liquids, C_p and C_v are practically the same, because, on heating, expansion in volume is very small.

82. To show that $C_p - C_v = \dfrac{R}{J}$:—

The specific heat of gas at constant pressure (C_p) is greater than the specific heat at constant volume (C_v) by an amount of heat equivalent to the external work done by unit mass of the gas when it is heated through 1° at constant pressure.

Let us take one gram of a gas at pressure P_0 dynes per sq. cm. in a cylinder fitted with a piston having a cross-section A sq. cms. Then the force on the piston $= P_0.A$ dynes. Suppose the gas is now heated at constant pressure through 1°*C*. due to which the piston moves outwards through a distance x cms. So the work done by expansion

$=$ force $\times$ distance $= P_0.A \times x$ ergs.

Now, the increase in volume of the gas for a rise of 1°*C*. $= A \times x$ c.c. Suppose the volume of 1 gm. of the gas at 0°*C*. and at pressure P_0 is V_0 c.c. Then, $A \times x = V_0/273$, by Charles' law.

Therefore the external work done by 1 gm. of the gas for a rise of 1°*C*. $= P \times P_0 \times A \times x = P_0V_0/273$ ergs. ... ... (1)

But from the gas equation, $P_0V_0 = KT_0$, where T_0 is the absolute temperature corresponding to 0°*C*. and is equal to 273.

$\therefore \quad K = P_0V_0/T_0 = P_0V_0/273$ ... ... (2)

Comparing (1) and (2), it is found that the external work done by 1 gm. of the gas for rise of temperature 1°*C*. is K ergs. or K/J calories. In other words, $C_p - C_v = K/J$.

If C_p and C_v are taken for a gm.-molecule of gas, the gas constant K will be represented by the universal gas constant R, and we have, $C_p - C_v = R/J$.

83. Consequences of high Specific Heat of Water :—From a table of specific heats it will be seen that mercury has a very low specific heat (0 033), which is one of the advantages of using mercury as a thermometric substance, because it will absorb only a very small amount of heat from the temperature bath and so can lower the temperature of the bath only slightly. *Water has a higher specific heat than any other liquid or solid.* So, a larger amount of heat is necessary to raise the temperature of a given weight of water through a certain range than is required by an equal weight of any other substance and that *is* why *water is not suitable as a thermometric liquid. Moreover, its specific heat varies with temperature.*

The sea is heated more slowly than the land by the rays of the sun, the specific heat of sea water being higher than that of land ; so during mid-day, the temperature of the coast will be greater than the temperature of the sea, but after sun-set, the case will be just the reverse, because the sea cools more slowly than the land For example, taking the specific heat of air to be 0 237, it is found that 1 gm. of water in losing one degree of temperature would raise the temperature of 1 0 237 gms (i.e. – 4 2 gms.) of air through one degree. Again, because water is 770 times heavier than air, one cubic foot of water in losing one degree of temperature would increase the temperature of $770 \times 4\ 2$ or 3231 cubic feet of air through one degree From the above consideration it is clear that islands have a more equitable climate owing to the influence of the sea, which prevents the occurrence of extremes of heat and cold, and so the sea is called a **moderator of climate.**

The effect of the difference in the specific heats of sea-water and land manifests itself in the setting-up of convection currents in nature producing land and sea-breezes (*vide* Chapter VIII).

Owing to its sp heat being high, water is preferably used in hot water bottles, foot-warmers and hot water pipes for heating purposes in cold countries. Moreover, it becomes less hot than any other liquid when kept in the sun.

84. Latent Heat :—It is found that when a solid substance fuses, *i.e.* changes from the solid to the liquid state, it absorbs heat *without rise of temperature* Similarly, a liquid during the process of solidification gives out heat *without fall of temperature. The heat absorbed, or given out, per unit mass* (1 gm or 1 lb.) *of a substance during change of state (i.e., say, from the solid to the liquid or from the liquid to the solid state) at the melting point of the substance* (Art 95), *is known as the* **heat of fusion** at the temperature which is a characteristic of the substance.

The word *latent* means hidden; that is, the heat which has got no external manifestation, such as rise of temperature, is called *latent heat*, but **when it raises temperature of the substance, it is called sensible heat.**

So, the **latent heat** of fusion *of a solid may be defined as the quantity of heat required to change unit mass of the substance at its melting point from the solid to the liquid state without change of temperature.* The same quantity of heat is also given out by unit mass of the substance at the same temperature in changing from the liquid state to the solid state without any change of temperature.

Latent Heat of Vaporisation.—Similarly, a liquid at its boiling point absorbs heat in order to be converted into vapour without rise of temperature. This heat is absorbed *only* to bring about the change of state.

The quantity of heat required to convert unit mass of a liquid at its boiling point to the vapour state without change of temperature is called the **latent heat of vaporisation** *of the liquid at that temperature.*

The same amount of heat is also given out per unit mass of the vapour of the liquid during condensation at the same temperature.

It has been found that 536 calories of heat are necessary to change one gram of water at 100°*C.* into steam *without change of temperature.* The same amount of heat is also given out by one gram of steam in condensing to water at 100°*C.*; or, in other words, the value of the latent heat of steam is 536 calories. This value will be 536 C.H.U. per lb. and in B.Th.U., $(536 \times \frac{9}{5}) = 964{\cdot}8$ B.Th.U. per lb.

85. Units of Latent Heat in Different Systems of Measurement:—Thus the amount of heat required to convert 1 gram of ice at 0°*C.* into water at 0°*C.* is called the *latent heat of fusion of ice*, or the *latent heat of water at* 0°*C.*, the value of which is 80 calories per gm. This is also the quantity of heat given out by 1 gram of water at 0°*C.* in transforming to 1 gram of ice at 0°*C.*

If the thermal unit be defined by using 1 lb. and 1°*C.* as units (C.H.U.) and 1 lb. be used as the unit of mass, the latent heat of fusion of ice will also be 80 C.H.U. per lb.

But in pound-degree-Fahrenheit units, the value must be larger in proportion to the ratio of a degree *C.* to a degree *F.*, *i.e.* 9 to 5. Hence the latent heat of ice in *British Thermal Units per lb.* $= (80 \times 9)/5 = 144$.

That is, for latent heats, the value in calories per gram must **be multiplied by $\frac{9}{5}$ to obtain the value in B.Th.U.** per lb.

"*The latent heat of fusion of ice is 80*" means that 80 calories of heat are necessary to convert one gram of ice at 0°*C.* from the solid to the liquid state *without change of temperature.*

Note.—This explains why, in cold countries, the thermometer may stand at 0°C. in winter without any ice being formed on the surface of a pond. The water must lose its latent heat before it can freeze.

86. Reality of Latent Heat :—The reality of latent heat may be shown by mixing 100 grams of water at 80°C. with 100 grams of water at 0°C., when the final temperature of the mixture will be 40°C. But, if 100 grams of water at 80°C., be mixed with 100 grams of ice at 0°C., the final temperature will be 0°C. All the heat given out by the hot water in coming to 0°C. will be used up to convert the ice at 0°C. to water at 0°C. So the final temperature will be 0°C.

Note.—The value of the latent heat of steam is rather high, and this explains why burns from steam are so severe. These burns are more painful than those from boiling water because of the heat given out by the steam in condensing.

87. Determination of the Latent Heat of Fusion of Ice :—Weigh a calorimeter and stirrer (w gms.). Half fill it with warm water at about 5° above the room temperature. Weigh the calorimeter with its contents again, whence the weight of water added is founded (m gms.). Note with a sensitive thermometer the initial temp (t_1°C.) of the water in the calorimeter. A block of ice is broken into small fragments which are washed with clean water and dried by means of blotting paper. Get some of them and drop them into the calorimeter holding them not with finger but with the blotting paper. Stir well until all the ice is melted. Note the lowest temperature attained by the mixture (t_2°C.), which should not exceed 5° below the room temperature. Weigh the calorimeter and its contents again, whence the wt. of ice added is found (M gms.).

The gain of heat takes place in two parts : (a) an amount of heat is necessary to melt the ice at 0°C. to water at 0°C., (b) a further amount of heat is required to raise the ice-cold water to t_2°C.

Heat lost by calorimeter and stirrer $= (ws + m)(t_1 - t_2)$ cals., where s = sp. heat of the material of the calorimeter.

Heat gained by ice in melting and by ice-cold water in rising to t_2°C. $= ML + M \cdot 1 \cdot t_2$ cals. where L = latent heat of fusion of ice.

$$\therefore ML + Mt_2 = (ws + m)(t_1 - t_2), \text{ whence } L = \frac{(ws+m)(t_1 - t_2)}{M} - t_2.$$

Errors and Precautic

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0·1 gm. of water (and ne

0·1 × 80 or 8 calories of heat in the calculation.

(2) The initial temperature of water is taken 5° above the room ... and final temperature 5° below it in order that any

gain of heat from the surroundings by the calorimeter after addition of ice may be exactly compensated for by the loss of heat due to radiation by the calorimeter before addition of ice (Rumford's method of compensation).

(3) The ice, during the process of melting, should be kept below the surface of water, and not allowed to float, otherwise the portion above the water surface will absorb heat from the outside air, instead of from the water in the calorimeter, and the calculations adopted above will not apply. For this, use a wire-gauze stirrer. Care should be taken so that no water particle accompanies the thermometer while removing it.

Examples.—*(1) Find the latent heat of fusion of ice from the following data : Weight of the calorimeter=60 gms. ; wt. of cal.+water=460 gms.*

Temperature of water (before ice is put in)=38°C. ; temperature of mixture=5°C. Weight of calorimeter+ice=618 gms. ; sp. heat of the calorimeter=0·1. *(C. U. 1918)*

Let L be the latent heat of fusion of ice ; mass of water $=(460-60)=400$ gms. and mass of ice $=(618-460)=158$ gms.

Heat lost by calorimeter and water $=60\times0{\cdot}1\times(38-5)+400\times(38-5)$ cal.

Heat required to melt 158 gms. of ice and to raise the temperature of the water formed to 5°C. $=158L+158\,(5-0)$ cal.

$\therefore\quad 158L+158\times5=(38-5)\,(6+400)$; whence $L=79{\cdot}8$ cals. per gm.

(2) A lump of iron weighing 200 gms. at 80°C. is placed in a vessel containing 1000 gms. of water at 0°C. What is the least quantity of ice which has to be added to reduce the temperature of the vessel to 0°C. ? (Sp. ht. of iron=0·112). *(All. 1926)*

Heat lost by iron in cooling to 0°C. $=200\times0{\cdot}112\times80=1792$ cal.

The vessel containing 1000 gms. of water was formerly at 0°C. Now to absorb 1792 calories of heat given out by the lump of iron, the mass of ice required $=1792/80=22{\cdot}4$ gms.

(3) Find the result of mixing equal masses of ice at −10°C. and water at 60°C. *(All. 1916)*

Let m gms. of ice be mixed with m gms. of water ; m gms. of ice in rising to 0°C. from −10°C. will require $m\times0{\cdot}5\times10=5m$ calories (sp. ht. of ice=0·5). Again m gms. of ice at 0°C. in changing to water at 0°C. will require $80m$ calories. But the heat supplied by m gms. of water in cooling from 60°C. to 0°C. is only $60m$ calories. Out of this amount $5m$ calories are required to increase the temperature of ice from −10°C. to 0°C., and the rest, *i.e.* $55m$ calories, can turn only $\frac{55}{80}\,m$ or $\frac{11}{16}\,m$ gms. of ice into water at 0°C. The remaining portion, *i.e.* $\frac{5}{16}\,m$ gms. of ice must remain as such. Thus, the result of the mixture is that $\frac{11}{16}$ parts of ice will be melted into water and $\frac{5}{16}$ parts will remain as ice at 0°C.

(4) What would be the final temperature of the mixture when 5 gms. of ice at −10°C. are mixed up with 20 gms. of water at 30°C. ? The sp. ht. of ice is 0·5. *(C. U. 1926)*

Let the final temperature be t°C. Heat gained by ice in going up to t°C from -10°C $= 5\times 0{\cdot}5\times[0-(-10)]+5L+5(t-0)$.

Heat lost by water $=20(30-t)$ calories. Taking $L=80$ units,

we have, $25+5\times 80+5t=20\times(30-t)$ $\therefore$ $t=7$°C

(5) The specific gravity of ice is 0·917; 10 gms. of a metal at 100°C are immersed in a mixture of ice and water, and the volume of the mixture is found to be reduced by 125 c.mm. without change of temperature. Find the specific heat of the metal. (Pat 1924)

We know that volume varies inversely as density; so when V c.c. of ice is changed into V' c.c. of water, we have,

$$\frac{V}{V'}=\frac{1}{0{\cdot}917}\quad(\because\ 0{\cdot}917=\text{sp. gr. of ice})=1{\cdot}09\text{ c.c.}$$

$\therefore$ 1·09 c.c. of ice becomes 1 c.c. of water (wt. = 1 gm.) at the same temperature or, in other words, 1 gm. of ice in melting is reduced in volume by 0·09 c.c., and this requires 80 calories of heat.

In the example, we have, the heat lost by the metal

$= 10\times s\times(100-0)$ cal $=(1000\,s$ cal.). (s = sp. ht. of the metal).

The reduction in volume of the mixture $=125$ c.mm. $=\frac{125}{1000}=\frac{1}{8}$ c.c.

$\therefore$ The amount of ice melted $=\frac{1}{8}\div 0{\cdot}09=\frac{25}{18}$ gm.

The amount of heat required to melt $\frac{25}{18}$ gm. of ice $=\left(\frac{25}{18}\times 80\right)$ calories.

By the example, we have $1000s=\frac{25\times 80}{18}$. $\therefore$ $s=\frac{25\times 80}{18\times 1000}=0{\cdot}11$

(6) What would be the result of placing $4\frac{1}{2}$ lbs. of copper at 100°C in contact with $1\frac{1}{2}$ lbs. of ice at 0°C? (Sp. ht. of copper = 0·095 and latent heat of fusion of ice = 79). (All. 1918)

$4\frac{1}{2}$ lbs. of copper at 100°C in cooling to 0°C give out $4\frac{1}{2}\times 0{\cdot}095\times 100=42{\cdot}75$ pound-degree °C heat units (C.H.U.).

To melt one pound of ice at 0°C 79 pound-degree °C heat units are required.

$\therefore$ The amount of ice melted by 42·75 heat units $=42{\cdot}75/79=0{\cdot}54$ lb.

Hence the amount of ice remaining unmelted $=1\frac{1}{2}-0{\cdot}54=0{\cdot}96$ lb.

So the result is 0·54 lb. of water at 0°C, and 0·96 lb. of ice at 0°C.

88. High Latent Heat of Water :— The latent heat of water being high, the change from water to ice or from ice to water is a very slow process, and during the time the change takes place, much heat is given out or absorbed. Had the latent heat of water been low, (*a*) the water of the lakes and ponds would have frozen much sooner, thus destroying the lives of aquatic animals living therein. (*b*) Ice-bergs on the mountains would have melted very rapidly on rise of temperature, thus causing disastrous floods in the neighbouring countries. The rise of temperature of a place is delayed by the presence of ice-bergs near it and so the climate of the place is greatly influenced by formations of ice-bergs in the neighbourhood.

89. Ice-Calorimeter :— The fact that a certain quantity of ice in melting always absorbs 80 calories of heat for each gm. of it has

oeen applied in the construction of ice-calorimeters for the determination of specific heats.

Black's Ice-calorimeter.—In the simplest form of an ice-calorimeter as used by Black, a large block of ice is taken, a cavity is formed in it, and a slab of ice is taken to cover the cavity (Fig. 40). The solid (w gms.) of which the specific heat (s) is required, is weighed and heated to a constant temperature (t°C.) in a steam-heater. On removing the slab, the water inside the cavity is soaked dry with a sponge, and the solid is quickly dropped into the cavity and covered by the slab. The solid melts some ice into water until its temperature falls to 0°C. After a few minutes, the water formed in the cavity is removed by a pipette and the mass determined (m gms.).

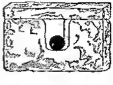


Fig. 40—Black's Ice-calorimeter.

Heat gained by ice in melting to water at 0°C.

$=mL$, where L is the latent heat of fusion of ice. Heat lost by the solid$=w.s.t$

$\therefore \quad mL=w.s.t.$ That is, $L = \dfrac{w.s.t.}{m}$.

The method may also be used to determine the sp. heat (s) of the solid, in which case the value of L is to be assumed.

Note.—Though in this method there is no loss of heat by radiation, still it is not a very accurate method, for

(a) the water formed in the cavity cannot be completely taken out ; and

(b) during the time taken for dropping the solid inside the cavity some ice may melt by absorbing heat from the atmosphere.

Example. *A litre of hot water is poured into a hole in a block of ice at 0°C., which is immediately closed by a lid of ice. After a time the whole is found to contain a litre and a half of ice-cold water. What was the original temperature of the water ?*

Let t°C. be the original temperature.

Mass of hot water=mass of a litre or 1000 c.c. of water=1000 gms.

Mass of ice melted=mass of 500 c.c. water=500 gms.

Heat lost by water=1000 (t−0) cal. Heat required to melt 1 gm. of ice at 0°C. to water at 0°C. is 80 calories.

Hence heat gained by ice=500×80 cal. $\therefore$ 1000t=500×80 ; or, t=40°C.

90. Bunsen's Ice-calorimeter :—1 gm. of ice at 0°C. in melting to water at 0°C. decreases in volume by about 0·09 c.c. Bunsen has utilised this change of volume in the construction of a very delicate calorimeter (Fig. 41). A thin-walled test tube B is fused into a wider tube A, which is attached to a bent tube C, as shown in the

figure. The other end of the bent tube is fitted with a cork D through which passes a fine capillary tube T of uniform bore having a scale S along its horizontal part. The upper part of A is filled with pure and air-free distilled water and the rest of A and the communicating tube C with mercury.

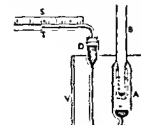


Fig 41—Bunsen's Ice calorimeter

The apparatus is kept in a box, surrounded as completely as possible with melting ice. A mixture of some solid carbon dioxide and ether is placed in B to freeze some of the water in A, forming a sheath of ice round its lower part. Now some amount of water is introduced into B and the calorimeter is allowed to stand for a long time until the whole of it is at 0°C., when the position of the mercury meniscus in T is read on the scale S [illegible]

volume, and the mercury meniscus is found to move towards D. By knowing the area of cross-section (a) of the capillary tube, the specific heat s of the metal can be calculated as follows :—

When the metal has cooled to 0°C., the heat lost by it $= mst$ cal. This amount is sufficient to melt mst/L gm. of ice, where L is the latent heat of fusion of ice

Now, 1·09 c.c. of ice becomes 1 c.c., *i.e.* contracts in volume by 0·09 c.c., when turned into water whose mass is 1 gm

Now L calories of heat will melt 1 gm. of ice into 1 gm. of water at 0°C., *i.e.* will cause a contraction of 0·09 c.c.

$\therefore$ For a contraction of 1 c.c., the amount of heat required $= \dfrac{L}{0{\cdot}09}$ cal. If the mercury meniscus has moved a distance, say, d cm. the decrease in the volume is $a \times d$, and for this, the amount of heat necessary $= \dfrac{a \times d \times L}{0{\cdot}09}$ cal. This amount has been supplied by the metal.

$$\therefore \quad m.s.t. = \frac{a \times d \times L}{0{\cdot}09} \text{ ; or, } s = \frac{a \times d \times L}{0{\cdot}09 \times m \times t}.$$ If s is given, latent heat of fusion of ice can be determined by this method.

Advantages and Disadvantages.—The disadvantage of this method is that it is difficult to set up the apparatus, but it is advantageous for the following reasons :—(*a*) The arrangement is very sensitive ; (*b*) there is no loss of heat due to radiation ; (*c*) no calorimeter or thermometer is necessary ; (*d*) the specific heat of a solid *available in a very small quantity* can be determined by this method.

Examples. *(1) Determine the specific heat of silver from the following data :—*
Weight of silver dropped=0·92 gm. ; Temperature of silver=98°C.
Distance travelled by the mercury thread=6 mm.
Area of cross-section of capillary tube=1 sq. mm.

The diminution in volume of the mercury thread$=0{\cdot}01\times0{\cdot}6=0{\cdot}006$ c.c.

Therefore, from the above relation, we have $s=\dfrac{0{\cdot}006\times80}{0{\cdot}09\times0{\cdot}92\times98}=0{\cdot}0591$.

(2) 20 gms. of water at 15°C. are put into the tube of a Bunsen's ice calorimeter and it is observed that the mercury thread moves through 29 cms. ; 12 gms. of a metal at 100°C. are then placed in the water and the mercury thread moves through 12 cms. Find the specific heat of the metal. *(All. 1920)*

The heat given out by 20 gms. of water at 15°C. in cooling to 0°C.$=20\times15=300$ cal. This produces a movement of 29 cms. of the mercury thread.

$\therefore$ Heat required for movement of 1 cm.$=\dfrac{300}{29}$ cal. and for a movement of 12 cms.$=\dfrac{12\times300}{29}$ cal. This amount has been supplied by the metal, which$=12\times100\times s$, where s is the sp. ht. of the metal.

$$\therefore\quad 12\times100\times s=\frac{300\times12}{29},\ \text{or,}\ s=\frac{3}{29}=0{\cdot}1\ \text{(approx.)}.$$

(3) The diameter of the capillary tube of a Bunsen's ice-calorimeter is 1·4 mm. On dropping into the instrument a piece of metal whose temperature is 100°C. and mass 11·088 gms., the mercury thread is observed to move 10 cms. Calculate the specific heat of the metal ; given the latent heat and density of ice to be 80 and 0·9 respectively. *(All. 1925)*

Mercury thread moves 10 cms. ; hence the volume of the mercury thread $=\pi\times(0{\cdot}07)^2\times10$ c.c.$=0{\cdot}049\pi$ c.c.
Mass of 1 c.c. of ice$=0{\cdot}9$ gm. $\therefore$ The volume of 1 gm. of ice$=1/0{\cdot}9=1{\cdot}11$ c.c.

But the volume of 1 gm. of water$=1$ c.c. $\therefore$ The diminution in volume when 1 gm. of ice is melted, *i.e.* changed into water$=1{\cdot}11-1=0{\cdot}11$ c.c. Hence to produce a diminution of $0{\cdot}049\pi$ c.c., the mass of ice melted$=\dfrac{0{\cdot}049\pi}{0{\cdot}11}$ gm. and the heat required for this$=\dfrac{0{\cdot}049\pi}{0{\cdot}11}\times80$ cal.

This is equal to the heat given out by the metal, which $=11{\cdot}088\times100\times s$ cal.

$$\therefore\quad 11{\cdot}088\times100\times s=\frac{0{\cdot}049\pi}{0{\cdot}11}\times80.\quad\therefore\quad s=\frac{0{\cdot}049\times22\times80}{11{\cdot}088\times100\times0{\cdot}11\times7}=0{\cdot}1.$$

91. Determination of the Latent Heat of Vaporisation of Water :—Take a clean and dry calorimeter (Fig. 42), and weigh it together with a stirrer made of the same material (w gms.). After filling it with water up to about two-thirds, weigh it again whence the mass of water (m gms.) is obtained. The steady temp. (t_1°C.) of the water is taken with a sensitive thermometer T inserted vertically. Boi some water in the boiler B, whose mouth is closed by cork through which a bent delivery tube A passes. The free end of the delivery tube is introduced into a ***steam trap*** which is really a water-separator.

It is a wide glass tube open at both ends which are closed by steam-tight corks. The delivery tube extends well into the trap. Through the cork at the bottom, two tubes pass, one a drain-off tube C for removing the collected water, and the other is an exit, being a straight tube D ending in a nozzle which dips into the water contained in the calorimeter. The screen P protects the calorimeter from direct heating by the boiler.

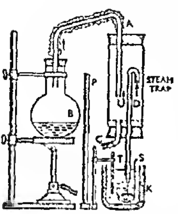


Fig 42

Bring the calorimeter under the exit tube D such that the nozzle goes well into the water in it. After some time take away the nozzle quickly and note the highest temperatur ($t°C$.) attained by the water. Remove the thermometer and allow the calorimeter and its contents to cool. Weigh the calorimeter with its contents again. The difference between the last two weighings gives the mass of steam condensed (M gms.)

Calculation.—Let L be the latent heat of steam and s the sp. heat of the material of the calorimeter. Then,

heat lost by steam in being condensed to water at $t°C$

$=ML+M\,1.\,(100-t)$ calories, assuming the temperature of steam to be $100°C$ and,

heat gained by the calorimeter and its contents in being raised from $t_1°C$. *to* $t°C$.

$$=(w.s.+m)\ (t-t_1).$$

Assuming heat loss equal to heat gain,

$$ML+M(100-t)=(w.s.+m)\ (t-t_1).$$

That is, $$L=\frac{(w.s+m)\ (t-t_1)}{M}-(100-t).$$

Errors and Precautions.—If some part of the steam is condensed before entering into the calorimeter, the value of L will be low. The steam-trap is used in order that any condensed steam may not pass into the calorimeter. Moreover, due to sudden absorption of steam by the cold water in the calorimeter, if any water, from the calorimeter is sucked back, it is arrested by the steam-trap and not allowed to get into the boiler B. As a precaution against condensation of the steam in passing along the delivery tube and the steam-trap, *both the delivery tube and the steam-trap should be carefully lagged with non-conducting materials* like cotton-wool or asbestos.

(2) To reduce the effect of radiation, the water in the calorimeter should be initially cooled a few degrees below the room temperature and steam passed till the temperature rises through the same amount above the room temperature (*cf.* Rumford's method of compensation).

(3) To protect the calorimeter from direct heating, a screen P is to be placed between the boiler and the calorimeter.

(4) The temperature of the water in the calorimeter after mixture should not be allowed to increase by more than 15°*C.*, otherwise much water (and therefore much heat), will be lost by vaporisation.

(5) If the issue of steam is too rapid, some water may be lost by splasing.

(6) The temperature of the steam should be determined in each case and cannot be taken as 100°*C.* without pressure correction.

92. Joly's Steam Calorimeter :—In 1886, Prof. Joly devised a very simple and accurate method of determining the specific heat of a substances with a steam calorimeter by the condensation of steam on the substance. His apparatus (Fig. 43) consists of a metal enclosure, A, called the steam chamber, into which steam is supplied through a tube T near the top, the exit tube K being placed at the bottom. From one arm of a balance a fine vertical wire passes through a small hole H into the steam chamber carrying a small pan P at the lower end. The body B whose specific heat is required is placed on the pan P and its mass M is determined by placing weights on the other pan of the balance. The temperature t of the body, that is, of the air in the chamber, is taken after B is placed for some time inside the chamber A. Then steam is admitted into the chamber which condenses on the body and the pan. The mass m_1 of water condensed on the body and the pan is determined by placing weights on the other pan of the balance to counterpoise them. The final steady temperature t_1 of the chamber is taken after steam is passed for sometime. The body is now taken out and the enclosure is allowed to cool down to room temperature when the pan is dried. Steam is again passed into the chamber when it condenses on the pan only, and the mass m_2 of the condensed steam is also determined as before. Then the mass of steam condensed on the body only is (m_1-m_2).

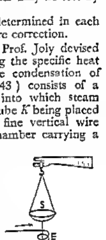


Fig. 43—Joly's Steam Calorimeter.

Now, if s be the specific heat of the body, the heat gained by it$=Ms(t_1-t)$. Heat lost by steam in condensation over the body $=(m_1-m_2)L$, L being the latent heat of steam. Then, we have,

$$Ms(t_1-t)=(m_1-m_2)L, \text{ whence } s=\frac{(m_1-m_2)L}{m(t_1-t)}.$$

It is clear from the above experiment that the latent heat of steam L can also be determined, if s, the specific heat of the body is known.

In order that steam might not condense on the suspending wire the wire is passed along the axis of a small spiral E of a platinum which is heated by passing an electric current through it.

When the *specific heat of a liquid* is required, it is enclosed in a small metal sphere and the experiment is carried out as before. In this case the mass of the sphere and the specific heat of the metal should be known for calculating the specific heat of the liquid contained in the sphere.

For determining *the specific heat of a gas* at constant volume Joly modified his calorimeter by suspending in the same steam chamber two hollow copper spheres of equal size from the opposite arms of the balance One of the spheres was filled with the gas, while the other was exhausted The mass of the gas is found out from the weights placed on the upper pan on the other side and the mass of steam condensed due to the enclosed gas is obtained by the difference in weights of the steam condensed on the two pans after the temperature inside the chamber becomes constant The calculation is made as before

92(A) Experimental Determination of C_v, by Joly's differential Steam Calorimeter :—The apparatus used [Fig. 43(A)] is similar in construction to the Joly's steam calorimeter described in Art. 92 with the difference that here the thermal capacity of the pans PP or catch-waters, as they are called, is eliminated by a differential weighing method

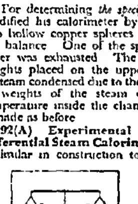


Fig 43(A)

From the balance pans, pp, two pans or catch-waters PP are suspended in this apparatus in a double-walled steam chamber A (to keep the steam dry). There are two heating coils cc and the plaster of Paris coatings at the holes, HH, in order to reduce condensation of steam on the suspension of the pans. In addition, there are shields fitted above PP to prevent the falling of the condensed water from the roof of the steam chamber on to the pans. I and O indicate the inlet and outlet for the steam in the steam chamber A. Two hollow copper spheres, aa', of identical size, weight and thermal capacity are taken on the pans, PP, and counterpoised when no steam is allowed to enter the steam chamber A. Now one of the spheres, a', is completely evacuated

and the other, a, is filled up with the experimental gas under high pressure. Again the balance is counterpoised. The difference in weights gives the weight of the gas enclosed in the sphere, a. Let m be this weight expressed in gm. molecules (*i.e.*, wt. in gms. divided by the molecular weight). Now steam is introduced in A through the inlet I and allowed to pass into it till the condensation is complete. This condensation is evidently due partly to the thermal capacity of the spheres and partly to that of the gas contained in a. Let θ_1°C. and θ_2°C. be the temperatures of the steam chamber before the introduction of the steam and after the completion of condensation respectively. These are recorded by a very delicate thermometer. When a steady value of θ_2°C. is obtained, the rate of steam flow is slowed down and the balance is again counterpoised and the new change in weight w gms. is noted, which is evidently the weight of the excess steam condensed on a. This w is due to excess thermal capacity of a arising out of the enclosed gas.

If C_v be the gm. molecular specific heat of the gas at constant volume, the heat required to raise the enclosed gas from θ_1°C. to θ_2°C. is given by $mC_v(\theta_2-\theta_1)$ calories. This heat has been given out by w gms. of steam during condensation.

$\therefore \quad mC_v(\theta_2-\theta_1)=wL$ where L is the latent heat of steam.

$\therefore \quad C_v=wL/m(\theta_2-\theta_1)$.

In determining C_v by the above method corrections are to be introduced for : (*i*) the expansion of the sphere a due to rise of temperature and increase of internal pressure ; here as the volume changes, some external work is done in expanding to this volume ; (*ii*) the unequal thermal capacities of the spheres ; (*iii*) the increased buoyancy of the sphere due to the increase in volume at the higher temperature. In addition, a further correction arises due to the fact that the weight of excess condensed steam on a is taken in a moving medium (steam). So w in steam must be reduced to its corresponding value in vacuum.

92 (B). **Determination of C_p by Regnault's method** :—The principle of Regnault's apparatus for determining the specific heat of

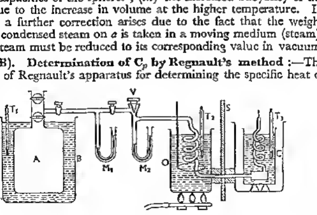


Fig. 43(B)

a gas at constant pressure is given in Fig. 43(B).

This amount will turn y gm. of water at 50°C. into steam, which will require $[(100-50)y+536y]$ calories.

Hence $130x=586y$; but from (1), $x=1000-y$; whence $x=818.5$ gms and $y=181.5$ gms.

93. Joseph Black (1728—1799) :—An Irish scientist. He was born at Bordeaux and had spent his childhood in France. He graduated from the Glasgow University and was awarded the doctorate of Medicine for his researches on the physiological effects of quick-lime and caustic potash on the human body. He joined this university in 1756 as Professor of Analytical Chemistry. Here his name spread wide as an eminent teacher. James Watt, David Hume and Adam Smith were the result of his inspiration. His outstanding research work relates to the absorption of energy during change of state. The term '*latent heat*' is due to him, and he measured the latent heat of fusion of ice by means of a calorimeter, which bears his name. In 1766 he joined the Edinburgh University as Professor of Chemistry where he served till he died in 1799.

Questions

1. Define calorie and "B.Th.U." (C. U. 1951, '52, G. U. 1949)
State the relation between them. (G. U. 1949)

2. Distinguish between the thermal capacity and the water equivalent of a body. State the units used in expressing them. (C. U. 1952, '53)

3. Define 'specific heat'. How is the specific heat of a solid determined? (C. U. 1940, '49, G. U. 1949)

Does the specific heat of a substance depend on the unit of heat chosen? (G. U. 1951)

4. A brass weight of 100 gms. is heated so that a particle of solder placed upon it just melts. It is then put into 100 c.c. of water at 15°C. contained in a calorimeter of water equivalent 12. If the final temperature of the water is 35°C., what is the melting point of the solder? (Sp. ht. of brass=0.88) (Pat. 1935)
[*Ans.* 289.5°C.]

5. A body of mass 100 gms. at 120°C. is plunged into 300 gms. of water at 20°C. contained in a copper calorimeter of mass 50 gms. The final temperature attained is 30°C. Find the sp. heat of the material of the body (sp. heat of copper=0.09)
[*Ans.* 0.34]

6. An alloy consists of 92% silver and 8% copper. Calculate the final temperature when 50 gms. of the alloy at 100°C. are mixed with 50 gms. of oil of specific heat 0.46 at 20°C. (The sp. heats of copper and silver are 0.095 and 0.056 respectively.)
[*Ans.* 29.1°C.]

7. Define unit of heat capacity for heat and specific heat. A piece of iron weighing 100 grams is warmed through 10°C. How many grams of water could be warmed 1°C. by the same amount of heat? The specific heat of iron is 0.10.
[*Ans.* 100 gms.]

8. Describe how you can measure the temperature of a furnace by applying calorimetric principle. (Pat. 1929)

9. A ball of platinum, whose mass is 200 gms. is removed from a furnace and immersed in 153 gms. of water at 0°C. Supposing the water to gain all the heat

the platinum loses and if the temperature of the water rises to 30°*C*., determine the temperature of the furnace. (Sp. ht. of platinum=0·031.) (C. U. 1936)

[*Ans.* 770·3°*C*.]

10. The calorific value of coke is 13,000 British Thermal Units per pound. Find the minimum amount of coke which would have to be burnt in order to heat 30 gallons of water from 60°*F*. to 130°*F* for use in a bath. (1 gallon of water weighs 10 lbs.).

[*Ans.* 21/13 lbs.]

11. If 90 grams of mercury at 100°*C*. be mixed with 100 grams of water at 20°*C*., and if the resulting temperature be 22°*C*., what is the specific heat of mercury ? (C. U. 1925)

[*Ans.* 0·0285.]

12. 10 gms. of common salt at 91°*C*. having been immersed in 125 gms. of oil of turpentine (sp. ht. 0·428) at 13°*C*., the temperature of the mixture is 16°*C*. ; supposing no loss or gain of heat from without, find the specific heat of common salt. Can you do this experiment with water instead of turpentine ? (C. U. 1938)

[*Ans.* 0·214.]

13. The temperatures of three different liquids *A*, *B*, and *C* are 14°*C*., 24°*C*., and 34°*C*. respectively. On mixing equal masses of *A* and *B*, the temperature of the mixture is 31°*C*. Supposing equal masses of *A* and *C* were mixed, what would be the temperature of the mixture ? (Pat. 1955)

[*Ans.* 29·6°*C*. nearly.]

14. A copper calorimeter weighing 10 gms. is filled first with water whose weight is 7·3 gms., and then with another liquid whose weight is 8·7 gms. ; the times taken in both cases to cool from 40°*C*. to 35°*C*. are 85 and 75 seconds respectively. Taking the specific heat of copper to be 0·095, calculate the specific heat of the liquid.

[*Ans.* 0·7275.]

15. A calorimeter whose water equivalent is 10 gms. is filled with 50 gms. of water at 80°*C*., and the time taken for the temperature to fall to 75°*C*. is 4 minutes. When filled with another liquid, the weight being 40 gms. the time taken for the same fall is 130 seconds. Find the sp. heat of the liquid. (U. P. B. 1947)

[*Ans.* 0·5625.]

16. A calorimeter, whose water equivalent is 5 gms. is filled with 25 gms. of water. It takes 4 minutes to cool from 25°*C*. to 17°*C*. When the same calorimeter is filled with 30 gms. of liquid it takes 180 secs. to cool through the same range. Calculate the sp. heat of the liquid. (R. U. 1953)

[*Ans.* 0·58.]

17. Supposing you were given a thermometer reading only from 50°*C*. to 100°*C*., and some water of which the temperature was below 20°*C*., describe an experiment showing how, without using another thermometer, you could determine roughly the temperature of the water. (C. U. 1953)

[**Hints.**—Take some water in another vessel whose mass is a little greater than that of the quantity given. Boil this water ; mix the two, and note the resultant temperature t°*C*. by the given thermometer which will be a little over 50 *C*. Let m be the mass of cold water, θ its temperature, and m' the mass of hot water ; then we have, $m'(100-t)=m(t-\theta)$. Hence calculate θ.]

18. Describe how the specific heat of a liquid is determined by the method of cooling. (U. P. B. 1947 ; R. U. 1949)

19. Account for the difference between the specific heat of a gas at constant volume and that at constant pressure ; and find the difference between them. (All. 1931 ; *cf.* 1944, '46 ; R. U. 1951)

20. Distinguish between specific heat of a gas at constant pressure and that at constant volume.

21. How would you show that the specific heat at constant pressure is greater than the specific heat at constant volume ? (R. U. 1948)

22. Deduce the relation $J(C_p - C_v) = R$, where the symbols have their usual significance. (R. U. 1918 ; M. B. B. 1932)

23. "Water has a higher specific heat than any other liquid or solid." How will this fact affect (a) determination of temperature by a water thermometer over ranges for which its use is permissible, and (b) the climate of islands and places on the sea-coast ? (Pat. 1931)

24. The latent heat of water is 80 calories. By what number will the latent heat be represented if the pound is taken as the unit of mass and the temperatures are measured on the Fahrenheit scale ? (Pat. 1933)

[*Ans.* 144]

25. What is meant by the statement that the latent heat of steam is 536 ? What number will represent the latent heat if the unit of mass is a pound and temperatures are measured on the Fahrenheit scale ? (Pat. 1941)

[*Ans.* 964.8.]

26. On what factor does the latent heat of a substance depend ? If the calorie be defined as the quantity of heat required to raise the temperature of one pound of water through one degree Farhenheit, what would be the value of the latent heat of vaporisation of water in such calories, if its value in the gramme-centigrade system is 580 ? (Pat. 1931)

[*Ans.* Value of latent heat in units as defined $= 580 \times \frac{9}{5} = 1044$]

27. Explain the meaning of "latent heat" (C. U. 1909, '13, '17 ; Pat. 1916)

28. Find the result of mixing 2 lbs. of ice at 0°C. with 3 lbs. of water at 45°C. (C. U. 1931)

[Hints.—The amount of heat given out by 3 lbs. of water at 45°C. in cooling to 0°C. $= 3 \times 45 = 135$ pound-degree °C. heat-units, and 80 such heat-units are necessary to melt 1 lb. of ice. So the amount of ice melted by this quantity of heat $= \frac{135}{80} = 1.69$ lbs. The result is $(3 + 1.69)$ or 4.69 lbs. of water at 0°C. and $(2 - 1.69)$ or 0.31 lb. of ice at 0°C.]

29. Dry ice at 0°C. is dropped into a copper can at 100°C., the weight of the can being 60 grammes and the specific heat of copper 0.1. How much ice would reduce the temperature of the can to 40°C. ? (C. U. 1924)

[*Ans.* 3 grams.]

30. What would be the final temperature of the mixture when 5 gms. of ice at -10°C. are mixed up with 22 gms. of water at 30°C. ? The sp. ht. of ice is 0.5

[*Ans.* 7°C.] (C. U. 1926 ; G. U. 1949)

31. Some ice is placed in a glass vessel held over a spirit-lamp and melts to water at 0°C. in 2 minutes ; how long will it take (a) before it reaches the boiling point ; (b) before it is all boiled away, assuming there is no escape of heat ?

[*Ans.* (a) $2\frac{1}{2}$ min. ; (b) $(2\frac{1}{2} + 13\frac{1}{2})$ min.]

32. A ball of copper of mass 30 gms. was heated to 100°C. and placed in an ice-calorimeter. In cooling down it evolved sufficient heat to melt 3.54 gms. of ice. If the latent heat of fusion of ice is 80, what is the specific heat of copper ? (Dac. 1937)

[*Ans.* 0.0944]

33. Explain how the specific heat of a solid may be determined by means of the ice-calorimeter. (C. U. 1913, '35 ; Pat. 1945)

34. Describe Bunsen's ice-calorimeter. Explain its use in determining the specific heat of a substance. What are the merits of the method ? (R. U. 1949, '52)

35. A spherical iron ball is placed on a large block of dry ice at 0°C. into which it sinks until half submerged. What was the temperature of the iron ?

(Density of iron=7·7 gms./c.c.; density of ice=0·92 gms./c.c.; sp. heat of iron=0·12; latent heat of fusion of ice=80 calories per gm.)

[*Ans.* 39·8°C. neglecting heat lost by radiation.]

36. If a gramme of ice at 0°C. contracts by 0·091 c.c., calculate the sp. heat of the substance when 40 gms. at 60°C. dropped into an ice-calorimeter cause a change in volume of 0·273 c.c. (latent heat of fusion of ice=80 cals./gm.) (Rajputana, 1948)

[*Ans.* 0·1]

37. A substance was heated to 100°C. and 0·8 gm. of it is dropped into a Bunsen's ice-calorimeter, due to which the thread of mercury in the capillary tube of 1 sq. mm. section moved through a distance of 6·9 mm. Calculate the specific heat of the substance (given that 1 gm. of water on freezing expands by 0·091 c.c.). (*cf.* Nagpur, 1952)

[*Ans.* 0·0758.]

38. Describe any method of determining the latent heat of steam in the laboratory. State the precautions that should be taken.

(C. U. 1931; All. 1918; Pat. 1935, '49; Dac. 1921)

39. A copper vessel, weighing 190 gms., containing 300 gms. of water at 0°C. and 50 gms. of ice at 0°C. Find the quantity of steam, at 100°C., that must be passed into the vessel to raise its temperature and that of its contents to 10°C. (Pat. 1949)

Sp. heat of copper=0·1; L (steam)=537 cals./gm.; L (ice)=80 cals./gm.

[*Ans.* 12·27 gms.]

40. Into a calorimeter containing 175 gms. of water and some ice, steam of mass 10 gms. and temp. 100°C., is passed. The temperature of the contents rises to 10°C. If the water equivalent of the calorimeter is 5 gms., calculate the mass of ice initially present. (Given latent heat of water=80 cals./gm.; latent heat of steam=540 cals./gm.). (Andhra, 1952)

[*Ans.* 50 gms.]

41. A copper ball 56·32 gms. in weight and at 15°C. is exposed to a stream of dry steam at 100°C. What weight of steam will condense on the ball before the temperature of the ball is raised to 100°C.? (Sp. ht. of copper=0·093; Latent heat of steam=536 cals.). (Dac. 1928)

[*Ans.* 0·83 gm.]

42. Alcohol boils at 78°C., its latent heat of evaporation is 202 cals./gm. and its mean sp. heat when liquid is 0·65. Calculate the least quantity of water at 10°C. needed to condense 100 gms. of alcohol vapour at 78°C. into liquid at 15°C. (Utkal 1954)

[*Ans.* 4859 gms.]

43. A copper vessel of water equivalent 60 gms. contains 600 gms. of water at 30°C. A bunsen burner, adjusted to supply 100 calories per second is used to heat the vessel. Neglecting all losses, calculate (*a*) the time required to raise the water to boiling point and (*b*) the time required to boil away 50 gms. of water (latent heat of steam=540 cals.) (C. U. 1952)

[*Ans.* (*a*) 7 mins. 42 secs.; (*b*) 12 mins. 12 secs.]

44. Describe Joly's Steam Calorimeter. How will you use the instrument to find the specific heat of a gas at constant volume?

(Nagpur, 1950; Rajputana, 1945, '49; U. P. B. 1952)

45. Describe, with necessary theory, how the specific heat of a gas at constant pressure is determined by Regnault's method.

CHAPTER VI

CHANGE OF STATE

94. Fusion and Solidification :—When a substance changes from the solid to the liquid state, the process is known as *fusion*, and when it changes from the liquid to the solid state the process is called *freezing* or *solidification*.

95. Melting Point :—For every substance there is a particular temperature at which it changes from the solid to the liquid state at a given superincumbent pressure. This fixed temperature is known as the *melting point* of the solid. *It remains constant throughout the process of melting*, i.e. the temperature remains constant until the whole of the solid is melted, if the pressure on it remains constant, although heat is applied all the time. The temperature will rise only after the last particle of the solid has melted. The *melting point* is different for different substances and for each substance it slightly varies when the superincumbent pressure varies.

Similarly, during the process of solidification at constant pressure, temperature remains constant until the whole of the liquid is solidified, although heat is withdrawn all the time. The temperature will begin to fall only when the last drop of the liquid has solidified. This fixed temperature is called the *freezing point* or the *solidification temperature* of that particular liquid and is different for different liquids and slightly changes with pressure. It is the same as the melting point of the substance.

The *normal* **melting point** *of a substance is a definite temperature at which it melts or solidifies at a pressure of one atmosphere*

If the cooling process be continued very slowly and without disturbance, then many liquids can be cooled below their normal solidification temperature. This phenomenon is known as **super-cooling,** or *superfusion* or *surfusion* and the liquid in this condition is called a supercooled liquid. This condition is not stable, for if the liquid is disturbed, or a particle of the substance in the solid form dropped into the liquid, solidification at once begins and the temperature quickly rises to the solidification point. The phenomenon is a delicate one and is possible only if the liquid is absolutely pure and free from any suspended foreign matter.

The amount of heat given up by a substance in solidification is equal to the latent heat of fusion. In the case of water, every gram of it must give out 80 calories before solidification takes place at 0°C., and for this reason water does not freeze at once when cooled down to 0°C. Conversely, every gram of ice must absorb 80 calories

at $0°C.$ before fusion takes place. For other substances the value of the latent heat is much smaller. So water can be called a storehouse of heat. For example, 1 cu. ft. of water weighs 62·5 lbs. which in freezing, gives up $62{\cdot}5\times80=5000$ C.H.U. of heat, which, again, can raise 50 lbs. of water from the freezing point to the boiling point $(50\times100=5000)$.

96. Viscous State :—Some substances, solid at ordinary temperatures such as iron, glass, pitch, wax, etc. have got no definite melting point. They gradually change from the solid to the liquid state passing through a plastic or viscous state intermediate between solid and liquid. This state may extend over a considerable range of temperature depending on the nature of the substance. Again, some substances, liquid at ordinary temperature, such as glycerine, acetic acid, and also some other organic acids and oils, pass through the intermediate *viscous* state in changing from the liquid to the solid state. Such liquids have no fixed solidification temperatures.

97. Sublimation :—Some substances, such as camphor, iodine, arsenic, sulphur, etc. change directly from the solid to the gaseous state without passing through the intermediate liquid state. They are called *volatile substances,* and such change of state is known as sublimation. Ice and snow also sublime slowly even when below the freezing point.

98. Change of Volume in fusion and Solidification :—Most substances increase in volume by fusion, but a few substances, such as ice, cast iron, antimony, bismuth, brass, etc. contract on melting and expand on solidification. In the first case, the solid sinks in the resulting liquid while in the other, the solid floats on the corresponding liquid. A lump of cast iron floats on the liquid metal just as ice floats on water, and it is for this reason that these metals can be used for sharp castings, since on solidifying they must expand and fill up every nook and corner of the mould.

It has already been stated in Art. 271, Part I, that on freezing, the volume of water increases by about 9 per cent, *i.e.* 11 c.c. of water at $0°C.$ becomes 12 c.c. of ice at the same temperature, and so ice floats on water with $\frac{11}{12}$ of its volume below the surface of water and $\frac{1}{12}$ above it. Thus, the volume of water formed by the melting of ice is less by $\frac{1}{12}$th of the volume of ice.

A great force is exerted by the expansion of water on freezing, which sometimes may cause great trouble. It does a good deal of damage by bursting water pipes in cold weather and by the splitting of rocks and soils, etc. On the other hand, the effect would have been still more disastrous if water would contract on freezing, as in that case ice formed would have been heavier and so would sink to the bottom of lakes or ponds, and soon the whole mass of water

would transform into a solid block of ice, and thus all aquatic animals would ultimately perish (*vide* also Art. 41).

Again, ice is a poor conductor of heat. In cold countries, when the surface of any lake or pond is frozen into ice, the ice prevents the flow of heat from the water below to the space above which is at a temperature lower than 0°C. So however severe the cold may be, water cannot freeze below a certain depth. Even in regions near the North Pole, the thickness of ice formed on the ocean reaches only about 4 or 5 metres, and this thickness changes by only a metre or two during the course of a year.

On the other hand, ice once formed, melts only slowly by the sun's rays which must supply the latent heat required for melting. If any latent heat of fusion were not necessary for the melting of ice, ice and snow would melt very rapidly and *disastrous floods would result.*

In summer water formed at the surface of ice being heavier sinks down and a fresh surface of ice is always exposed to the sun which helps in melting more. Thus the expansion of water on solidification serves two purposes : it prevents accumulation of much ice in winter and also helps the melting of ice in summer.

99. Determination of the Melting Point of a Substance :—

Two methods are given below for the determination of the melting point of a solid like naphthaline (which expands on melting and contracts on solidifying).

(1) Cooling Curve Method.—This method is used when an appreciable quantity of the substance is available. Put the substance in a test tube and melt it by heating in a water bath. Place a thermometer in the liquefied substance, take the tube out of the bath, dry its outside, surround it by a large vessel to protect it from air currents, and take readings at intervals of one minute as the cooling proceeds. The reading will remain constant during the process of solidification after which it will fall. Take temperature readings until, sometime after, solidification is observed to be complete.

Now, plotting a graph with time and temperature, a part of the curve will be seen to be parallel to the time-axis. The temperature corresponding to this part is the melting point of the substance, and Fig. 41 is the general form of the cooling curve for a pure single chemical substance like naphthaline. The part which is parallel to the time axis shows no variation of temperature with time and it corresponds to a purely liquid state, and the portion below this represents the solid state of the substance.

[N.B. If the substance is heated and a heating curve (time-temperature curve) is plotted in a similar way as above, the graph

will rise first and then a part of the curve will remain parallel to the time-axis, and then it will rise again. The horizontal parts of the cooling curve and the heating curve will be almost coincident if the substance is a pure single substance.]

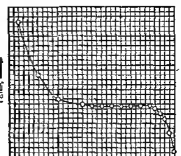


Fig. 44—Cooling Curve.

In the melting point curve of a substance which is a *mixture* of different substances, such as paraffin wax, or any fat, solidification takes place over a range of temperature, and there is no definite melting point. (The melting point curves for a mixture of substances have several horizontal steps corresponding to the melting points of the different constituents.) For substances like glass, sealing wax, etc. there is no abrupt change from the solid to the liquid state and they remain plastic over a range of temperature between the solid and the liquid state. As glass remains plastic over a wide range of temperature, so it can be worked and moulded. After taking a sharp bend, as in Fig. 44, the slope of the curve in these cases changes continuously and does not become horizontal, that is the thermometer-readings do not remain constant for several minutes.

(ii) Capillary Tube Method.—This method is used when only a small amount of the substance is available. Heat a piece of glass delivery tubing in a blowpipe flame and quickly draw it out, when soft, to form a capillary tubing of about $\frac{1}{2}$ mm. diameter and with very thin walls. Take about 10 cms. of this tube A. Melt some naphthaline, suppose, in a dish and suck up about 4 cms. length of it into the capillary tube. Now, seal off the lower end of the tube, and attach it by a thin band to the bulb of a mercury thermometer T, which is mounted so that the bulb and the tube dip into a beaker of water with the top of the substance just below the water surface (Fig. 45). Now carefully heat the water stirring it all the time. After some time, the opaque solid will change to a transparent liquid on melting ; note this temperature. Now remove the burner and allow the liquid to cool, stirring the water all the time ; note the temperature when naphthaline becomes opaque, *i.e.* it solidifies.

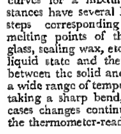


Fig. 45—Capillary Tube

The mean of these two temperatures gives the melting point of the substance. Repeat this experiment two or three times so as to get a very good result.

Note.—Generally the temperature at which a solid melts is the same as that at which the corresponding liquid freezes. But for certain fats like *butter*, this is not the case. For example, *butter* melts at about 33°C., but it solidifies at about 20°C.

100. Melting Points of Alloys :—In the case of alloys, the melting points are usually lower than those of the constituents, and it is for this reason that 'flux' is added to a substance with a high melting point in order to make it melt at a lower temperature.

There are other alloys like **Wood's metal,** which is an alloy of tin, lead, cadmium and bismuth, having a melting point of 60·5°C. ; and **Rose's metal**—an alloy of tin, lead, and bismuth,—having a melting point of 91·5°C. These alloys are readily fusible and so they find many applications in our daily life. They are used in *automatic sprinklers for buildings*, so that when a fire breaks out, a plug, made of one of these alloys and inserted in a water pipe, melts and thus the water rushes out from the mains. Fusible plugs are also used in closing fire proof doors automatically in the event of a fire, and *fuse* in electrical circuits are also made of these alloys.

101. Effect of Pressure on the melting Point :—The melting points of substances like ice, iron, etc. which *contract* on melting, are lowered, and the melting points of those such as paraffin, etc. which *expand* on melting are *raised* by increase of pressure. The melting point of ice at 0°C. is lowered by about 0·0073°C., for an increase of pressure of one atmosphere. Paraffin wax, which expands on melting, melts at about 54°C. at a pressure of one atmosphere, and it will melt at a higher temperature if the pressure be increased.

From a simple consideration we would also expect the above facts. For, in the case of ice, any increase of pressure tends to diminish its volume and thus it helps the process of melting and so the melting point will be lowered under increased pressure. In the case of paraffin, which expands on melting, any increase of pressure which tends to diminish the volume, will oppose the process of melting and so the melting point in this case will be increased under a increased pressure.

Regelation.—The fact that by exerting pressure the melting point of ice can be lowered, may be shown by pressing two pieces of ice against each other and then releasing the pressure, when it will be found that the two pieces are frozen into one. Such phenomenon of melting by pressure and refreezing on withdrawal of pressure is known as *regelation* (L. *re*, again ; *gelare*, freeze). The pressure lowers the melting point, and so water is formed at the surface of contact. On removal of the pressure, the melting point rises, water freezes again, and thus the two pieces are joined together, provided the temperature of the ice is not below 0°C., in which case the pressure applied by

the hands will not be sufficient to reduce the melting point below the actual temperature of the ice and so the pieces of ice will not be joined together. It has been found that a pressure of about 1000 atmospheres will be necessary to melt ice when the air temperature is $-7{\cdot}6°C$.

The phenomenon of regelation is demonstrated by the following experiments :—

(1) Bottomley's Expt.—A large block of ice rests at its two ends on two supports (Fig. 46). A turn of a thin metallic wire with a heavy weight attached is placed round it. In about half an hour the wire cuts its way right through the block of ice but the block of ice remains as one piece. The pressure of the wire causes the ice under it to melt and the wire passes through the water formed, which being relieved of the pressure then freezes into ice again.

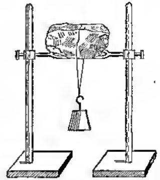


Fig. 46—Bottomley's Expt.

It is to be noted that the ice melting beneath the wire requires heat for melting and the water above the wire gives out heat at the time of freezing, which is conducted through the wire to help the ice below in melting. So the above process is helped if a metallic wire is used, for a metal is a good conductor of heat. Hence a *twine is not suitable* in this case and a copper wire will work more quickly than a steel wire.

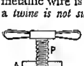


Fig. 47—Mousson's Apparatus.

Experiments have proved that if the block be in an ice-house where the temperature is below 0°C., the wire cannot cut through the block ; the temperature of the surrounding air must be above 0°C.

(2) Mousson's Apparatus.—The lowering of the melting point of ice by increased pressure can also be shown by means of the apparatus shown in Fig. 47, which is known as **Mousson's Apparatus.**

Expt.—The apparatus consists of an iron cylinder *AB* closed at one end with a strong screw plunger *P*. The cylinder is partly filled with water which is then frozen by keeping it inside a mixture of ice and salt. A small metal ball *C* is now placed on the top of the ice in the cylinder which is then closed by the screw plunger. The whole is then surrounded by ice and the pressure is increased by driving the screw plunger in. On opening the cylinder at the bottom, the metal

106. (a) Evaporation and Ebullition (or Boiling) :—

Evaporation.—If a shallow dish containing water be left in a room, the water will gradually disappear. Such gradual change from the liquid to the gaseous state which takes place quietly from the surface of the liquid and goes on at all temperatures is known as *evaporation*.

That is, *evaporation is the gradual and slow change of a substance from the liquid to the vapour state which takes place at the surface of the liquid at all temperatures.*

Factors governing Evaporation.—

(i) *The temperature of the liquid*: The higher the temperature, the faster is the formation of vapour.

(ii) *The nature of the liquid*: A quantity of ether will disappear faster than the same quantity of water under the same conditions, *i.e.* a liquid having a low *boiling point* will be evaporated quickly.

(iii) *The renewal of air over the liquid surface*: The rate of evaporation increases by renewing air over the liquid surface. That is why wet linen dries up more quickly on a windy day than on a calm day.

(iv) *The pressure of the air*: The less the pressure of air on the liquid, the greater is the rate of evaporation. So the rate of evaporation is maximum in vacuum. Evaporation in vacuum is used in chemical works for preparing extracts from solutions.

(v) *The area of the exposed surface*: The greater the area of the surface of a liquid exposed to the air, the greater is the evaporation. So hot tea is taken in a flat dish to get it cooled quickly.

(vi) *The pressure of vapour in contact with the liquid*: The rate of evaporation becomes slower, if there is vapour of the liquid in contact. That is why evaporation is quicker in dry than in moist air. Wet linen and muddy roads dry up more quickly in the winter than in the rainy seasons.

(b) Boiling.—If a liquid is continuously heated under a given superincumbent pressure, vapour is given off at the initial stages from the surface of the liquid but finally a stage comes when the vaporisation takes place throughout the mass of the liquid in a rapid and vigorous way. This stage is called the *boiling* of the liquid. The bubbles of the vapour always originate at the heated surface.

As long as the boiling takes place, the temperature of a liquid remains constant if the superincumbent pressure does not change. This constant temperature, which is different for different liquids, is called the **boiling point** of a liquid corresponding to the superincumbent pressure. If the superincumbent pressure is one atmosphere the temperature of boiling is called the **normal boiling point** of a liquid and is ordinarily designated as its *boiling point*.

Factors governing Boiling Point.—

(*i*) Boiling point increases or decreases according as the superincumbent pressure on the liquid increases or decreases.

(*ii*) The presence of any dissolved impurity increases the boiling point. So the boiling point of a solution is always greater than that of the pure solvent.

(*iii*) The boiling point depends, though to a small extent, on the material of the boiler, its roughness and the degree of cleanness of its inner surface.

(c) Distinction between Evaporation and Boiling.—The difference between evaporation and boiling (ebullition) is that the former takes place at the *surface* of the liquid at *all temperatures*, whereas the latter takes place *throughout the mass* of the liquid at a *particular temperature depending on the superincumbent pressure.* Moreover, the former is a *slow* process while the latter is a *rapid* one.

107. Cold caused by Evaporation :—*Evaporation produces cooling.* When the evaporation of a liquid takes place, the temperature of the liquid falls, because the latent heat necessary for vaporisation is supplied by the liquid itself and so it goes down in temperature.

This is the reason of the cooling effect of the wind on moist skin, or of the wind coming through *khas-khas* screens in summer months. One gram of water, say, at 15°C., would require about 536 calories to change it into vapour at that temperature. At these rates, heat is absorbed from the skin, or *khas-khas* when evaporation takes place. The wind accelerates the rate of evaporation.

The cooling effect will be rapid if a few drops of ether or alcohol are placed on the skin instead of water, because, the rate of evaporation of these liquids at the room temperature is very rapid.

The bulb of a thermometer wrapped with a piece of muslin will show a rapid fall in temperature, when a few-drops of ether are poured over the muslin.

(1) A porous pot keeps water cooler than a non-porous pot.—

In hot countries, water is put into earthen vessels which are porous. The water which oozes out of the pores are evaporated and thus the water inside is kept cool. Water in this case will be much cooler than the water kept in a glass or metallic vessel of equal size, because, in the first case, the evaporation takes place all over the vessel, while in the other case, it takes place only from the surface of water at the mouth of the vessel.

(2) The watering of the streets in summer not only settles down the suspended dust but produces a cooling effect by evaporation.

(3) In drinking hot milk or tea, it is generally poured in a shallow saucer before drinking, in order to expose a large surface of the liquid to the air so that evaporation can take place more rapidly.

(4) In summer, dogs are seen to hang out their tongues in order to expose a surface to air for evaporation so that they may enjoy the cooling effect caused by it.

(5) The reason of using a fan in summer is to increase the rate of evaporation of the perspiration coming out of the pores of our skin. Generally, the vapour formed out of the perspiration clouds over the skin due to which the rate of evaporation becomes slow, but when a fan is used, the wind produced by the fan removes the layers of vapour and this renewal of the air in contact with the skin increases the rate of evaporation. This causes greater absorption of heat from the skin due to which cold is produced.

108. Experiments on Absorption of Heat by Evaporation :—The absorption of heat, and the consequent production of cooling, by an evaporating liquid, may be shown by the following experiments, where it will be seen that it is possible even to freeze a liquid by the loss of heat caused by its own evaporation

(1) A few drops of water are placed on a block of wood and a thin copper calorimeter containing some ether is placed on the water. The ether is now made to evaporate rapidly by blowing air through it by foot bellows. The ether in rapidly evaporating takes heat from the water, under the beaker, which will ultimately freeze, and the beaker will be fixed to the wood by a layer of ice formed between them

(2) **Wollaston's Cryophorus.**—This apparatus illustrates the above principle of cooling by evaporation It consists of a bent glass tube having a bulb at each end containing a little water and water vapour only, but no air All the water is transferred to the bulb P and the bulb A is surrounded by a freezing mixture (Fig 48) The vapour in A condenses, the pressure inside falls and more water evaporates from P, the water in which is gradually cooled and ultimately may be frozen into ice.

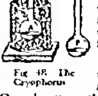


Fig 48 The Cryophorus

(3) A shallow metal dish containing a little water and another dish containing strong sulphuric acid are placed under the receiver of an air-pump On exhausting, the pressure inside falls, the water of the dish rapidly evaporates, and the vapour formed is absorbed by the sulphuric acid and thus the pressure inside is always kept low So the water continues to evaporate rapidly, whereby the temperature of the water falls and ultimately a thin layer of ice forms on the surface of the water This is known as **Leslie's Experiment.**

109. Refrigeration :—*It is the science of artificially maintaining an enclosure at a desired constant temperature much lower than that of the surrounding atmosphere.*

At temperatures above 50°F, bacteria multiply at an increasingly rapid rate. Food articles such as fish, meat, potato, eggs, fruits, etc.

for this reason go bad in hot weather. If kept within a cool hold they keep well for a long time. Many medical products such as vaccines, injectibles, etc. also behave similarly. In fact, the scope of refrigeration is very wide ranging from the small domestic refrigerator in which a temperature of 40° to 45°*F.* is aimed at, to cargo vessels in which refrigerated holds are maintained many degrees below the melting point of ice for the transport of frozen meat. It also covers ice-making plants. Some **ice-machines** produce even several hundred tons of ice per day. Ice-plants form an indispensable equipment for the fishing fleets; the refrigeration of the catch is no less important than act of catching, for such fleets report to the shore sometimes a few days after. The word "commercial refrigeration' is ordinarily used to indicate in general the technique of preservation of goods at low temperatures. Commerical refrigeration is already an important trade in the United States of America, U.K., and some other advanced countries of Europe. It is so bound to grow in this country, especially because ours is a tropical country. A refrigerator, besides being used for cold storage purposes as stated above, is also used for industrial purposes. A refrigerating device forms the most important part of a Summer Air-conditioning plant with which modern Public Halls such as Lecture Halls, Theatres, Picture Houses, Hospitals, etc. are fitted, or of Air-conditioners used in Research Laboratories, Spinning rooms in Textile Mills, Rubber Factories, etc.

In the act of refrigeration the principle which is commonly utilised is that of cooling a liquid by rapid evaporation. The liquid which produces cold by evaporation is called the **refrigerant.** A refrigerant should have a high latent heat of vaporisation, and a low boiling point, besides other secondary qualities. Some common refrigerants are—ammonia, sulphur-dioxide, carbon-dioxide, methyl chloride, ethyl chloride, Freon (CCl_2F_2), etc. Freon, for various considerations, is rightly regarded as an ideal refrigerant.

In a refrigerator the hold is maintained at a lower temperature than that of the surrounding atmosphere. This means that, to start with, heat has to be removed from the given enclosure to the hotter surroundings at such a rate that the temperature falls to the desired value and at this temperature heat is to be continuously transferred from it at a rate at which it will enter from outside such that the temperature of the enclosure may remain constant. The act of such removal requires the expenditure of some energy. Two distinct types of refrigerators have come into existence which differ from each other in respect of the nature of supply of the energy.

(1) The **Electrolux Refrigerator** (or the **Absorption type** refrigerator).—In it the working energy is supplied in the form of heat energy by burning a fuel such as coal gas, kerosene, etc.

(2) **Frigidaire** type or the **Compression type.**—In it the working energy is supplied in the form of mechanical energy by a

be increased and there will be further depression of the mercury column.

On continuing this process, a stage will be reached when there will be no more evaporation and so there will be no further depression of the mercury column. At this stage if a little liquid be introduced, it will collect as a thin layer on the surface of mercury. This shows that a confined space has only a limited capacity to hold a vapour at a given temperature. Let the top of the mercury stand at G at this stage, when the depression of the mercury column is greatest. When the depression of the mercury column is greatest, the enclosed space above the mercury top G *is said to be saturated with the vapour, or is said to be full of saturated vapour.* Hence in a closed space if a vapour is in contact with its liquid, it is a visible indication that the space is saturated with the vapour. Before this stage, the space is *unsaturated*, or is full of *unsaturated vapour*. Since no further depression of the mercury column occurs after the vapour becomes saturated, it is evident that the vapour in this condition exerts the *maximum pressure possible at that temperature*, i.e. *the saturation pressure is the maximum pressure of a vapour at a given temperature.*

In the above experiment the difference in height between the initial level B and the final level C (when the mercury column in the tube Y is depressed most) gives a measure of the saturation pressure of the vapour at the temperature of the experiment.

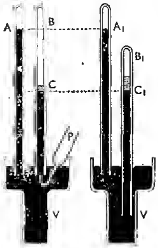
Thus, *a mass of vapour is said to be saturated at a given temperature when the pressure it exerts is the maximum for it at that temperature and this maximum pressure is called the Saturation Vapour Pressure (S. V. P.) of the liquid at that temperature; the vapour is said to be unsaturated when the pressure it is exerting is less than the saturation vapour pressure of the liquid at that temperature.*

111. Change of Volume at Constant Temperature :—

(a) Unsaturated Vapour.—Take two simple barometers, each about a metre long standing in the same trough F of mercury and then proceed as in the last article to find the *saturation vapour pressure* of water at the room temperature. Note the difference of levels, BC [Fig. 51(1)] which represents the S V P. of water so determined.

Next remove the experimental tube, refill it with mercury and again invert it into the same trough when it will be ready for a fresh set of observations. Let Fig. 51(2) represent the apparatus when the second set of observations is taken. Introduce two to three drops of water into the tube and observe the depression of the mercury column as the water evaporates. The vapour formed is, in all probability, unsaturated. To be completely sure, raise the tube some way up. In this way a mass of unsaturated vapour at the room

temperature is formed and enclosed above the mercury-top in the experimental tube. Note the volume of the vapour and the difference in level between the mercury-tops in the two limbs, which gives the corresponding pressure of the vapour. Gradually raise the experimental tube (taking care that the lower end of the tube remains under mercury) and note that as the volume of the enclosed vapour is thus increased, the mercury column in that tube also increases in height, showing that the pressure of the vapour decreases. At this stage note the volume, and the pressure which is given by the difference between the mercury-tops in the two limbs. Mark that the product of the pressure and volume at each stage is approximately constant.



(1) Fig. 51 (2)

Next push the tube gradually into the trough when the volume of the vapour will be decreased. Note that as the volume is decreased, the height of the mercury column in that tube also decreases, showing that the pressure of the vapour correspondingly increases. This goes on until the enclosed space is so diminished, as shown in Fig. 51(2), that a thin layer of water deposits on the surface C_1 of the mercury, which indicates that the space is no longer unsaturated, but is saturated with water vapour at that temperature. The difference A_1C_1 of the mercury levels in the two tubes at this stage will be the same as BC determined in the first part of the experiment. Any further depression of the tube does not tend to depress the mercury coulmn any more ; that is, the pressure attained has reached a maximum value and any further decrease in volume instead of increasing the pressure will gradually condense the vapour into liquid. Almost up to this stage, *i.e.* up to when saturation is reached, the product of pressure and volume will be constant, and equal to that when the volume was increased in the previous part of the experiment. The product will also be constant, if any other liquid instead of water is taken in the experiment.

Hence at constant temperature the product of pressure and volume of any unsaturated vapour is approximately a constant. That is, **unsaturated vapour obeys Boyle's law approximately.**

(b) Saturated Vapour.—Take the same apparatus as in Fig. 51 (2) and proceed as in the last part of the last article when gradually pushing the experimental tube into the trough a stage will finally be reached such that a thin layer of the liquid deposits on the clean

To start with, open S_1 and S_2 and gradually raise the open tube KG till the mercury in AB rises above S_1. This is how the air in AB is driven out. Close S_1 and lower KG until the mercury-top in AB recedes well below so as to leave a suitable vacuous space under S_1. The difference in level between the mercury-tops in AB and KG now gives the barometric height at the time of experiment.

Close S_2 and then open S_1 and pour some liquid, say water, into the funnel F. Then close S_1 and open S_2 when the water between S_1 and S_2, which is a small quantity of water, runs down into the vacuous space below and gets vapourised immediately. Mark that there is a depression of the mercury level in AB. *This is due to the pressure of the vapour formed.* In all probability, the enclosed space in AB is unsaturated. To be fully sure of it increase the volume of this space by lowering the arm KG some way down when the mercury-top in AB also will sink down. Fix the tube KG and note the mercury level in AB and that in KG. The difference between them will now be less than barometric height observed already. The deficit gives the pressure of the unsaturated vapour at the volume it now occupies in AB.

Next pass steam into the water bath and raise its temperature to a definite value by regulating the steam. Mark that the mercury column in AB goes down. Raise KG till the mercury level in AB goes up and reaches the initial position. This is necessary in order that the vapour may occupy the same volume while the temperature is changed. After the volume is thus restored to the original value find the difference in the mercury levels in AB and KG. Observe that the difference in levels has decreased. This shows that the pressure of the vapour has increased due to rise in temperature. Continue the above operations, raising the bath to gradually higher and higher temperatures. *It will be found that the increase of pressure with increase of temperature at constant volume follows the pressure law* (Art. 50) *which is a form of Charles' law.* The experiment may be repeated for temperatures lower than the room temperature by adding ice to the water bath, when it will also be found that the pressure decreases with decrease of temperature (volume remaining constant) according to the same pressure law as noted already. This reduction of pressure with decrease of temperature proceeds till at a certain temperature the vapour becomes saturated when it will begin to deposit as water. After this stage the pressure falls very quickly, being always equal to the saturation vapour pressure of the liquid at the corresponding temperature.

Thus unsaturated vapour obeys Charles' law.

114. Distinction between Saturated and Unsaturated Vapour :—

(1) When a space contains the maximum amount of vapour it can possibly hold at a given temperature, it is said to be saturated with the vapour and the pressure exerted by the vapour then is the

maximum pressure at that temperature, called the saturation vapour pressure. In a closed space, when a vapour is in contact with its liquid, it is a visible indication that the space is saturated.

A space is unsaturated at a given temperature if the maximum amount of vapour is not present in the space, *i.e.* if further liquid is introduced into the space it is evaporated. In a closed space a vapour is in all probability unsaturated, unless it is in contact with its liquid.

(2) If the temperature of saturated vapour in contact with its own liquid is increased, more liquid evaporates and consequently the pressure increases till the maximum pressure at the raised temperature is attained, *i.e.* the pressure attained is always the saturated vapour pressure at the higher temperature. On decrease of temperature, condensation of the vapour takes place at such a rate that the residual vapour at each lower temperature saturates the space at that temperature. The changes of saturation vapour pressure due to changes of temperature do not, however, follow Charles' law.

In case of unsaturated vapour, the increase of pressure due to increase of temperature takes place approximately according to Charles' law. On decrease of temperature, the pressure decreases according to the same law up to a stage, but finally at a certain temperature the space may be saturated with the vapour, and on further lowering of the temperature, more and more vapour condenses out, the pressure being maintained at the saturation vapour pressure corresponding to the lower temperature. Thus *saturated vapour does not obey Charles' law but unsaturated vapour does, though approximately.*

(3) Keeping the temperature constant, if the volume of saturated vapour, in presence of its own liquid, be increased, more vapour will be formed, and if diminished, some will be condensed but the pressure will always remain constant corresponding to the temperature (*vide* Art. 111) at which the experiment is done.

If no liquid be present when the volume is increased, the vapour becomes unsaturated and the changes of pressure and volume will take place according to Boyle's law. Thus *saturated vapour does not obey Boyle's law while unsaturated vapour does.*

The cases of saturated and unsaturated vapours can be compared to the solution of a soluble solid, *e.g.* sugar in water. When the solution contains the maximum amount of sugar possible at that temperature, it is called a *saturated* solution like the 'solution' of the maximum amount of water vapour in air. If the sugar solution is cooled, some sugar crystallises out ; so also if air saturated with water vapour is cooled, part of the water vapour condenses out. Again, by increasing the temperature of the sugar solution, more sugar can be dissolved, and similarly, the warmer the air the more water vapour will it hold in suspension.

the pressure due to saturated water vapour in presence of air at atmospheric pressure at the room temperature.

Next, to determine the saturation vapour pressure of water at the same temperature, when the vapour is produced in vacuum, open both S_1 and S_2 and gradually raise KG until the mercury in AB exceeds the level S_1. Close S_1. Then proceed as in Art. 110, to find out the saturation vapour pressure of water, when the water vapour is produced in vacuum.

If the experiment is correctly done, it will be found that the pressure of vapour in vacuum is the same as that found in the first experiment, even though the volume of the vapour in the second experiment may be different from that in the first experiment.

This shows that the saturation vapour pressure of water at the room temperature is independent of the volume of the air, as also of the presence of air, and depends only on the temperature. If any other liquid is taken or the gas taken is other than air, or any other constant temperature be used, the experiment reveals the same truth.

If, in the first experiment, the volume of the gaseous mixture in AB *when saturated with water vapour*, be increased or decreased (from the initial volume of the enclosed air) by lowering or raising the tube KG, the total pressure will be different. But if the alteration of pressure due to the change in volume of the air, as may be found from a Boyle's law experiment with the same mass of air enclosed, be taken into account, the pressure due to the vapour alone remains the same if it is saturated. If, however, the space is unsaturated at every stage, the change of pressure with change of volume of the mixture will follow Boyle's law.

Using the above apparatus as a Boyle's law tube, temperature being maintained constant at the room temperature or any other definite temperature, draw a P-V graph with air as the enclosed gas. Similarly, draw another P-V graph with a small quantity of water vapour (unsaturated) alone in the vacuum space of the tube AB. Next introduce the same quantity of water into the same volume of air at atmospheric pressure within the enclosed space above the mercury in AB and repeat a similar experiment as above and obtain a P-V graph for the gaseous mixture (unsaturated). Find that for the same volume, if the pressures obtained from the first two graphs be added it becomes equal to the pressure of the gaseous mixture at the same volume.

The above verifies Dalton's second law for saturated or unsaturated vapours.

Examples. (*1*) *A certain quantity of vapour of a liquid mixed up with air is contained in a vessel of constant volume. The pressure shown at 20°C. is 80 cms. of mercury and at 40°C. it is 100 cms. Given that at 20°C. the vapour pressure of liquid is 15 cms. calculate the same at 40°C.*

At 20°C. the total pressure is 80 cms., but that due to the vapour being 15 cms., we have from Dalton's law,

Again, if P_0 be the pressure, at 0°C., we have,

$$\frac{P_0}{273}=\frac{438\cdot64}{273+100};\ \therefore\ P_0=321\cdot04 \text{ mms.}$$

116. Critical Temperature : Gas and Vapour : Permanent gases :—

Critical Temperature.—*There is for every substance in the gaseous state a certain temperature such that if the substance be below this temperature, it can be liquefied by the application of a suitable pressure, and if above this temperature, it cannot be liquefied, however great the pressure applied may be. This temperature for a substance is called its critical temperature.*

The pressure which will liquefy the substance at the critical temperature is called its **critical pressure.**

Dr. Andrews found the critical temperature for carbon dioxide to be 31·1°C., and its critical pressure nearly 73 atmospheres. So above this temperature carbon dioxide is not liquefiable.

Gas and Vapour.—There is no hard-and-fast line of difference between these two terms ; one is often used to denote also the other. Strictly speaking, however, the term *gas* should be used to denote a substance in the gaseous state when the temperature is *above its critical temperature* ; whereas, the term **vapour** should be used when the same is at any temperature *below* its *critical temperature.*

Commonly, however, the term *vapour* is used in a restricted sense. It is used for substances in the gaseous state which at ordinary temperatures do not require any very large pressure to liquefy them, *e.g.* ether vapour, etc, ; for, a pressure of about half an atmosphere is sufficient to liquefy ether vapour at 12° to 15°C.

Permanent Gases.—At temperatures of *freezing mixture*, certain substances in the gaseous state, like ammonia, sulphur dioxide, chlorine, etc. can be liquefied with moderate pressures. Faraday in 1823 succeeded thus in liquefying many ordinary substances in the gaseous state, but found that substances like hydrogen, oxygen, nitrogen, air, etc. could not be liquefied in that way. So he called this class of gases, **permanent gases.** Some subsequent experimenters also similarly failed to liquefy these gases at temperatures of *freezing mixtures* by applying enormous pressures too. The reason for such failures at liquefaction was pointed out in 1863 by Dr. Andrews as a result of his celebrated experiments on carbon dioxide. He asserted that the temperature of the substance must be brought below the *critical temperature* before any pressures could liquefy it. It is now known that the critical temperatures of the so-called *permanent gases* are extremely low. This explains why Faraday failed to liquefy them. All known gases have already been liquefied and so the term *permanent gases* has no meaning to-day excepting its historical interest.

They only indicate in general those substances which do not liquefy at ordinary temperatures and pressures.

Substance	Normal Boiling Point °C.	Critical Temperature °C.
Sulphur dioxide	−10·1	157
Methyl Chloride	−24·09	143·3
Ammonia	−33·35	131·9
Carbon dioxide	−78·6	31·1
Oxygen	−182·983	−118·82
Nitrogen	−195·800	−147·13
Hydrogen	−252·78	−239·91
Helium	−268·92	−257·84

117. Boiling by Bumping :—When water is heated in a glass vessel, bubbles will appear in the body of the vessel and they rise to the surface with increase of temperature. These are air-bubbles dissolved in water. After a time, bubbles of steam formed at the bottom of the vessel while rising above towards the colder layers, collapse due to condensation. This produces a peculiar 'singing' sound. On further rise of temperature, the steam-bubbles rise vigorously to the surface and boiling begins.

If *pure* water, which has been previously boiled to drive away dissolved air, be heated in a clean vessel, bubbles will not be formed for some time and the temperature will rise above the boiling point. This phenomenon is called the **superheating** of the liquid. Then suddenly large bubbles will be formed which will burst forth with explosive violence and there is a tendency for the whole liquid to be thrown out. The temperature of the liquid now comes down to its normal boiling point. This phenomenon is called **boiling by bumping.**

Bumping may be prevented by introducing some rough materials, say, a few fragments of glass or porcelain, into the liquid, as the presence of the crevices will facilitate boiling.

118. Condition for Boiling :—*A liquid boils at a temperature at which the pressure of its vapour is equal to the superincumbent pressure, i.e. the pressure to which the surface of the liquid is exposed.*

Expts.—(1) A barometer tube T is filled with mercury and inverted over a trough B of mercury (Fig. 54). The tube is completely surrounded with a jacket G through which steam can be passed. Introduce some water into the tube by means of a bent pipette, and gradually pass steam into the jacket. As the temperature rises, more and more water vapour is formed at the top of the mercury column, which depresses the mercury column until, if there be sufficient liquid present, the mercury inside the tube is at the

same level as that in the trough. This means that the pressure of the water vapour at the temperature of the steam, *i.e.* at the boiling point, is the same as the outside pressure, which is the atmospheric pressure ; or, in other words, *water (or any other liquid) boils at a temperature when its vapour pressure is equal to the pressure on the surface of the liquid.*

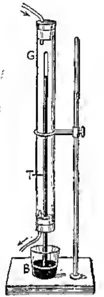


Fig. 54

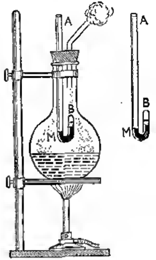


Fig. 55

Consulting the table of vapour pressures of water, it will be seen that the maximum pressure of water vapour at 100°*C.* is 760 mms. ; so water boils at 100°*C.* when the external pressure is 760 mms. ; similarly, water boils at 90°*C.* when the external pressure is 525.5 mms.

(2) Take a bent tube *AB* closed at *B*, as shown is Fig. 55. The small arm contains only some well-boiled water below which is mercury *M* which also partly rises in the longer arm. The level of mercury in the longer arm is below that in the other. Now introduce the tube into a flask containing some water such that the tube is above the surface of water in the flask. Boil the water and allow the steam, which surrounds the lower part of the tube, to escape through an exit tube. In a short time it will be found that the mercury assumes the same level in the two arms, showing that the

maximum vapour pressure at the boiling point is equal to the atmospheric pressure.

119. Boiling Point depends on the Pressure :—From the experiments already described it follows that *the boiling point of a liquid will change, if the pressure to which the surface of the liquid is exposed, changes.* Thus water will boil at a temperature higher than 100°C., if the atmospheric pressure is higher than 760 mms. and similarly, it will boil at a lower temperature if the pressure is lowered. So, on the top of a high mountain, water will boil at a temperature lower than 100°C.

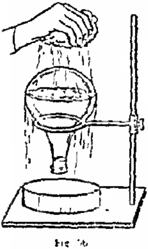


Fig. 56

(1) Boiling under Reduced Pressure.—This is demonstrated by the following experiments :—

(a) Franklin's Expt.—Boil some water in a strong glass flask until all the air is expelled. Now remove the burner, and invert it after it is tightly corked (Fig. 56). The space above the surface of water contains saturated water vapour. When boiling ceases, pour some cold water on the flask. This condenses some vapour inside the flask and thus reduces the pressure over the surface of water, and so the water boils again. This shows that boiling is possible at a temperature below 100°C. by reducing the pressure on the liquid.

(b) The same result can be produced by placing a beaker containing some boiling water in the receiver of an air pump. On pumping out some air (as soon as boiling ceases), the water will again be found to be boiling.

(2) Variation of Boiling Point with Pressure.—A liquid can be boiled at different temperatures by changing the pressure of air above the surface of the liquid. The arrangement is shown in Fig. 57. The liquid is placed in a boiler *A* which is connected with a large air-reservoir *B* through a Liebig's condenser *C*. The reservoir *B* is connected with a mercury manometer *M* and an air pump. The liquid is heated until it boils under a given pressure, and the boiling point is read by means of a sensitive thermometer *t*, the bulb of which is placed in the vapour, and in the liquid. The reason for this is that liquids sometimes may boil irregularly such that the temperature may rise several degrees above the true boiling point. The condenser condenses the vapour and restores it back to the boiler *A*. The reservoir *B* containing air is surrounded by water to keep its tempera-

ture constant. The pressure in B is adjusted to a definite value by connecting it with a compression or exhaust pump as is required for increasing or reducing the pressure. Take the reading of the thermometer when it becomes stationary after boiling commences, and record

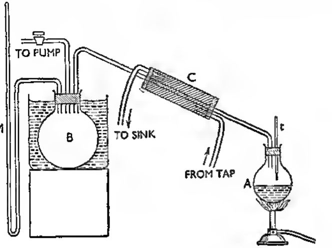


Fig. 57

the manometer reading at the same time. When the liquid boils the pressure of its vapour is equal to the superincumbent pressure which is indicated by the manometer M. By altering the pressure to a new value, a new boiling temperature is obtained.

By this means Regnault determined saturation pressures of water vapour up to 230°*C*., the pressure at the last temperature being $27\frac{1}{2}$ atmospheres, and he used this method for determining vapour pressures of water between 50°*C*. and 230°*C*.

On the top of a mountain the pressure is less than that at sea-level; so the boiling point of water there is less than 100°*C*. For example, water boils at 93·6°*C*. at Darjeeling, which is about 7,200 ft. above the sea-level; and at Quito (in S. America), the highest city in the world (9,520 ft. above the sea-level), the normal height of the barometer is 52·3 cms. and water boils at 90°*C*. At the top of Mont Blanc (15,781 ft.) water boils at 85°*C*.

It has been found that the boiling point of water decreases by 1°*C*. for every 960 ft. increase in elevation above sea-level, or in other words, *for a reduction of pressure of 26·8 mms. the boiling point falls by 1°C.*

119. (a) Papin's Digester :—The cooking power of boiling water depends upon the temperature at which it boils; hence on the top

of a very high mountain it is impossible to cook food in an open vessel. But, by increasing the pressure, water can be made to boil at any higher temperature. So for cooking food on the top of a very high mountain a specially closed vessel provided with a safety valve is used, the pressure within which can be raised to about 760 mms. This special contrivance is named *Papin's Digester*. Ordinarily, by closing a pot with a lid the difficulty of cooking etc. can be solved to some extent.

Boiling under increased pressure is useful for the manufacture of artificial silk; for the preparation of pulp (used in paper-making) by boiling wood with caustic soda etc.

Boiling under diminished pressure also has its uses. For instance, in the preparation of condensed milk, much of the water of the milk is driven off at a low temperature in order to keep the food value of the milk unaltered. Sugar is also refined by similar process.

120. Boiling Points of Solutions:—What has been said so far regarding boiling points is confined to pure liquids only, such as water, ether, etc. The law, namely, a *liquid boils at a temperature at which its vapour pressure is equal to the pressure on its surface*, is also obeyed by the boiling points of solutions, but the vapour pressure of a solution at a particular temperature is always less than that of the pure solvent at the same temperature, so the temperature of the solution has got to be raised above the boiling point of the pure solvent before it will boil. So, (*a*) *the boiling point of a solution is always higher than that of the pure solvent, and* (*b*) *the amount by which the boiling point is increased is proportional to the concentration of the solution.*

Besides the effect of pressure, *the boiling point of a liquid is also affected by the presence of substance dissolved in it.* For example, the boiling point of sea-water is about 104°C. while that of pure water is 100°C., and it has already been said that the increase of the boiling point depends upon the weight of the substance dissolved. So, *the purity of a liquid can be tested by its boiling point.*

121. Laws of Ebullition:—

(*1*) *Every liquid has got a definite boiling point at a particular pressure; by increasing or decreasing the pressure the boiling point is raised or lowered.*

(*2*) *A liquid boils at its boiling point when the maximum pressure of its vapour is equal to the atmospheric pressure.*

(*3*) *The temperature at which a liquid boils remains stationary until the whole of the liquid is evaporated.*

(*4*) *The temperature during boiling is constant so long as the pressure is constant. A definite quantity of heat, known as the latent heat of vaporisation, is absorbed by unit mass of the liquid in changing from the liquid to the vapour state at the same temperature.*

122. Ebullition and Fusion Compared :—

(*a*) The temperature remains stationary throughout each process, when the corresponding latent heat is absorbed.

(*b*) As there is super-cooling of a liquid under some conditions, so there may be super-heating, that is, the liquid may be heated above its boiling point without boiling.

(*c*) Both the freezing and the boiling points of a liquid are changed with pressure, though in the first case it is very small.

(*d*) For both the processes there is generally an increase in volume.

(*e*) In the case of a solution, the freezing point of a solution is *lower*, but the boiling point is *higher* than that of the pure solvent.

123. Change of Volume of Water with Change of State :— When heated, water is changed from the solid at 0°C. to the liquid state, its volume decreases up to 4°C., after which it gradually increases up to 100°C., and when it is changed into steam at 100°C. at atmospheric pressure, its volume is increased more than 1670 times ; that is, a cubic inch of water produces about a cubic foot of steam. The curve (Fig. 58) shows diagrammatically (not according to scale) the changes in volume when 1 gram of ice at

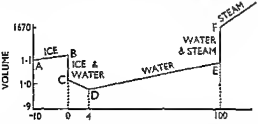


Fig. 58

-10°C. is heated to steam. The portion *AB* of the curve represents the expansion of ice as its temperature increases from -10°C. to 0°C. The portion *BC* represents the state of melting of ice when the volume diminishes, the temperature remaining constant at 0°C. The portion *CD* shows the diminution in volume of water as its temperature rises from 0°C. to 4°C. when it attains the minimum volume, after which the volume of water increases as its temperature rises from 4°C. to 100°C. at which the water begins to boil. This is represented by the portion *DE*. The portion *EF* shows the state when water boils and changes into steam, the temperature remaining constant at 100°C., but the volume of the steam formed is enormously increased being about 1670 times the volume of water taken. The portion beyond *F* shows the increase in volume of the steam with the rise of temperature.

[Hints.—In the glass vessel evaporation takes place only over the small surface exposed to the outside air through the neck of the bottle, while in the other case evaporation takes place through the pores of the whole vessel ; hence there is greater fall of temperature in its case.

If there is no difference of temperature, it shows that the atmosphere is saturated with water vapour.]

12. Distinguish between evaporation and boiling, and discuss the factors governing them.

Describe an experiment to show the cooling of a liquid by evaporation and explain the observed effect.

Do you know of any machine in which the above principle has been utilised ? (C. U. 1930)

13. Explain the construction and action of some kind of practical freezing machine that does not require the freezing mixture. (Pat. 1931)

14. Write a note on 'Refrigerators'. (U. P. B. 1933)

15. How would you find out whether a space is saturated or not ? (C. U. 1929, '32 ; Pat. 1931 ; Dac. 1931)

16. What is meant by maximum vapour pressure of water vapour ? Describe an experiment to determine it from the laboratory temperature up to 100°C. (C. U. 1921 ; cf. 1916, '17, '24, '34 ; cf. Dac. 1931 ; U. U. 1932)

17. Two barometers stand side by side. A few drops of water are introduced into the vacuum of one and a little air into the other. What would be the effects on the errors of the barometer readings thus produced for (a) change in the atmospheric pressure, (b) a change in temperature. (C. U. 1939)

18. What is saturated vapour pressure ? Under what conditions is a vapour able to exert such pressure ? What happens when unsaturated vapour is compressed till further compression is impossible ?

If water boils at 99°C when the pressure is 733 mm, what is the saturated pressure at 101°C. ? Explain briefly. (Pat. 1929)

[*Ans.* $760+(760-733)=787$ mm.]

19. Distinguish between saturated and unsaturated vapours and discuss their behaviour as regards change contemplated by Boyle's and Charles' laws. (Pat. 1931, '42)

20. Describe the behaviours of saturated and unsaturated vapours when the pressure exerted on them is varied. (C. U. 1934)

21. Explain how the maximum tension of aqueous vapour is determined at temperatures below and above the normal boiling point. (C. U. 1917 ; Utkal, 1952)

[For determining the maximum tension of aqueous vapour from 0°C to 52°C, *vide* Art. 112 (Fig. 53), and from 52°C upwards, *vide* Art. 119 (Fig. 57).]

22. Water is sprinkled in a room containing a barometer. State how will the barometer be affected under the following conditions :—

(a) The doors and windows are closed and the room is gradually heated.

(b) The room is heated but with doors and windows open. (Pat. 1916)

23. Into a cylinder exhausted of air and provided with a piston, there is introduced just enough water to saturate the space at 20°C. Describe what happens under the following conditions :—

(a) The volume of the space is increased by pulling up the piston.

(b) The volume is diminished by pushing the piston down.

(c) The volume remaining as at first, the temperature is increased to 30°C.

(d) The temperature falls to 10°C. (C. U. 1910, '23, '31)

24. 50 c.c. of a gas are collected in an inverted tube over water. The height of the barometer is 77 cms., the temperature of the room is 17°C. and the water level inside the tube is 7·6 cms. above that outside. What is the volume of the dry gas at 0°C. and at 76 cms. pressure ? The maximum pressure of aqueous vapour at 17°C. is 14·4 mms.

[*Ans.* 46·5 c.c.]

25. Enunciate Dalton's laws of partial pressure. (All. 1920 ; Pat. 1926, '40)

26. A mass of air is saturated with water vapour at 100°C. On raising the temperature 200°C. without change in volume, the mixture exerts a pressure of 2 atmospheres. What was the pressure due to air alone in the initial condition ?

[*Ans.* 438·6 mms.] (Pat. 1938)

27. Distinguish carefully between a gas and a vapour.
(Pat. 1926, '44 ; C. U. 1927 ; Utkal, 1951)

28. Describe an experiment to show that the vapour pressure of a liquid exposed to air at its boiling point is equal to the atmospheric pressure.
(C. U. 1915 ; Pat. 1931 ; G. U. 1953)

29. Explain the statement, "the vapour pressure of a liquid at its boiling point is equal to the superincumbent pressure". How is this verified experimentally ?
(C. U. 1952)

30. Distinguish between boiling and evaporation. What conditions determine whether a liquid will boil or evaporate ?

(C. U. 1914, '25, '41 ; Pat. 1928, '41, '44 ; *cf.* Dac. 1931)

[**Hints.**—A liquid evaporates as long as the vapour pressure at the temperature of the liquid is less than the atmospheric pressure, and it boils when these two pressures are equal.]

31. Explain how a knowledge of the boiling point of water would enable you to determine the barometric pressure.

Into the Torricellian vacuum of a barometer, water is introduced drop by drop till some water is left over. From the depression of the mercury column it is possible to determine the temperature of the room. How ? (C. U. 1913, '20)

[**Hints.**—A liquid boils when its vapour pressure is equal to the superincumbent pressure. Knowing the boiling point we can find out the vapour pressure from the Reganult's table which will be the same as the barometric pressure.

From the depression of the mercury column the maximum vapour pressure at room temperature is known. Now by consulting Regnault's table, the temperature corresponding to this vapour pressure, is known, which is the same as the temperature of the room.]

32. Why does it take a longer time to cook food on the top of high mountains ? At Darjeeling the barometric height is found to be about 23" only. At what temperature will you expect water to boil there ? (Pat. 1919)

[**Hints.**—There is a change of 0·04°C. in the boiling point for a change of 1 mm. (or 0·04 inch) in pressure.]

[*Ans.* 93°C.]

33. Define boiling point of a liquid. Describe suitable experiments to show that water can be made to boil at temperatures greater or less than 100°C. At Darjeeling the barometric height is about 23 inches. If there is a change of 0·04°C. in the boiling point of water for a change of 1 mm. of Hg., at what temperature will water boil there ? (Dac. 1942)

[*Ans.* 92·97°C.]

34. Define boiling point of a liquid. Describe suitable experiments to show that water can be made to boil at temperature greater or less than 100°C.
(C. U. 1930, '41 ; Dac. 1932)

35. State the laws of boiling. How is it that things cannot be cooked properly on a high mountain. How can water be made to boil at any temperature above 100 C. ? (Del. U. 1913)

36. Heat is continuously applied to a mass of ice at −10 C. until it becomes steam at 100 C. If the temperature is taken at intervals of time and a graph is plotted of the temperature against time, what would be the shape of the curve obtained ? Give reasons for this. (Pat. 1933 ; C. U. 1922)

37. Explain how you are able to determine (approximately) the height of a mountain by finding the boiling points of water at its top and bottom. (C. U. 1915 ; *cf.* Pat. 1913)

CHAPTER VII

HYGROMETRY

125. Hygrometry :—Aqueous vapour is more or less always present in the atmosphere, for, evaporation takes place constantly from the surface of water such as from the seas, rivers, lakes, the moist earth, the vegetations, etc. *Hygrometry* is that part of Physics which deals with the measurement of the amount of aqueous vapour present in a given volume of air. The formation of cloud, mist or fog, dew, etc. proves that water vapour is present in the atmosphere.

On a warm damp day the outside of a tumbler of *cold* water soon becomes covered with dew owing to condensation of water vapour from the air.

It has also been observed that on a cold night water vapour condenses on the inside of the glass panes of a sitting room window. The room receives water vapour from the breathing of the persons in it, but this vapour cannot saturate the warm air of the room. The glass of a window being thin is cooled to a lower temperature by the cold air outside. The air in contact with the glass is cooled to a low temper[illegible] ,and becomes saturated with water vapour which is con[illegible]. This shows the existence of water vapour in t[illegible] The doors and windows [illegible]

(b) The room is heated but with[illegible]rdinarily the quantity of water vapour

23. Into a cylinder exhausted of air[illegible] sufficient to produce saturation,
introduced just enough water to saturate the [illegible] vapour is less than the saturation
under the following conditions :—

(a) The volume of the space is increased [illegible] but the same quantity of vapour

(b) The volume is diminished by pushing [illegible]n at a lower temperature. If

(c) The volume remaining as at first, the [illegible]he pressure remains constant

(d) The temperature falls to 10°C. [illegible] pressure ; the air contracts

in volume and more air enters from the surrounding regions, but the pressure does not change. In case of the aqueous vapour in the air, the same statement is true as long as the air is not saturated. This pressure also remains constant until, on cooling, a temperature is reached at which the air becomes saturated ; the pressure of the vapour at this temperature is the same as it was originally. If the air be further cooled, some of the vapour gets condensed as moisture and so the pressure falls. The temperature at which such condensation starts is called *the dew-point, and the saturation vapour pressure at the dew-point is equal to the pressure of the aqueous vapour under the original condition.*

Definiton.—*The temperature at which a mass of air is saturated with the aqueous vapour it contains is called the* **dew-point.**

It is clear from above that the pressure of aqueous vapour may be found by determining the dew-point and then finding from the Regnault's table of vapour pressures the saturation pressure at that temperature.

Relative Humidity.—For meteorological work, the *degree of saturation* of the atmosphere is more important than the actual amount of water vapour in the air. This is known as the *Relative Humidity* or the *Hygrometric State of the Air*. Relative Humidity may be defined as—

$$\frac{\text{the mass of water vapour actually present in any volume of air at } t^\circ C.}{\text{the mass of water vapour necessary to saturate the same vol. at } t^\circ C.} \quad \ldots \quad (1)$$

$$= \frac{\text{pressure of water vapour actually present in the air at } t^\circ C.}{\text{pressure of water vapour necessary to saturate the air at } t^\circ C.}{}^* \quad \ldots \quad (2)$$

$$= \frac{\text{saturation vapour pressure at the dew-point}}{\text{saturation vapour pressure at the temperature } (t^\circ C.) \text{ of the air}} \quad \ldots \quad (3)$$

Relative Humidity is generally expressed as a percentage and is calculated by applying either of the expressions (1) and (3).

That is, **Relative Humidity (R. H.)**

$$= \frac{\text{mass of water vapour actually present in any vol. of air at } t^\circ C \times 100}{\text{mass of water vapour necessary to saturate the same volume at } t^\circ C.} \text{ per cent.}$$

$$= \frac{\text{saturation vapour pressure at dew-point} \times 100}{\text{saturation vapour pressure at air temperature } (t^\circ C.)} \text{ per cent.}$$

*[Water vapour obeys the gas laws fairly well even up to the saturation stage. Suppose the (partial) pressure of water vapour in a volume V of air is p. If the absolute temperature of air is T and the mass of water vapour present in volume V of air is m, we have

$$m = \frac{PV}{KT} \quad \ldots \quad \ldots \quad (1)$$, where K corresponds to

unit mass of water vapour. Let P be the saturation vapour pressure of water at the same temperature as that of the air. Assuming the

equation to be still true, the mass M of water vapour which is required to saturate the air at the given conditions will be given by,

$$M = \frac{PV}{kT}, \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad (2)$$

Dividing (1) by (2), $\frac{m}{M} = \frac{p}{P}$.

A table is given below wherefrom it will be actually found that the water vapour in a given volume of air is nearly proportional to the pressure it exerts, where m represents the mass of water vapour necessary to saturate one cubic metre of air at the temperatures shown and p the saturation pressure of water vapour at those temperatures.

Absolute Humidity is defined as the mass of water vapour actually present in a given volume of air. This is generally expressed as the mass of water vapour in grams per cubic metre of air.

Example.—*On a certain day the dew point was found to be 12°C, when the temperature of the air was 15°C. Calculate relative humidity of the air.*

By consulting the table of vapour pressures it will be seen that the saturation pressure of aqueous vapour at 12°C = 10.51 mms and at 16°C = 13.62 mms.

$\therefore$ Relative humidity $= \frac{10.51}{13.62} = 0.77$ or 77 per cent.

MASS AND VAPOUR PRESSURE TABLE

Temperature (°C)	0	5	10	15	20	25
Mass m in gms.	4.9	6.8	9.4	12.8	17.2	22.8
Pressure p in mm	4.6	6.5	9.2	12.8	17.5	23.7

127. Dryness and Dampness :—Our *sensations of dryness or dampness* do not depend only *on the actual quantity of water vapour present*, but also on the *quantity of vapour necessary to saturate* the air at that temperature. It is on the *ratio of the above two quantities, i.e. on the relative humidity*, that our sensations of dryness or dampness chiefly depend. It is found that on a cold misty day in winter, when the air seems to be quite 'damp', the actual amount of water-vapour in a given volume of air is often less than that on a hot day in summer when we feel the air is 'dry', because in the former case the amount of vapour the atmosphere contains is a larger fraction of the amount required for saturation. The dampness or dryness of the air is *judged by the rate* at which evaporation goes on, and this depends upon *how far the air is from the saturation state*, i.e. how much more vapour it can take up, and does not depend only upon how much water vapour the air already contains.

Things such as wet clothes will be dried more quickly when the relative humidity of the atmosphere is low, because in such cases the atmosphere can readily take up more water vapour. Also the evaporation of moisture from such things as wet clothes will be more rapid if the air in contact with them is constantly renewed.

The ventilation of buildings is necessary for two reasons—to remove the carbon dioxide exhaled by us and also to remove the water vapour evaporated from our lungs and bodies.

Our bodies are constantly emitting water vapour, and this fact is very important from the standpoint of health. We know how difficult it is to work in a stuffy room. This is because the air in the room contains a lot of water vapour; that is, the air is nearly saturated with moisture due to which normal evaporation from our skin cannot go on, and this produces a feeling of uneasiness.

This is particularly the case when the temperature of the atmosphere is high, as the feeling of easiness depends upon evaporation from the body so that its temperature may not rise above the normal value. Hence in India the weather near about Bengal during the wet season is more oppressive than that in other parts where the temperatures may be even 10° to 20°*F.* higher, because the atmosphere is drier.

If the relative humidity of air is about 100 per cent., we perspire and the weather feels sultry and very oppressive.

Relative humidity is determined regularly at meteorological stations, because it affords information as to the likelihood of rain. We can expect rain when air contains a considerable amount of water vapour. This *damp air is lighter than dry air*, because water vapour is lighter than air. *The density of water vapour relative to dry air is 5/8.*

The record of the relative humidity is useful to the Public Health Department as certain diseases thrive in damp atmosphere. It is also important for certain industries; for example, cotton weaving and spinning can be conducted satisfactorily only when the air is comparatively damp. For this reason the damp climate of Lancashire has been found suitable for the development of the cotton industry.

128. The Hygrometers:—Hygrometers (Gk. *hygros*, wet + *metron*, a measure) are instruments used for the determination of the hygrometric state of the air at any place and time. The hygrometric state is given by the relative humidity.

The hygrometers can be divided into the following classes:—

(1) *Dew-point Hygrometers*— (*a*) *Daniell's Hygrometer.* (*b*) *Regnault's Hygrometer.*

(2) *Wet and Dry Bulb Hygrometer.*

(3) *Chemical Hygrometer.*

(4) *Hair Hygrometer.*

(I) Dew-point Hygrometers :—

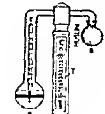


Fig. 59—Daniell's Hygrometer

(a) Daniell's Hygrometer.—It consists of two bulbs A and B (Fig. 59) bent downwards connected by means of a wide tube. One of the bulbs A contain ether, and the other bulb B with the tube connected to it is full of ether vapour, the air having been expelled before the apparatus was sealed up. There is a delicate thermometer t inside the bulb A containing ether. The bulb is silvered, or gilt within, while the other is covered with muslin. Another thermometer T placed on the stem C indicates the temperature of the air.

To determine the dew-point, some ether is poured on the muslin which, on evaporating, cools the bulb and condenses a portion of the ether vapour inside. The pressure inside being thus reduced, ether from the other bulb A evaporates and so it becomes colder. The temperature is reduced until the dew-point is reached. The temperature of the thermometer inside the bulb is noted as soon as the first film of dew appears on the silvered surface. The cooling process is discontinued by allowing the muslin to dry up and again the temperature is noted when the film just disappears. The mean of these two temperatures is the dew-point.

Sources of Error. This form of hygrometer is rather defective for the following reasons: (i) Ether evaporating outside B contaminates the air and thus effects the hygrometric state of the air, (ii) It is rather difficult to observe the exact moment of appearance or disappearance of dew as there is no comparison standard (*vide* Regnault's hygrometer), (iii) Inside the bulb A, ether evaporates mostly at the surface of the liquid which is thus cooled more rapidly than the interior and thus actual dew-point is not observed. (iv) Because glass is a bad conductor, temperature outside A is not the same as that inside.

Precaution.—With any hygrometer, observation ought to be taken either (i) by a telescope, or (ii) by placing a piece of glass between the observer and the apparatus, so that the result may not be effected by the heat from the body or breath.

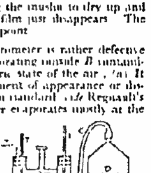


Fig. 60. Regnault's Hygrometer

(b) Regnault's Hygrometer.—This is a better form of hygrometer. This consists of a test tube having a side tube (Fig. 60), the lower part E of the test tube being made of silver. The mouth of the tube

is closed by a cork through which passes a delicate thermometer T. A glass tube A also passes down through the cork nearly to the bottom of the tube.

To work the instrument, some ether is placed in the test tube. The side tube is connected to a vertical brass tube, which again is connected to the rubber tube C with an aspirator. The vertical brass tube is supported in a clamp. A second glass tube similar to the first, fitted with a thermometer t inside it, is also mounted by the same support. By opening the aspirator, which is full of water in the beginning, a current of air is drawn through the tube E which causes rapid evaporation and sufficient fall of temperature to condense the water vapour on the outside of the silver tube E. The temperature is noted and the aspirator is shut off when dew is first observed. Condensation of water vapour is ascertained from the loss of brightness of the surface E. The temperature is again noted as soon as the dew disappears. The *mean* of these two temperatures is the *dew-point*. The aspirator must not be placed too close to the hygrometer, for the water released from the aspirator may then alter the humidity of the space around. The other tube is not an essential part of the instrument, and serves only as a standard of comparison of brightness of the two silver surfaces E and F. The thermometer t inside the other tube gives the temperature of the air. The relative humidity is then given by,

$$\text{Relative Humidity} = \frac{\text{saturation vapour press. at the dew-point}}{\text{saturation vap. press. at the tem. } (t°C.), \text{ of the air.}}$$

Advantages of Regnault's Hygrometer :—

(*i*) By regulating the flow of water of the aspirator, the rate of evaporation of ether in the tube can be better controlled than in the Daniell's Hygrometer.

(*ii*) Silver being a very good conductor of heat, the temperature of the ether, indicated by the thermometer, is practically the same as that of the silver surface which is in direct contact with the ether and the atmosphere.

(*iii*) The presence of the dummy tube facilitates the observation of the appearance and disappearance of dew on comparing the brightness of the two silver surfaces.

(*iv*) The continuous agitation of ether by the bubbling of air through it keeps the temperature uniform throughout its mass.

(*v*) Observations being taken from a distance by a telescope, the result is not affected by breath or heat from the body.

(2) Wet and Dry Bulb Hygrometer : (*Mason's Hygrometer* or *Psychrometer*).—The humidity of the atmosphere can also be judged by observing the rate of evaporation. When the atmosphere is dry, evaporation goes on more rapidly than when it is nearly saturated. Depending on this principle, the Wet and Dry bulb hygrometer is constructed.

It is a reliable apparatus used for the determination of *relative humidity* without necessitating the dew-point to be determined.

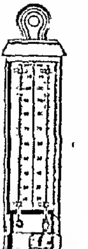


Fig 61 - Dry and Wet Bulb Hygrometer.

The hygrometer consists of two mercury thermometers, placed vertically side by side on a board which can be hung up against a vertical wall ; the bulb of one of the thermometers is covered with muslin, which is always kept moist by dipping its free end into water contained in a small vessel (Fig. 61). The continuous evaporation from the wet bulb keeps its temperature always lower than the other thermometer which is quite dry. The difference between the two temperatures indicates the humidity condition of the air. The drier the air, the quicker is the evaporation and the more rapid is the cooling, and so the difference between the *readings of the two thermometers will be* great and hence the *dew-point will be low*. When the difference is small, it indicates that evaporation from the wet bulb is very slow, and this is due to the presence of considerable water vapour in the air and hence the *dew-point is high*. If the air is already saturated, no evaporation will take place, and the two thermometers will give the same reading

Determination of Relative Humidity :—

(a) By Tables.—The dew-point and relative humidity can be found by means of an experimental table given below

t°C.	0	1	2	3	4	5	6
10	9·2	8·1	7·0	6·0	5·0	4·0	3·1
11	9·8	8·7	7·6	6·5	5·5	4·5	3·5
12	10·5	9·3	8·2	7·1	6·0	5·0	4·0
13	11·2	10·0	8·9	7·6	6·5	5·5	4·5
14	11·9	10·7	9·4	8·3	7·1	6·1	5·0
15	12·7	11·4	10·1	9·0	7·8	6·6	5·5
16	13·5	12·2	10·9	9·7	8·4	7·3	6·0
17	14·4	13·0	11·7	10·4	9·1	8·0	6·7
18	15·4	13·9	12·5	11·2	9·6	8·6	7·5
19	16·3	14·9	13·4	12·0	10·7	9·4	8·1
20	17·4	15·9	14·3	12·9	11·5	10·2	8·3

The first column gives the temperature (*t°C.*) of the dry bulb thermometer, and the second column the corresponding vapour pressure of water in millimetres. *The numbers* 1, 2, 3, *etc. at the* top of the other columns indicate the difference of temperatures in

degrees centigrade between the dry and wet bulb thermometers. The use of the table will be clear from the example given below :

Example. *The reading of a dry bulb thermometer is 18°C. and that of the wet bulb 16°C. Find the relative humidity of the air, and the dew-point.*

The difference in dry and wet bulb temperatures$=18-16=2°C.$

In the first column, we find 18°*C* and on the same level in the second column we find 15·4. Then 15·4 mms. is the vapour pressure at 18°*C.* Now at the same level in the column headed "2"—the difference of the two temperatures, we find 12·5. Then 12·5 mms. is the vapour pressure at the dew-point.

Hence the relative humidity $=\frac{12\cdot5}{15\cdot4}=0\cdot81$, or 81 per cent.

The dew-point is the temperature at which 12·5 mms. is the saturation vapour pressure. From the second column we find that 12·7 mms. is the vapour pressure at 14°*C.* and 11·9 mms. at 14°*C.* So the dew-point is little below 15°*C.* We observe from the table that there is a change of 0·8 mm. in the vapour pressure for a change of 1°*C.* in temperature from 14° to 15°; so, for an increase of (12·5−11·9) mm. in vapour pressure, the change in temperature$=0\cdot6/0\cdot8=0\cdot70°C.$

∴ The actual dew-point is $(14+0\cdot75)=14\cdot75°C.$

(b) By Formula.—The relative humidity and dew-point can also be calculated by determining the pressure f (in mm.) of aqueous vapour from the formula, $f=F-0\cdot00077\ (t-t')\times H$, where t is the reading of the dry bulb and t' that of the wet bulb thermometers on the centigrade scale, F the saturation pressure of the aqueous vapour at $t°C.$ and H, the atmospheric pressure (in mm.).

(c) By Glaisher's Formula.—The dew-point can also be determined from the **Glaisher's Formula.** If t_0=dew-point, then $t-t_0=F(t-t')$, where F is the Glaisher's factor.

The following table gives the values of the Glaisher's Factor corresponding to different Dry Bulb (D.B.) temperatures.

GLAISHER'S FACTOR TABLE

D.B. Temp. °C.	Glaisher's Factor (F)	D.B. Temp. °C.	Glaisher's Factor (F)
4	7·82	12	1·99
5	7·28	14	1·92
6	6·62	16	1·87
7	5·77	18	1·83
8	4·92	20	1·79
9	4·04	22	1·75
10	2·06	24	1·72
11	2·02	26	1·69
		28	1·67
		30	1·65
		32	1·63
		34	1·61
		36	1·59

(3) Chemical (or Absorption) Hygrometer.—The mass of water vapour present in a given volume of air can be measured directly in the following manner:—

The apparatus consists of an aspirator A (Fig. 62) filled up with water and provided at the bottom with an outlet tap. It is connected with a bottle B, called the trap bottle, containing concentrated H_2SO_4, which is connected successively to the drying U-tubes C and D filled with phosphorous pentoxide or anhydrous calcium chloride.

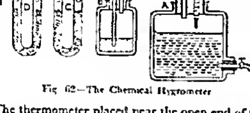


Fig. 62—The Chemical Hygrometer

The thermometer placed near the open end of the tube D registers the temperature of the air. In an expt., the tubes C and D are detached and weighed (W'_1). Water is then allowed to run out from the aspirator by opening the exit tap whereon a slow current of air is drawn into the aspirator. When a considerable amount of water has passed out, the tap is closed and the water level in the aspirator marked. The tubes C and D are taken out and weighed again (W'_2). *The difference in the two wts. gives the quantity of moisture absorbed from the air at a certain temperature.* In the second part of the expt. a tube charged with pumice stone soaked in water is then connected to the tube D. The aspirator is again filled up with water up to the same level as in the beginning of the first expt., the tap opened and water allowed to come out till its level again falls as before. In this case, the same volume of air saturated with water vapour is sucked in. The wt. of the tubes C and D is again taken (W'_3). *The difference between the second and the third weights gives the mass of moisture absorbed from an equal volume of saturated air at the same temperature.* Then relative humidity

$$= \frac{W'_2 - W'_1}{W'_3 - W'_2}$$

(4) Hair Hygrometer.—The principle of the Hair Hygrometer is very simple. Hair when moist slightly increases in length. This change of length with moisture is utilised in the working of this hygrometer.

A fine hair formerly treated with caustic soda solution so as to be free from grease and then washed and dried is stretched as shown in Fig. 63. Its one end is fixed at E while the other end passing ultimately round a grooved wheel F is attached to a

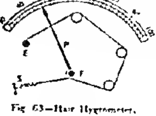


Fig. 63—Hair Hygrometer.

spring S which keeps it taut. When the length of the hair increases, the grooved wheel moves a pointer P attached to it over a scale which gives the relative humidity values directly, being previously calibrated by comparison with a standard hygrometer.

The advantages of the instrument is that it reads the humidity value directly when simply put in an enclosure.

129. Mass of Aqueous Vapour (*Mass of Moist Air*) :—It is often required to find the mass of water vapour present in a given volume of moist air. Assuming that the vapour obeys the gas laws and knowing that the density of water vapour compared with that of air is 5/8 (or 0·62) at the same temperature and pressure, the mass of a given volume of moist air can be calculated as follows :—

Suppose we want to find the mass of 1 litre of moist air at $t°C.$, when the height of the barometer is P mm. and the vapour pressure obtained from dew-point observation is f mm. According to Dalton's laws, the pressure of the air alone is $(P-f)$ mm. The volume V of 1 litre reduced to N.T.P. becomes $V = 1 \times \dfrac{273}{273+t} \times \dfrac{(P-f)}{760}$ litre.

The mass of 1 litre of air at N.T.P. is 1·293 gm.

$\therefore$ Mass m_1 of V litre of air at N.T.P. [which is the same as 1 litre at $t°C.$ and $(P-f)$ mm.]$=1·293 \times \dfrac{273}{273+t} \times \dfrac{P-f}{760}$ gm.

Again, the pressure of water vapour is f mm. ; hence its mass

$$m_2=0·62\times1·293 \times \frac{273}{273+t} \times \frac{f}{760} \text{ gm.} \qquad \qquad (1)$$

$\therefore$ The mass of 1 litre of moist air$=m_1+m_2$

$$=1·293 \times \frac{273}{273+t} \times \frac{(P-f)}{760} + 0·62\times1·293 \times \frac{273}{273+t} \times \frac{f}{760} \text{ gm.}$$

$$=1·293 \times \frac{273}{273+t}\left(\frac{P-f+0·62f}{760}\right) \text{ gm.}$$

$$=1·293 \times \frac{273}{273+t} \times \frac{P-0·38f}{760} \text{ gm.} \qquad \qquad (2)$$

Examples. (*1*) *Find the mass of a litre of moist air at 32°C. and 758·2 mm., the dew-point being 15°C. The maximum pressure of aqueous vapour at 32°C. is 12·7 mm.*

The whole gaseous mass may be divided into two portions,—one litre of dry air at 32°C. and (758·2—12·7) mm. or 745·5 mm., and one litre of water vapour at 32°C. and 12·7 mm. (*cf.* Dalton's law). 1 litre of dry air at 32°C.

and 745·5 mm. reduced to N.T.P. becomes $= 1 \times \dfrac{273}{273+32} \times \dfrac{745·5}{760}$ litre.

The mass of this air$=1·293\times\dfrac{273}{305}\times\dfrac{745·5}{760}=1·1352$ gm., since 1 litre of dry air weighs 1·293 gm.

The mass of aqueous vapour $= \frac{5}{8} \times 1{\cdot}293 \times \frac{273}{303} \times \frac{12{\cdot}7}{760} = 0{\cdot}0121$ gm.

$\therefore$ The mass of 1 litre of moist air $= 1{\cdot}1552 + 0{\cdot}0121 = 1{\cdot}1673$ gm.

(2) *A cubic metre of air at 30°C, of which the relative humidity is 0·8 is cooled to 5°C. Find the quantity of vapour which will be condensed into water. The maximum pressure of aqueous vapour at 30°C = 31·6 mm. and at 5°C = 6·5 mm.*

Relative humidity $= \dfrac{\text{mass } m \text{ of vapour present in 1 cu. m. of air at } 30°C}{\text{mass of vapour necessary to saturate 1 cu. m. at } 30°C}$

$\therefore \quad 0{\cdot}8 = \dfrac{m}{\frac{5}{8} \times 1293 \times \frac{31{\cdot}6}{760} \times \frac{273}{303}}$, whence $m = 22{\cdot}9$ gm. = mass of vapour present at 30°C.

Again, mass required to saturate 1 cubic metre of air at 5°C = $\frac{5}{8} \times 1293 \times \frac{6{\cdot}5}{760} \times \frac{273}{278}$ gm. $= m_1$ gm. (say) $= 6{\cdot}8$ gm. $\therefore$ Vapour condensed $= (m - m_1)$ gm. $= (22{\cdot}9 - 6{\cdot}8)$ gm. $= 16{\cdot}1$ gm.

(3) *If 200 gms. of water are allowed to evaporate in a room containing 50 cubic metres of dry air at 30°C and 760 mm., what will be the relative humidity of the air in the room?*

If f be the pressure of the vapour formed, we have (see Ex. 2)

$$200 = 50 \times \frac{5}{8} \times 1293 \times \frac{f}{760} \times \frac{273}{273+30} \qquad f = 4{\cdot}17 \text{ mm.}$$

The maximum pressure of aqueous vapour at 30°C is 31·6 mm. $\quad$ R.H. $= \frac{4{\cdot}17}{31{\cdot}6} = 0{\cdot}13$

(4) *The temperature of the air in a closed space is observed to be 15°C, and the dew-point 8°C. If the temperature falls to 10°C, how will the dew-point be affected? (Press. of aq. vapour in mms. of mercury, at 7°C = 7·49, at 8°C = 8·02.)*

(Pat. 1925, '31, '40, '41; G. U. 1947)

If the volume of the space be reduced, then, when the space is saturated with vapour, some vapour will be condensed but the pressure will remain constant, and if the space be not saturated, then there will be no condensation on reducing the volume, and pressure will be increased instead of remaining constant.

As it is a closed space (i.e. volume is constant), the pressure is proportional to the absolute temperature.

$$\frac{\text{Press. at } 10°C}{\text{Press. at } 15°C} = \frac{(10+273)}{(15+273)} = \frac{283}{288}$$

But the pressure at 15°C = maximum pressure at 8°C (the dew-point) = 8·02 mm.

$$\therefore \quad \text{Press. at } 10°C = 8{\cdot}02 \times \frac{283}{288} = 7{\cdot}88 \text{ mm. (approx.).}$$

Now, a temperature is to be found for which 7·88 mm. will be the maximum pressure. The temperature will be the dew-point corresponding to 10°C. From the data given it is seen that for a change of 1°C in temperature there is a change of (8·02 − 7·49) or 0·53 mm. in pressure.

Therefore for (8·02 − 7·88) or 0·14 mm. change in pressure, the change in temperature = ¼°C (approx.). So, at 10°C the dew-point is lowered by ¼°C.

130. Condensation of Aqueous Vapour:—The condensation of aqueous vapour in the air gives rise to the formation of cloud, rain, sleet, snow, hail, frost, fog or mist and dew.

(a) Clouds, rain, sleet, snow, hail, frost or hoar-frost.—

Clouds.—Due to constant evaporation from the water-covered areas of the earth's surface, the moist earth and the vegetations, the lower layers of the atmospheric air are always charged with water vapour the quantity of which differ from one region to another due to local conditions, temperature, etc. The moist warm air, saturated or unsaturated, being lighter than dry air, rises to higher levels where the pressure is less. The temperatures of the higher layers are also lower and lower, the higher the layers are, till the limit of the troposphere is reached. The rising moist air is gradually cooled down by expansion to lower and lower pressures and also due to the temperatures of the higher and higher layers being lower and lower. If the moisture-laden air is chilled below the saturation point, the excess of moisture is deposited into tiny droplets at some height. Clouds are nothing but formations of such droplets floating in the air. They remain stationary or may be moving with the wind.

Clouds show a great variety of forms. The variety is due to differences in the conditions under which the clouds are formed. When a big column of warm moist air ascends into the upper colder air, a region of condensation at the top of the ascending column is formed with a copious supply of vapour and we have a form of clouds known as **cumulus** clouds. When currents of moist air at different temperatures meet, the layers in contact may become regions of condensation and clouds of the **stratus** type appear. When the clouds are formed at great heights where the temperature is very low, the droplets in the condensed phase may be turned into tiny ice crystals forming what are known as **cirrus** clouds. The dark rain-clouds, known as **nimbus** clouds, are nothing but very dense *cumulus-like* formations at comparatively lower heights.

Rain.—If the lower layers of the atmosphere are saturated with water vapour, the small cloud particles in the condensed phase may collect into drops by coalescing with each other and fall by gravity as rain. As a rain drop falls, water vapour in the succeeding layers condenses on the cold drop which thus grows in size as it falls. The drops vary in size, and so is the velocity of the fall, for they pass through viscous air.

Sleet.—If the falling rain freezes before it reaches the ground, it is called *sleet.*

Snow.—If the cold at a layer of the saturated atmosphere is sufficiently intense to freeze the minute particles before they collect into rain drops, a fall of snow takes place.

Hail.—If the rain drops already form, and are then frozen, the result is *hail.* Due to violent air currents accompanying thunder-storms, the condensed moisture is carried up and down through

regions of snow and rain and so hail-stones with alternate layers of white snow and clear ice are formed.

Hoar-frost.—If the temperature of the earth's surface and of the objects on it rapidly fall below 0°C. before the air reaches the *dew-point*, the water vapour in contact with the surfaces directly turn into ice crystals and are fast deposited as *frost* or *hoar-frost*. Thus the frosting of surfaces is caused by direct freezing of water in contact and is not due to frozen dew.

(b) Fog (or mist).—The distinction between *fog* and *mist* is in the degree of condensation. Thick mist is *fog*. Fog or mist is a cloud formed at or near the earth's surface. The cloud is formed by the condensation of water vapour in the air on *hygroscopic particles of dust or dirt*. Ordinarily dusts do not serve as condensing nuclei—they must be appropriate particles. Thus in large towns and industrial areas, dotted with smoking chimney's the dense fogs that are formed are due to condensation of water vapour on particles of soot and other particles of dirt. Though clouds are formed much above the earth's surface and a fog or mist on or near it, the mode of fog formation is practically the same as that of a cloud, the difference being only in that in the formation of fog the moisture-laden mass of air must be at rest or at most in very slow motion, while in the case of clouds it need not be so. A fog is more stationary and fixed in form than a cloud.

The vapour-laden air must be cooled below the dew-point for the fog or mist to appear. When warm saturated air comes in contact with a mountain top which is very cold, the air is cooled below the dew-point and the mountain peak is enveloped with a thick mist. During a cold still night, cold air runs down a hillside into the valley and may so cool the air in the valley that its water vapour condenses into a thick *mist*. Such mists often begin to develop over damp meadows or marshy lands after sunset and fill the whole valley by early morning. A mist like this is quickly dispersed with the rising of the sun.

Fog generally disappears before noon. For, the atmosphere warms up and tends to be unsaturated when the condensed phase re-evaporates and the fog disappears.

(c) Dew.—During the day, the air in contact with objects which are heated by direct radiation from the sun, is charged with an amount of water vapour which remains unsaturated due to high temperature. During the night, cooling takes place and objects which radiate their heat well, cool below the temperature of the surrounding air and in consequence, the air in contact with them becomes saturated with the vapour it contains. With further cooling, a portion of the vapour is deposited as dew on the surfaces of the cold bodies. Green plants are good radiators of heat; so dew is deposited copiously on green leaves and grasses.

The conditions favouring the formation of dew are (*i*) *a clear sky* for free radiation from the heated objects ; (*ii*) *absence of wind* in order that air in contact with any object may remain there to be cooled below the dew-point ; (*iii*) the objects on which dew will be formed must be (1) *good radiators* so that they may cool rapidly, (2) *bad conductors* so that their loss of heat by radiation may not be compensated for by a gain of heat from the earth by conduction, (3) *placed near the earth*—if situated very much up above the earth's surface, the air in contact being chilled will become heavier and sink towards the earth and will be replaced by warm air from above, and so none of the air will be cooled enough so as to deposit its vapour as dew.

The above theory of formation of dew is due to Wells. *According to some experimenters, dew is formed not only out of the vapour present in the air, but also from vapour arising from the earth and the vegetation on which the dew is formed.*

130 (a). Rain-gauge :—It is an instrument for measuring the amount of rainfall in a locality. The instrument in common use is known as "*Symon's Rain-gauge*", which consists of a funnel provided with a circular brass rim having a diameter of five inches. It is fitted to a collecting vessel, which is generally a bottle *B*, placed within a metal cylinder (Fig. 64). The funnel *F* is kept one foot above the ground. The rain passing through the funnel collects into the bottle and the quantity collected in a certain period is measured by a glass cylinder graduated to hundredths of an inch. The rainfall of a place is expressed in inches or cm. per annum. An inch/cm. of rainfall means that the amount of water collected would fill to the depth of one inch/cm. a cylinder with its base equal to the rim of the funnel.

Fig. 64—Rain-gauge

In the Indian Union we have the greatest amount of rainfall in Cherapunji. The average value of annual rainfall in Bengal is about 75 inches, in Bihar and Orissa 52 inches, in Bombay 45 inches, and in Cherapunji 500 inches.

Questions

1. Why does a glass tumbler 'cloud over' on the outside when ice-cold water is poured into it ? (C. U. 1930 ; Dac. 1929)

2. Write a short essay on 'Hygrometry'. (U. P. B. 1948)

3. Explain the formation of dew. Show that the pressure of unsaturated vapour in a room is equal to the saturation pressure at dew-point.

Define Relative Humidity. On what factors does it depend ? Obtain an expression for its determination. (Pat. 1932 ; All. 1945 ; *cf.* G. U. 1949)

3. (*a*) A hot day in Puri causes greater discomfort than an equally hot day in Delhi. Why ? (C. U. 1948)

4. What is meant by Relative Humidity ? Explain how the determination of the dew-point enables you to calculate the relative humidity of a particular place. (All. 1946 ; A. B. 1952 ; C. U. 1953)

CHAPTER VIII

TRANSMISSION OF HEAT

131. Modes of Transmission :—There are *three* distinct processes by which *heat* may be *transferred* from a place to another. These processes have been named *conduction, convection* and *radiation.*

(1) Conduction.—In *conduction* heat passes along a substance from the hotter to the colder parts, or from a hotter body to a colder one in contact, without any transference of material particles. *When one end of a metallic rod is put into a furnace, the other end is heated by conduction.* A material medium only can pass heat by conduction.

(2) Convection.—In *convection* heat is transferred from the hotter part of a material medium to the colder parts by the bodily movement of hot particles. *When a vessel containing a liquid is heated from below, the upper layers of the liquid are heated mostly by convection.*

(3) Radiation.—In *radiation* heat is conveyed from one body to another, entirely separated from it, without heating the intervening medium which may be material or vacuous. *The heat of the sun is received on the earth's surface by radiation.*

132. Conduction :—When a body is heated, the molecules there vibrate vigorously, and this increased agitation (*i.e.* the increased heat energy) is passed on by collision from particle to particle. Consider the *mechanism of conduction of heat* to the other end of a metal bar heated at one end. Here heat is first communicated to particles of the bar in contact with the source of heat. These particles, as a result, vibrate more vigorously about their respective mean positions of rest and transfer the energy to adjacent particles by collision; and these the next layer of particles, and so on. The energy of vibration so conducted from layer to layer means the heat transferred by conduction. Some substances conduct heat better than others. Metals are generally good conductors, while substances like glass, mica, ebonite, felt, etc. are all bad conductors of heat. Air and other gases are bad conductors of heat.

Good and Bad Conductors : Expts.—(1) Prepare a small vessel of *thin* paper. Place a piece of copper wire-gauze on a tripod stand

and then place the paper vessel on it. Now carefully put some water into the vessel and heat the water gently from below the wire-gauze. After sometime the water will begin to boil. As the paper is *very thin*, the heat is conducted rapidly through the paper to the water and so the temperature reached is not sufficient for the paper to be charred. The temperature of water does not rise above 100°C.

(a) (b)

Fig 65

(2) Lower a piece of wire-gauze upon the flame of a Bunsen burner. The flame burns *below* the gauze and does not pass through the meshes of the gauze [Fig. 65(*a*)]. Now put out the gas, and holding the gauze about two inches *above* the top of the burner, turn the gas on. Light the gas above the gauze. It burns, but the flame does not travel down the gauze [Fig. 65(*b*)]. No combustible substance will burn, even in presence of air unless it is raised to a certain temperature known as the '*temperature of ignition*' for that particular substance. The reason why the flame does not pass through the meshes of the gauze is that metal wires conduct away the heat so rapidly that the temperature of the gas on the other side of the gauze does not rise high enough to ignite the gas.

133. Thermal Properties of some Materials :—

(a) Davy's Safety Lamp (Fig. 66) used in mines is an example in which the high conductivity of a metal has been utilised. It consists of an oil lamp, the flame of which is surrounded by a cylindrical wire gauze of close mesh. Even if the lamp is surrounded by an explosive gas, the heat is conducted away so rapidly that it prevents any flame from passing from the inside to the outside, and, when brought in the atmosphere charged with an explosive gas the danger is indicated by the *character of the flame.*

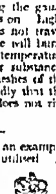


Fig. 66— Davy's Safety Lamp

(b) Other Illustrations.—The advantage of the bad conductivity of glass is often taken in opening a glass stopper which is stuck tight in the neck of a bottle. If the neck of the bottle is gently and carefully heated, the neck expands before the stopper which becomes loose thereby.

Our feeling of warmth or cold on touching different bodies depends to a *great extent* on conductivity. Thus, if we touch iron and flannel (both being placed in the *same* room) the temperature of which is *below* that of the hand iron appears to be colder, because it rapidly conducts the heat from the hand, and flannel being a bad conductor conducts very little heat. If the temperature of iron and flannel be *above* that of the hand, as

when kept in warm air (or rooms), iron appears to be warmer, because it rapidly conducts more heat to the hand than the flannel.

This is the reason why a pice of metal appears hotter to the touch than a piece of wood when both have been lying long in the sun ; and for the same reason a marble floor appears colder than an ordinary cemented floor.

(c) Use of Bad Conductors.—In summer ice is packed with felt or saw-dust which being bad conductors do not conduct heat to the ice from outside. We use woolen dress in winter because it conducts *very slowly* the heat of our bodies to the outside air, and thus the feeling of warmth is maintained. Again, the handles of kettles and tea pots are very often made of wood, or of vulcanite, in order that the heat from the hot water or tea may not pass through them as much as through a metal handle. Besides this it should be noted that the very low conductivity of cotton, wool, felt and other fabrics of open texture is largely due to the *low conductivity* of air enclosed in the fabric. For this reason wool is preferred to cotton for preparing warm clothings as the texture of wool is more loose and so it contains more air. It should be noted that the bad conductors not only keep *in* heat, but they also keep *out* heat.

134. Comparsion of Conducting Properties :—The conducting property of different substances can be compared by an experiment of the following type :—

Expt.—Take a cylinder one-half of which is brass and the other half wood. Wrap a piece of thin paper tightly round the cylinder, and hold the middle portion on a Bunsen flame (Fig. 67). It will be seen that the paper over the wooden portion is scorched long before any effect is produced on the other half.

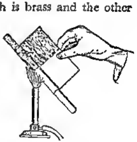


Fig. 67

The brass being a good conductor, conducts away the heat so rapidly that the paper is not scorched ; while wood, being a bad conductor, is not able to do this.

135. Thermal Conductivity :—

If Q=total quantity of heat conducted through a plate, then it is found, $Q \propto A$, the area of the plate,

$\propto (\theta_1-\theta_2)$, when θ_1, θ_2 are respectively the temperatures of the hotter and colder faces of the plate,

$\propto t$, the time in seconds in which the quantity Q passes.

$\propto 1/d$, d being the thickness of the plate.

$$\therefore \quad Q \propto \frac{A(\theta_1-\theta_2)t}{d}\text{; that is, } Q=\frac{K.A.(\theta_1-\theta_2)t}{d},$$

where K is a constant and characteristic of the material of the plate. This constant K is called the *thermal conductivity*, or the *coefficient of conductivity* of the material.

If in the above equation we take $A=1$ sq. cm., $d=1$ cm, $(\theta_1-\theta_2)=1°C$, $t=1$ sec., then we have $K=Q$ cals. cm.$^{-1}$ $C.^{-1}$ sec.$^{-1}$.

That is, **thermal conductivity** *of a material is the amount of heat which passes in one second through the opposite faces of a unit cube (i.e. 1 cm. cube) of it, the difference of temperatures between the opposite faces being 1°C.*

Note. The quantity $(\theta_1-\theta_2)/d$, or, in other words, the fall in temperature per unit length is called the *temperature gradient*.

136. Thermal Conductivity and Rate of Rise of Temperature :—

Let us consider a metal bar whose one end is held in fire. As the temperature at this end rises, the layer of the metal next to it receives heat by conduction. Of the transferred heat this layer absorbs a part on account of which its own temperatures rises, loses another part by radiation from its surface and convection of gases around it, and passes on the remainder to the next layer. This continues for some time. Then a stage comes when each layer attains a stationary or steady temperature, i.e. does not absorb any more heat passed on to it by the preceding layer. This state is known as the **stationary state.** After this state is reached, the passage of heat down the bar depends only on the conductivity of the bar. The state previous to the stationary state is called the **variable state**, for during the state each layer absorbs some heat from what it receives and rises in temperature. In this state both *absorption* and *conduction* of heat take place.

In the *variable state*, the rate of increase of temperature depends not only on the thermal conductivity of the substance but also on its *specific heat*, which is the quantity of heat required to raise unit mass of the substance through unit temperature. The quantity of heat reaching any portion of the rod will depend on the thermal conductivity, but the rise of temperature produced by that amount of heat will depend on the specific heat of the material. If the specific heat is low, the temperature of any portion of the rod rises quickly until the *stationary state* is reached, even if the conductivity of the substance is not high, because in this case, only a small amount of the heat that comes along is necessary to raise the temperature. But on the other hand, if the spec[illegible]

[illegible]

(i.e. volume = 1 c.c.) of the material of the rod. [illegible] Consider a unit cube

Let d = density of the material, i.e. mass of unit volume, s = specific heat of the material, $t°C.$ = rise of temperature per second, and Q = quantity of heat reaching the volume per second.

We have, then, $d.s.t.=Q$ or, $t=Q/d.s.$

That is, *the rise of temperature during the variable state produced in a unit volume of the rod is directly proportional to the quantity of heat reaching the volume, and so, to the thermal conductivity and inversely proportional to the product of the density and specific heat, that is, the thermal capacity per unit volume.*

So, the rate of rise of temperature depends on the ratio of,

$$\frac{K}{d.s} = \frac{\text{thermal conductivity}}{\text{thermal capacity per unit volume}}.$$

The ratio $K/d.s$ has been termed by Lord Kelvin as *diffusivity* (or *thermometric conductivity*) of the substance.

Taking the case of iron and bismuth, we have the thermal capacity of unit volume of iron ($7{\cdot}8 \times 0{\cdot}11 = 0{\cdot}858$) much greater than that of bismuth ($9{\cdot}8 \times 0{\cdot}03 = 0{\cdot}294$) and so, if we take a rod of bismuth and a rod of iron in Ingen Hausz's experiment (Art. 137), the rate of melting of the wax (*vide* Fig. 68) at the beginning will be much lower for the iron. But the thermal conductivity of iron being 7 times greater than that of bismuth, a longer length of wax will be ultimately melted along the iron.

Thus, it is clear that both the *thermal conductivity* and *specific heat* play important part during the *variable state*; but when the *sationary state* is reached, no more heat is absorbed, and then the flow of heat depends on *thermal conductivity* only. Therefore, in comparing the thermal conductivities of different substances, *we should wait until the stationary state is reached.*

137. Comparison of Thermal Conductivity :—

Ingen Hausz's Expt.—A number of metal or other rods, of the same length and diameter are introduced into the holes in front of a metal trough (Fig. 68). All the rods are previously covered with a uniform coating of wax, and the metal trough is then filled with boiling water. Heat is carried along each rod, and at the proper temperature, wax melts. After sometime a steady state (Art. 136) is reached, when there is no further sign of melting of the wax. It will be observed that the wax melts up to different distances along different rods showing that the conducting power of different substances is different.

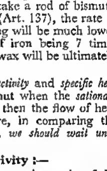


Fig. 68—Ingen Hausz's Method.

It can be proved from theoretical considerations that after the steady state is reached *the thermal conductivity of the different rods are proportional to the squares of the lengths of the wax melted on the rods.*

Thus, if l_1, l_2, l_3, etc. are the lengths of the rods, and if k_1, k_2, k_3, etc. are their thermal conductivities, we have, $k_1 : k_2 : k_3 \ldots\ldots\ldots = l_1^2 : l_2^2 : l_3^2 \ldots\ldots\ldots$

N.B. Before the steady state is reached in the Ingen Hausz's experiment, heat diffuses through the different rods at different rates depending on the diffusivity of the substances and so the time-rate of the melting of wax gives the measure of diffusivity along a rod.

138. Determination of Thermal Conductivity of Solids :—

The same method is not applicable to all solids for the determi- [illegible] metallic bars can be compared, as already described, by Ingen Hausz's method

Searle's Method for a good Conductor.—G. F. C. Searle, of Cambridge University, has devised the following method for a good conductor like copper, brass, etc. supplied in the form of a bar or rod.

A thick bar R of the specimen is taken and is well lagged with layers of wool or felt (Fig. 69). A chest P for passing steam is constructed around one end of the bar. Two holes E and H, 8 to 10 cms. apart, are drilled into the bar at the middle and are filled with mercury such that thermometers inserted into them may be in good thermal contact with the sections of the bar at E and H and record their temperatures truly. A copper tube C is wound round the bar at the other end and soldered to it. A steady flow of water is passed through it, water entering at M and leaving at N, and the inlet and outlet temperatures of the water are measured by means of thermometers introduced there.

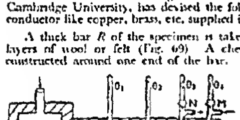


Fig. 69—Searle's Apparatus

Steam is turned on into the chest when the four thermometers show gradual rise of temperatures. After some time the temperatures become stationary when the *steady state* (Art. 136) is reached. Readings are to be taken after this state is reached.

Suppose the readings of the four thermometers, as shown in the figure, from left to right, are θ_1, θ_2, θ_3, θ_4. The steady flow of water in the tube C is collected in a beaker and suppose m gms. of water are collected in t secs. The time is measured by a stop-watch.

If Q=quantity of heat flowing through the bar per sec. at the steady state.

$\therefore Q=m(\theta_3-\theta_4)/t$, where θ_3 and θ_4, as already stated, are temperatures of the water at the outlet and inlet respectively.

But $Q=KA\left(\frac{\theta_1-\theta_2}{d}\right)$, where K=thermal conductivity of the bar, A=area of cross-section of the bar, d=distance between E and H, where the temperatures are θ_1 and θ_2 respectively as already stated. Hence K can be calculated from the following :

$KA\left(\frac{\theta_1-\theta_2}{d}\right)=m\left(\frac{\theta_3-\theta_4}{t}\right)$, all other quantities in the equation being known.

Examples.—*(1) If conductivity of sandstone is 0·0027 C.G.S. units and if the underground temperature in a sandstone district increases 1°C. for 27 metres descent, calculate the heat lost per hour by a square kilometre of the earth's surface in that district.*

We know that, $Q=\frac{KA(\theta_1-\theta_2)t}{d}$.

Here, $K=0{\cdot}0027$; $A=1$ sq. kilometre$=10^6$ sq. metres$=10^{10}$ sq. cms. ;

$(\theta_1-\theta_2)=1°C.$; $d=27$ metres $=2700$ cms. ; $t=3600$ secs.

$$\therefore \quad Q=\frac{0{\cdot}0027\times10^{10}\times1\times3600}{2700}=3{\cdot}6\times10^7 \text{ calories.}$$

N.B. The area given in sq. metre must be reduced to sq. cm., if the value of thermal conductivity is given in C.G.S. units.

(2) An iron boiler 1·25 cms. thick contains water at atmospheric pressure. The heated surface is 2·5 sq. metres in area and the temperature of the underside is 120°C. If the thermal conductivity of iron is 0·2 and the latent heat of evaporation of water 536, find the mass of water evaporated per hour. (Pat. 1930, '41 ; R. U. 1946)

Here, $K=0{\cdot}2$; $A=2{\cdot}5\times100\times100$ sq. cms. ;

$\theta_1=120°C.$; $\theta_2=100°C.$ ($\because$ The boiling point of water at atmospheric pressure is 100°C.) ; $t=60\times60$ secs. ; $d=1{\cdot}25$ cms.

$$\therefore \quad Q=\frac{0{\cdot}2\times2{\cdot}5\times10^4\times(120-100)\times3600}{1{\cdot}25}=288\times10^6 \text{ calories.}$$

The latent heat of evaporation of water is 536, i.e. 536 calories of heat are required to evaporate 1 gm. of water. Therefore, the number of grams of water evaporated by 288×10^6 calories of heat$=\frac{288\times10^6}{536}=537313$ gms.

(3) The absolute conductivity of silver is 1·53 ; its specific heat is 0·056, and its density is 10·5. Find (i) the thickness of silver plate 1 sq. cm. in area that would be raised in temperature through 1°C. by the quantity of heat transmitted in 1 second through another plate of silver of the same area and 1 cm. thick with a difference of temperature of 1°C. between its opposite faces ; (ii) the rise of temperature produced in a plate of silver 1 sq. cm. in area and 1 cm. thick by the same quantity of heat.

(i) Let x cm. be the thickness of the first plate, then its mass$=x\times1\times10{\cdot}5=10{\cdot}5x$ gm. Therefore the quantity of heat required to raise the temperature of this mass through 1°C.$=10{\cdot}5x\times0{\cdot}056$ cal.

But the heat which flows through the second plate in 1 second $= 1{\cdot}53$ cal. Hence $10{\cdot}5\,x \times 0{\cdot}056 = 1{\cdot}53$. $\therefore\ x = 2{\cdot}6$ cms.

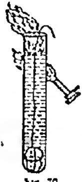


Fig. 70

(ii) Let θ°C be the rise of temperature produced in the plate of silver 1 sq. cm. in area and 1 cm. thick by 1·53 cal. of heat, then

$1{\cdot}53 = ms\,\theta = 10{\cdot}5 \times 0{\cdot}056 \times \theta$. $\therefore\ \theta = 2{\cdot}6$°C.

139. Conductivity of Liquids and Gases :—

Expt.—Wrap a copper wire round a piece of ice so that it may sink in water. Place this in a test tube and pour water in the test tube (Fig. 70). Now heat the upper part of the water with a flame. It will be found that water can be boiled at the upper part without melting the ice.

Liquids are generally bad conductors of heat, but *mercury* is a good conductor of heat, and is an exception.

The conductivity of gases (excepting hydrogen and helium) is extremely low and its determination is complicated by the effects of convection and radiation.

140. Convection:—When liquids and gases are heated, the heat is carried from one part to another by the *actual movement* of hot particles. These movements arise from the difference in temperature between different parts of the same substance. When the temperature at some part of a liquid or gas increases, it causes a reduction in density, and the hotter portion being lighter rises, its place being taken by the colder and heavier portion from the sides. Thus, convection currents are set up which can easily be visible by heating some water in a flask in which some colouring matter is kept at the bottom of the flask (Fig. 71).

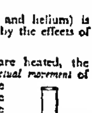


Fig. 71

141. Convection Currents in Liquids :—

Convection currents may also be illustrated by the apparatus shown in Fig. 72.

Expt.—A flask B (Fig. 72) and a reservoir A open at the top are connected by two glass tubes AB and CD. AB runs from the top of the flask to the top of the reservoir and CD runs from the bottom of the flask to the bottom of the reservoir. The whole apparatus is filled with water. The water heated in B ascends along the tube AB and the colder water in the upper vessel being lighter runs down the tube CD to fill the place. Thus a circulation is set up and finally all the water reaches

the boiling point. The motion becomes visible on dropping some dye into A, when the colour can be seen travelling down along the tube CD.

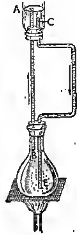


Fig. 72

The above experiment illustrates the principle applied in the *hot-water heating system* for buildings. In this case, a pipe rises from the upper part of the boiler to a reservoir at the top of the building and the downward pipe passes through a number of metal coils placed in various rooms and ultimately enters the boiler again. The water, in circulating through the pipe, is cooled and the heat is given out to the rooms.

This method of heating illustrates all *the three processes of transmission of heat*, viz. conduction, convection and radiation. It is by *conduction* that heat passes from the furnace to the water through the boiler; it is carried to the interior of the pipes by *convection*, and the whole system is a good example of a continuous water *convection current*. Heat is carried to the exterior of the pipes by conduction and it escapes into each room from the pipes and coils by *radiation*.

142. Convection of Gases :—The ascent of smoke up a chimney is a familiar example of convection. In the same way convection currents are produced in the chimneys of oil-lamps. Hot air above the fire rises up the chimney, its place being taken from below by cold air drawn from the room. Thus, a fire helps to ventilate a room. Winds are caused by convection currents in the atmosphere.

Warmth of Clothings.—The warmth of clothings depends to a large extent upon convection. A loosely woven thick cloth consists of wool fibres separated by air spaces. The heat of the body trying to escape to the outside must do so either by the zig-zag paths among the fibres or it must go through the shorter and more difficult path, partly through the non-conducting fibres, and partly across the air spaces, by setting up convection currents. Thus a loosely woven cloth is really warmer in cold air, which is at *rest*, than another cloth having the same amount of material but closely woven. The air should be at *rest*, otherwise the heat of our bodies will be lost by convection. For this reason closely woven cloth is necessary for people exposed to strong winds, that is, for aviators and motorists who can use leather cloth. So, our 'warm' clothes are not really warmer than other objects in the room.

143. Ventilation :—The ventilation of a room depends on merely establishing convection currents between the outside air and

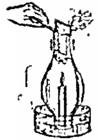


Fig 73

the air in the room. The following experiment will illustrate it :—

Expt.—Place a lighted candle on a saucer and pour water around it (Fig. 73). Put a lamp chimney over the candle. The flame after a while goes out as no fresh air can get in from below, and through the sides of the chimney.

Repeat the experiment and introduce a piece of T-shaped metal, or card-board sheet, down the chimney. The candle continues to burn. This is because the T-piece had divided up the chimney into two halves, one for up-draught to get rid of hot gases, and the other for down-draught to take in fresh air. The existence of these two currents can be shown by holding a piece of smouldering paper near the top of the chimney.

(a) Conditions necessary for proper ventilation in a room.—The things necessary for proper ventilation of a room are—an outlet for the warm and impure air near the top of the room, and an inlet for the cold pure air near the bottom of the room.

(b) Chimney.—The draught in a chimney of an ordinary lamp or over a furnace is due to convection. The heated air and smoke go up the chimney, while fresh cold air enters at the bottom and thus a convection current is set up. The *draught* is due to the *difference* in weight between the cold air outside and the hot air inside the chimney. The taller the chimney, the greater will be this difference in weight and so the greater will be the draught. So the factory chimneys are *tall*. But tall chimneys will be of no advantage unless there is enough fire at the bottom to keep the gas hot all the way up. In order that the descending currents may be prevented, narrow chimneys are better than wide ones.

(c) Gas-filled Electric Lamps.—The heat of the filaments of a gas-filled electric lamp, which contains a small quantity of some inert gas, such as argon or nitrogen, is carried away to the upper part of the bulb by means of convection current set up by the heated filament. As the heat from the filament is carried away, the filament can be raised to a higher temperature without any risk of melting than if surrounded by a vacuum. Besides this, the convection currents have another advantage ; they carry to the top of the bulb the tiny metal particles of the gradually disintegrating filament which cause the blackening of the lamp. Thus as the blackening, which would otherwise take place over the entire inside surface, is prevented, these lamps last longer than the vacuum type does (*vide* Ch. VIII, Part VIII).

144. Natural Phenomena :—

Winds.—Winds are due to convection currents set up in the atmosphere arising from unequal heating due to local reasons.

Land and Sea Breezes.—Convection currents account for land and sea breezes.

Sea Breeze.—During the day time land becomes more heated than the sea, firstly because of its greater absorbing power and secondly due to its lower sp. heat. In the evening, therefore, air above the land being more heated rises up and colder air from over the sea blows towards the land by convection, causing sea breeze [Fig. 74(a)].

Fig. 74(a)—Sea Breeze.

Land Breeze.—Since good absorbers are good radiators (Art. 151), during the night the land loses more heat than the sea. Sp. heat of land being lower, the temperature of the land in the early hours of the morning will be lower than that of the sea. So convection currents of air will flow from the land towards the sea [Fig. 74(b)], causing what is called the land breeze.

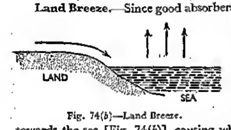


Fig. 74(b)—Land Breeze.

Trade Winds.—Heated air over the tropics rise up and cold air from the north and south moves towards the equator, but owing to the rotation of the earth from west to east, the wind gets a resultant velocity in the north-eastern direction in the northern hemisphere and south-eastern direction in the southern hemisphere. The first is known as *north-east trade wind* and the other as *south-east trade wind.*

145. Distinction between Conduction, Convection and Radiation :—(1) In conduction and convection heat is propagated in a material medium ; while in radiation no assistance of material medium, either solid, liquid or gas, is essential and heat energy passes through a vaccum, *without affecting the temperature* of the intervening medium ; but conduction and convection raise the temperature of the medium. In conduction there is no transference of material particles, while in convection the heated particles bodily move.

Heat energy s transferred to us from the sun through thousands of miles of so called vacuous space where there is no material medium. So heat is received by radiation from the sun. We also receive heat by radiation from a fire or any other hot body. If you hold your hand *below* an electric lamp, your hand will get warmer. This isniot due to conduction, for air is a bad conductor of heat, and it is

also not due to convection, for a convection current always has the tendency to move upwards. So it is due to radiation.

(2) A body emits radiation in *all directions* and in *straight lines* while in other processes it is not so. For this reason, a screen, placed between the source of heat and any body, cuts off the radiation.

(3) Transference of heat by radiation takes place almost *instantaneously*, while the other processes are comparatively *much slower*.

Radiant heat travels with the velocity of light, *i.e.* 1,86,000 miles per second.

146. Nature of Radiation (*Ether waves*) :—When we stand before a fire, we feel hot. It is obvious that we do not get heat from the fire by conduction because the air medium is a bad conductor; also the convection currents carry heat upwards and bring cool air from around to the fire. So the heat we feel is not due to convection. Again we know that we get something from the sun and fire that can give rise to sensation of heat and sometimes of light. This something is called *radiation*. Radiant energy reaches us from the sun, a distance of about 92,000,000 miles, in about 8⅓ minutes only. The atmosphere, which surrounds the earth, does not extend upwards indefinitely. How then, the radiant energy is communicated to us from the sun? To explain this, scientists have assumed the existence of a medium, called the *ether*, which is a very delicate medium and which is present everywhere, even in the interstices of the molecules of even the hardest solids, just as air is present everywhere between the leaves and branches of a tree.

Just as by disturbing the surface of water in a pond waves can be created, which spread outwards from the point of disturbance, so transverse waves are created in the ether by the rapid vibration of the molecules of a hot body, and these waves pass outwards in all directions with the velocity of light (186,000 miles per sec.) When these waves are stopped by a body, the molecules of the body are made to vibrate, producing heat in the body. There are some substances which allow these waves to be transmitted through them. These are called **dia-thermanous** substances; while the other substances which do not allow the waves to pass through them, are known as **adia-thermanous** or **a-thermanous substances**. A vacuum is perfectly dia-thermanous. Dry air, rock salt, carbon bisulphide are also good dia-thermanous substances. Wood, slate, metals, etc. are adia-thermanous. The latter class gets heated by absorbing radiant energy. It is to be noted that *radiant heat or* radiation, strictly speaking, *is not heat in the sense we understand it, but is energy which being absorbed by certain bodies manifests itself as heat, & during transit it is only the energy of the wave which passes through the ether.*

147. Radiant Energy :—Any form of energy transmitted by means of ether waves is called *radiant energy*. These ether waves

differ amongst themselves in **frequency** (*i.e.* in the number of vibration per second) and **consequently in the wavelength,** just as there are small ripples and big waves on the surface of the sea.

Waves of different lengths produce different effects. Very long ether waves carry *electro-magnetic waves energy*, and they are used for the transmission of wireless messages. Waves shorter than these give us *radiant heat* and still shorter waves affect our eyes, which we call **light.** The waves, which are still shorter, or rather too short to affect the eyes, can produce *chemical action* on photographic plates. These are alled **Ultra-violet rays.** Still shorter waves are known as **X-rays,** and waves still shorter than the X-rays are **Gamma rays** which are given out by radio-active substances.

A hot body at a low temperature is not visible in a dark room, as it emits only heat radiation. But at a sufficiently high temperature it becomes visible, when it emits also comparatively smaller waves, which can excite in our eyes the sensation of light in addition to that of heat; so, at a high temperature, it emits both kinds of radiation (heat and light). As water waves are produced by the vibration of water particles, so the ether waves are produced by the vibration of ether particles. Vibration of ether particles of certain degrees of rapidity produce mainly heating effects on bodies on which they fall; while certain others of higher degrees of rapidity can produce in our eyes the sensation of light. Longer waves are produced by slow vibration and shorter waves by rapid vibrations of ether particles. For example, vibrations between $3{\cdot}75\times10^{14}$ (*red*) and $7{\cdot}5\times10^{14}$ (*violet*) times per second producing ether waves of approximate lengths between 80×10^{-6} cm. (*red*) to 40×10^{-6} cm. (*violet*) can produce the sensation of vision. This is the range of **luminous radiations;** while the frequencies of **actinic radiations,** which can produce chemical changes are higher than $7{\cdot}5\times10^{14}$ times per second, *i.e.* beyond the violet end of the visible light. *So heat and light are both forms of radiant energy, and the difference between them is a difference in degree rather than in kind.* The waves producing thermal effect, and which do not affect our sense of vision, vary in lengths between 80×10^{-6} cm. to about 0·03 cm. These are called **Infra-red** waves. The waves which are smaller than 40×10^{-6} and produce actinic effects, *i.e.* produce chemical changes on plants and certain salts of silver due to which photography becomes possible, are called **Ultra-violet** waves. These vary in lengths between 40×10^{-6} to 1×10^{-5} cm. Waves smaller than these are popularly known as *X-rays*, the wavelengths of which vary between the limits 1×10^{-5} to 6×10^{-10} cm. There are also waves shorter than the X-rays which are known as *Gamma rays.*

On the other side beyond the Infra-red region there are very big radiant waves which do not affect any of our bodily senses. Very long ether waves whose lengths may range up to several miles are

known as '*wireless*' waves. The wireless waves can, however, also be small ; even waves of length 3 cms. have been used in wireless.

148. Instruments for Detecting and Measuring Thermal Radiation :—

(1) Ether Thermoscope.—The ether thermoscope (Fig. 75) contains some quantity of coloured ether and ether vapour, a chemical substance, the whole of the air from within having been expelled before sealing the instrument. One of the bulbs is coated with lamp-black which is a perfect absorber of thermal radiation. When thermal radiation falls on the black bulb, its temperature, and consequently that of the contained vapour, rises. This increases the pressure of the vapour on the ether inside the bulb. Hence the level of the ether in the black bulb is pushed down and that in the other bulb rises.

Fig. 75—Ether Thermoscope

(2) Differential Air Thermoscope.—This was first used by Leslie. It consists of a glass tube bent twice at right angles, terminating in two equal bulbs containing air. The tube contains coloured sulphuric acid up to a certain height and the quantity of air in the bulbs is so adjusted that the liquid stands at the same level in the two tubes when both the bulbs are at the same temperature. For a slight difference of temperature of the air in the bulbs, there is a small difference in level of the Liquid due to the expansion of air in the warmer bulb, which depresses the liquid column nearest to it and raises that in the other.

(3) Thermopile.—This is a very sensitive electrical instrument (*vide* Volume II) which is used by all modern experimenters (Fig. 76)

149. Radiant Heat and Light compared :—

(A) Similarity.—

(1) *Radiant heat and light travel in vacuum as well as in air in all directions with the same velocity.*

At the time of an eclipse of the sun when the moon comes directly between the sun and the earth, it is seen that heat and light from the sun are cut off at the same instant, showing that heat and light energy travel everywhere in all directions and with the same velocity (186,000 miles per second).

(2) *Radiant heat and light travel in straight lines.*

Two wooden screens are taken having a small hole in the middle of each. They are arranged parallel to each other, and a red-hot ('red-hot' at about 525°C) metal ball is placed opposite to the hole of one of the screens. If now the lamp-black-coated bulb of an ether thermoscope (Fig. 75), or a thermopile (Fig. 76), be placed far away

from the other screen and opposite to the hole in it, it will be observed that, when the two holes are in the same straight line with the ball, the thermoscope, or the thermopile, is greatly affected, while the effect is very little when the two holes are not in the same straight line. This proves that radiant energy travels in straight lines.

(3) *Heat rays can be reflected in the same way as light obeying the same laws as in the case of light.*

(*a*) *Reflection at a Plane Surface.*—Two tin-plate tubes are supported horizantally in front of a vertical polished tin-plate so as to be equally inclined to the plate. Now placing a hot metal ball near the end of one tube, and a thermopile, or the black bulb of an ether thermoscope, near the end of the other, the instrument is affected. The effect on the instrument will be much less when the tubes are placed unequally inclined to the plate. It will be found that the effect is a maximum when the tubes make equal angles on the opposite sides of the normal to the reflecting plate (*vide* Chapter IV, Part III).

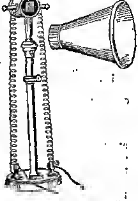


Fig. 76—The Thermopile.

(*b*) *Reflection at a Concave Spherical Surface.*—If two large concave metallic reflectors (*vide* Chapter IV, Part III) are placed co-axially facing each other at a little distance apart, then the blackened bulb of the thermoscope placed at the focus of one of them will be seen to be greatly affected by a red-hot ball placed at the focus of the other reflector. The difference in effect may be noticed by displacing the reflector a little towards the thermoscope.

(4) *Heat rays can be refracted in the same way as light, and they obey the laws of refraction of light.*

The rays from the sun, *i.e.* both the heat and light rays, can be concentrated at a point by means of a convex lens, and a piece of paper placed at the point may be easily ignited by the heat rays.

A better effect will be obtained by using a convex lens made of rock-salt, instead of glass, as rock-salt, being dia-thermanous to heat rays, absorbs only a small percentage (about 7 per cent) of them, while glass absorbs a considerable amount of heat rays.

(5) *The amount of heat received per second per unit area of a given surface, i.e. the intensity of radiation, by absorption of thermal radiation emitted by a source of heat at a constant temperature, is inversely proportional to the square of the distance between the source and the absorbing surface. This is known as the* **Inverse Square Law.**

(a) Emissive and Absorbing Powers of a Surface.—

Ritchie's Expt.—The apparatus consists of two cylindrical metal vessels *C* and *D* filled with air and connected by a glass tube bent twice at right angles in which some coloured liquid has been placed (Fig. 79). A large cylindrical vessel *AB* is supported between *C* and *D*. The surfaces *A* and *C* are coated with lamp-black while the other surfaces *D* and *B* are polished. When *AB* is filled up with boiling water, the level of the coloured liquid is found to remain the same, which shows that *C* and *D* are at the same temperature. The face *A* emits more than the face *B*, but the black face *C* absorbs more than the polished face *D*. As the level of the liquid remains the same, it shows that one vessel gain as much heat energy as the other, i.e. *the emissive power is equal to the absorbing power*.

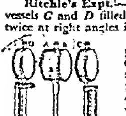


Fig. 79

As lamp-black is the best absorber, and the polished metallic surface the worst absorber, we conclude that **good absorbers are good radiators.**

A body which absorbs all the radiations incident on it is called a **'perfectly black body'**, or simply 'a black body'. *A black body* at any temperature emits **full radiation** for the temperature. A lamp-black surface though not a perfectly black body is the nearest approach to such a body as it emits or absorbs about 97% of the radiation.

(b) Radiometer.—This sensitive instrument was designed by Sir William Crookes for the detection of heat radiation. It consists of a glass bulb *B* almost completely evacuated [Fig. 79(a)]. It has four thin aluminium vanes *V* fastened to a vertical axis about which they can rotate freely. One surface of each vane is coated with soot while the other is polished.

The pressure of air inside the bulb being low, the molecules of air have better freedom of motion. When heat radiations fall on the vanes, the black surfaces absorb and radiate more heat than the bright surfaces and so air molecules colliding with black surfaces acquire higher kinetic energy and rebound with greater velocity than those from the bright surfaces. Thus every push received by a black surface from the air molecules is more vigorous and so it recedes, as a result of which their vanes rotate in a direction opposite to the direction of heat radiation.

The instrument is so sensitive that even a burning match stick held within a few inches from it will be sufficient to rotate the vanes.

Fig. 79(a)—Crooke's Radiometer.

152. Selective Absorption of Heat Radiation.—Different bodies, even when at the same temperature, will radiate, as also absorb, heat differently, and generally, bodies which can reflect heat radiation very well, are bad absorbers of heat. For example, *bad reflectors like lamp-black, ashes, etc. are good absorbers of heat.* It takes less time to boil water in an old kettle covered with soot than in a new one which is polished. The soot absorbs heat better than the polished metal and so water boils quickly in an old kettle. In winter, ice and snow kept beneath the ashes melt sooner than the ice and snow which are uncovered, because ashes are good absorbers of heat. *Good reflectors such as polished metals are bad absorbers and also bad radiators of heat.*

153. Some Practical Applications of Absorption and Emission :—In our everyday life we require for some purposes good reflectors of thermal radiation, while for other purposes good absorbers are necessary. A few examples are given below. Vessels such as tea-pots, calorimeters, etc. which are meant for retaining their heat are made with polished exteriors because polished bodies radiate less heat. For cooking purposes vessels should preferably be black with rough exterior. Black clothing is preferred in winter as it absorbs almost the whole of the heat rays falling on it and thus becomes warm, while white clothing is more suitable in summer as it absorbs very little of the sun's heat rays. The advantage of the white painted walls and roofs of a building is that they keep the building warmer in winter and cooler in summer than if they were painted with a dark colour. In order to cool down hot liquids quickly it is better to use a black stone vessel and not a metal cup with polished surface. Dry air absorbs very little heat radiation. It transmits nearly the whole amount of heat radiation falling on it, *i.e.* it is a *dia-thermanous substance*, while *moist air absorbs* heat radiation to a great extent. Thus, the moisture of the air helps us in two ways ; it prevents the earth from becoming too much heated during the day time by absorbing sun's rays and also from becoming too much cooled at night by absorbing the radiation escaping from the heated surface of the earth. We know that **clear night is colder than a cloudy night** *as clouds are partically opaque to the long heat rays radiated from the surface of the earth.*

Water transmits only 10 per cent of the incident heat radiation and alum transmits less. But when alum is mixed up with water, the transmitting power of the latter is increased.

Gases are bad radiators of heat ; so fire bricks, which are good radiators, are used in the construction of furnaces in which the hot gases are made to play on the fire-bricks, which are heated by contact and then radiate the heat freely.

(a) Green-House.—It is an example of selective absorption of heat by glass. The amount of heat transmitted through a substance

depends upon the temperature of the source of heat; for example, glass transmits about 50 per cent. of heat when the heat rays come from a source which is at a high temperature, *e.g.* the sun, or a hot fire. *Glass is adia-thermanous* to heat rays when the source is below 100°C. This is why heat accumulates in a green-house, the glass windows of which allow rays of heat from the sun to pass through them. These rays heat the objects, *i.e.* the plants and ground inside, but when the bodies inside, which are evidently at a temperature below 100°C., radiate their heat, the glass windows do not allow it to pass out. Glass thus serves as a *trap* to the sun-beams.

A glass fire screen is also an example of the above principle. It will absorb most of thermal radiations falling on it, while only a small part is transmitted along with the luminous portion. One, therefore, will see the fire while much of the heat is cut off. Ordinary *glass* not only *absorbs* the long *infra-red waves* but also the short *ultra-violet waves*. It *transmits* only the visible *light waves*. A special kind of glass has, however, been made which can transmit infra-red radiation. So these are used for camera lenses for long distance photography.

Quartz glass and *Vita glass* transmit ultra-violet portion of the radiation and they are often used for window-panes in hospitals.

(b) Temperature of the Moon's Surface.—Like glass, water is dia-thermanous to radiations from a hot source, but adia-thermanous to those from cold bodies. This fact has been applied to measure the temperature of the surface of the moon. It is known that the moon reflects the sun's radiations and also emits its own. These two different types of radiations have been separated by passing the radiations through water, when the sun's radiation will be transmitted, while those from the moon will be absorbed, by calculating the amount of which the temperature of the moon's surface can be determined.

154. Radiation Pyrometry :—It has been stated in Art. 15 that very high temperatures can be measured by a system of measurement known as *radiation pyrometry*. By this system very high temperatures of bodies like the sun or other heavenly bodies at great distances or of furnaces, can be measured from the radiation emitted by them.

155. Dewar's-Flask (Thermos-Flask or Vacuum Flask) :— It is a flask in which loss or gain of heat through conduction, convection and radiation has been reduced to a minimum. It is used for keeping a hot liquid hot and a cold liquid cold for a good length of time.

It consists of a double-walled glass flask B_1B_2 (*Fig.* 80) placed on a spring *S* within a metal or wooden casing *C*, its mouth being closed by a cork stopper *A*. The space between the flask and the outer casing C is preferably packed with a non-conducting material *D* like

felt. The space between the two walls of the flask is exhausted of air by pumping out the air through the nozzle at the bottom which is finally sealed off. The outer surface of the inner wall and the inner surface of the outer one are silvered. This vacuum belt around the liquid in the flask prevents any loss or gain of heat through conduction and convection, while radiation is reduced to a minimum due to the silvering of the surfaces. The non-conducting packing of felt reduces any sharing of heat by conduction through the walls. Conduction, convection and radiation, the three possible modes of exhange of heat being guarded, the liquid remains almost in a state of thermal isolation and it thus maintains its own temperature for a pretty long time.

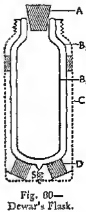


Fig. 80—Dewar's Flask.

156. Heat Loss by Radiation :—The rate at which a body loses heat by radiation depends on (*i*) the temperature of the body, (*ii*) the temperature of the surrounding medium, and (*iii*) the nature and extent of the exposed surface.

Newton's Law of Cooling.—*The law states that the rate of loss of heat from a body is proportional to the mean difference of temperature between the body and its surroundings.*

Verification of Newton's Law of Cooling.—Take some hot water in a calorimeter and note the temperature of the water at an interval of one minute for a period of about 20 minutes, carefully stirring the water all the time. Now, note the fall of temperature for a small interval of time, say, 2 minutes and also the mean difference of temperature between the water and the air of the room during the same two minutes interval. Calculate the ratio of the fall of temperature during this interval to the mean difference of temperature, and repeat the process taking the fall of temperature at various stages all over the period of 20 minutes. It will be found that these ratios are practically equal.

As the mass and specific heat of the liquid are constant, this experiment shows that the rate of cooling of the water is proportional to the mean difference in temperature between the water and its surroundings. This will be true for any other liquids, and this fact was first expressed by **the Newton's law of cooling.**

Again, taking two or three calorimeters and recording the time and temperature as before, it will be seen that *the amount of heat lost per second depends also on the extent and the nature of the radiating surfaces.* The rate of cooling does **not, however,** at all depend on the nature of the liquid.

Discussion of Newton's Law of Cooling.—The above law applies when a body cools in air due to loss of heat by radiation and convection only. Moreover, as Newton expressly stated, the body must be "*not in still air, but in a uniform current of air*". That is, the law applies to cases of loss of heat under *radiation and forced convection* and does not apply to natural convection as in still air. The law is true even for large differences of temperature provided that there is a uniform current of air in which the body is placed, as in forced convection. In natural convection of air, the rate of loss of heat by a body, it may be noted, is proportional to the $\frac{5}{4}$th power of the temperature difference.

In the laboratory generally we apply Newton's law of cooling to the case of a calorimeter placed in still air. The justification, if any, is that the error in doing so is quite small, if the temperature difference is small.

156(a). Prevost's Theory of Exchanges :—A hot body gives out *hot radiations* and a cold body *cold radiations*—these were the ideas until 1792. When a man stands before a block of ice, he feels cold. This occurs, the people in old days thought, *as though cold were radiated from the ice*. The true explanation came in 1792 when Prevost of Geneva propounded his Theory of Exchanges. According to this theory, which is also the modern conception, a body whether hot or cold, gives out only one form of radiations, namely heat radiation and these are radiated at all times provided the temperature of the body is above the absolute zero, irrespective of the presence of other bodies ; moreover, the higher the temperature, the greater is the amount of radiations.

Let us apply this theory to an enclosure in which, suppose, two bodies at different temperatures are placed. Each will radiate out heat according to its own temperature independent of the other and again will receive heat being placed in the field of radiation of the second. The one initially hotter of the two gives out more heat than it receives while the colder gives out less heat than it receives. As a result of exchange of heat, the hotter falls in temperature and the colder gains in temperature until a common temperature is attained by both. We then say that an equilibrium has reached. Even when the temperature is equalised, the exchange does not stop, each receives as much heat from the other as it itself gives out. The equilibrium is a *dynamic* one. The theory applies to any number of bodies at different temperatures at a time.

In the case of the man and the ice-block, as a result of the differential effect of exchange of heat between them, the man on the whole loses more heat while he stands before the ice-block than formerly, which makes him feel cold, while the ice-block gains heat and gradually melts down.

157. Air-conditioning :—It is the art of securing and maintaining the conditons of human comfort in an air enclosure. The exact conditions which produce a comfortable and healthful atmosphere differ from people to people and season to season. The findings of the *American Society of Heating and Ventilating Engineers* (ASHVE) have led to the following recommendations for America—

(*i*) Percentage Relative Humidity of enclosure—30 to 70 ;
(*ii*) Effective Temperature—

(*a*) between 63° and 71°F. in winter ;
(*b*) between 66° and 75°F. in summer ;

(*iii*) Ventilation Requirements.—The air must be kept fresh and free from all odours, notably tobacco, food and body odours. Its carbon dioxide content must be also low. Odour and gas should be controlled by diluting them to a harmless concentration by introducing sufficient fresh air ; 30 cu. ft. (10 cu. ft. outdoor fresh air plus 20 cu. ft. of room air) per person air movement in the room at a velocity of 15 to 50 ft. per minute is necessary. The corresponding data for India are yet to be collected. Engineers use regional data of their own in the different parts of the country.

It will be noted from the above that the essential factors which have to be controlled, besides any impurity liable to be present in the air, are the *relative humidity*, *temperature* and *ventilation*. So, to secure the above ends in India, a scientific *Summer Air-conditioning Unit* must arrange for cooling of the air, dehumidifying (for in summer humidity is high), cleaning of the air and adequate ventilation, while a *Winter Air-conditioning Unit* must arrange for heating, humidifying (for in winter humidity is small), air-cleaning and adequate ventilation.

Summer Air-conditioning.—The cooling coil of a refrigerator is put at some chosen spot of the enclosure. By means of a suction pump fresh air is drawn through a filter (for removal of particles of dust, smoke, etc. which are carriers of harmful bacteria) from one end of the coils to the other end, thus producing the desired circulation of a current of cooled air. If the moisture content of the room exceeds the *comfort limit*, it is precipitated on the coils and is drained out. This is de-humidification. The speed of the suction pump and rate of cooling of the refrigerator coil are adjusted by automatic methods. The load on the plant depends on the season and the amount of heat leaking through the roof and walls. For air-conditioning, therefore, a room carefully designed with suitable materials costs less.

Winter Air-conditioning.—A heating coil (an electric heater or a steam-piping) is installed at a suitable place. By means of a suction pump fresh air passed through a filter is drawn over the heater surface. Humidification is accompanied by trickling water on to the surface of the heater or by passing the steam pipes through a pan

containing water. There are automatic arrangements for adjustment of the rate of heating, sucking of air and supplying water for humidification to keep them within comfort limits.

Only the most elementary principles of air conditioning have been described above and the description are only illustrative.

Questions

1. Distinguish between conduction and convection of heat. Illustrate the difference by examples (C. U. 1918, '28, '33 ; Dac. 1933)

2. Point out the various ways in which a hot body may lose its heat. What methods would you adopt to reduce the rate at which heat is lost in each of these ways? (C. U. 1924 ; Pat. 1939)

3. Distinguish between conduction, convection and radiation of heat. Describe experiments to illustrate the distinctions. (Pat. 1930 ; cf. Dac. 1931, '33)

4. Of what importance are these (refer to the previous question) in calorimetric determinations and what arrangements would you make to eliminate their effects? (Pat. 1932 ; cf. '28)

5. What are the different methods for the transmission of heat from point to point? Clearly explain their difference with suitable examples. (C. U. 1931, '48 ; Pat. 1919)

6. Can you boil water in a paper vessel? If so, how? (Utkal, 1950)

7. Explain why water can be boiled in a vessel made of thin paper (Vis. U. 1955)

8. If you touch a piece of iron and a piece of wood lying exposed to the heat of the sun, which feels hotter and why? (Dac. 1930 ; Pat. 1913)

9. On a cold day a piece of wood and a piece of iron, when touched with fingers, appear to be different at different temperatures, though a thermometer placed successively against each gives the same reading. How do you account for this, and how would you verify your explanation by experiment? (Pat. 1943)

10. Explain the working of Davy's Safety lamp (C. U. 1929)

11. How will you show experimentally that different substances have different conductivities? (Pat. 1937, '43)

12. State briefly how you would compare experimentally the conductivities of a rod of copper and one of lead. (C. U. 1923)

13. Define thermal conductivity. Explain the statement that the thermal conductivity of glass is 0·002 C.G.S. units. (All. 1941 ; U. P. B. 1940)

14. The opposite faces of a cubical block of iron of cross-section 4 sq. cms. are kept in contact with steam and melting ice. Determine the quantity of ice melted at the end of 10 minutes, the conductivity of iron being 0·2 (latent heat of ice = 80 calories)
[Ans. 300 gms.] (Pat. 1925)

15. Find the difference in temperature between the two sides of a boiler plate 2 cms. thick, conductivity 0·2 C.G.S. units, when transmitting heat at the rate of 600 kilogram-calories per square metre per minute. (Pat. 1935)
[Ans. 10°C.]

16. Steam at 100°C is passed into an iron pipe 1 metre long, 15 mms. thick and whose circumference is 10 cms. Water at 100°C collects at the rate of 100 gms. per min. What is the temperature of the outside?
(Conductivity of iron = 0·2, latent heat of steam = 540 cals./gm.)
[Ans. 93·25°C.] (U. P. B. 1939)

17. Explain how heat is propagated through a given body by conduction and define coefficient of conductivity. (C. U. 1932)

18. Calculate the amount of heat lost through each square metre of the walls of a cottage, assuming that the walls are 42 cms. thick and that the conductivity of the material is 0·004 C.G.S. units, that the temperature is 10°C. higher than outside.
[*Ans.* 9·5 cals/sec.]

19. Find how much steam per minute is generated in a boiler made of boiler plate 0·5 cm. thick, if the area of the walls of the fire-chamber is 2 sq. metres; the mean temperature of the plate faces 200°C. and 120°C. respectively, the latent heat of steam 522, and the conductivity of the steel plate 0·164.
[*Ans.* 60321·8 gms.]

20. Heat is conducted through a slab composed of parallel layers of two different materials of conductivities 0·32 and 0·14, and of thickness 3·6 cms. and 4·2 cms. respectively. The temperatures of the outer faces of the slab are 96°C. and 6°C. Find the temperature gradient in each portion. (Pat. 1937)
[*Ans.* 6·67°C. and 15·24°C.]

21. A cubical vessel of 10 cms. side is filled with ice at 0°C. and is immersed in a water bath at 100°C. Find the time in which all the ice will melt. Thickness of vessel=0·2 cm. and the coefficient of conductivity=0·32. (U. P. B. 1946)
[*Ans.* 12·22 secs., assuming density of ice=0·9167 gms./c.c.]

22. Spheres of copper and iron of the same diameter and of masses 8 : 7 are both heated to 100°C. and placed on a slab of paraffin wax. It is found that copper sinks in more quickly than the iron, but in the end the iron is in level with the copper having melted the same amount of wax; give an explanation of this. (Pat. 1935)

[**Hints.**—Copper has less specific heat but greater conductivity than iron.]

23. One end of a metal bar is heated. Indicate clearly the factors on which the rate of rise of temperature at any point on it depends. (Pat. 1925; All. 1946; R. U. 1949; Utkal, 1951)

24. Two metal bars *A* and *B*, of the same size, but of different materials are coated with equal thickness of wax and placed each with one end in a hot bath. It is noted that at first the wax on *A* melts at a greater rate than that on *B*, but that when a steady state has been reached, a greater length of wax has been melted on *B* than on *A*. Explain this. (C. U. 1941)

25. Define 'thermal conductivity' of a material. You are given two metal rods of the same dimensions; describe an experiment to show which of them has the higher thermal conductivity. (Utkal, 1950)

26. Explain 'conductivity' and describe a method for determining it for a metal. (R. U. 1948, '51)

27. Explain why we get land breeze during night and sea breeze during day. (Utkal, 1947)

28. Discuss, as fully as you can, the grounds on which we conclude that radiant heat is but invisible light. (C. U. 1912, '33; *cf.* Pat. 1929)

29. Describe an experiment to show that the intensity of the radiation at a point due to a given source is inversely proportional to the square of the distance of the point from the source. (Utkal, 1951)

30. Describe an experiment showing that thermal radiations are transmitted in straight lines. Show how to prove experimentally that the radiant heat received by a given surface is inversely proportional to the square of the distance of the surface from the source of heat. (Pat. 1920)

31. Describe a convenient apparatus for investigating the laws of reflection and refraction of heat and give the general results arrived at. (All. 1932; Pat. 1945)

32. Describe and explain the use of Leslie's cube. (C. U. 1948)

CHAPTER IX

MECHANICAL EQUIVALENT OF HEAT : HEAT ENGINES

158. Nature of Heat (*Caloric Theory*) :—The old idea as to the nature of heat was that of a weightless invisible fluid, called *caloric*, which, according to the supporters of the caloric theory, is present in every substance in large or small quantities, rendering that substance hot or cold. The fluid is, according to them, to be given up by a hot body when placed in contact with a colder one. The heat produced by compression or hammering was explained by supposing that *caloric* was squeezed out of the body like water of a sponge. Again, the heat produced by friction, as for example, by rubbing two bodies together, was explained by stating that in addition to the *caloric* squeezed out, the thermal capacity of a substance was less in the powder form than when taken in large masses, and so the particles did not require so much *caloric* to maintain the former temperature ; so some heat was given up which raised the temperature of the fine particles and the rubbed bodies.

Rumford's Experiments.—The first blow to the caloric theory was given by Count Rumford in 1798, while superintending the boring of cannon at the Munich Arsenal. He observed that a large amount of heat was developed both in the cannon and in the drill, which was apparently unlimited. He arranged to revolve a *blunt* drill in a hole in a cylinder of gun metal weighing 113 lbs. by which the temperature rose up to 70°F., though the weight of the metallic dust rubbed off the cylinder was only 2 ounces. It occurred to him that the only other source from which heat could be received was air. So in order to avoid the effect of the atmosphere, he repeated his experiment by surrounding the cylinder with $2\frac{1}{4}$ gallons of water which began to boil after some time. It appeared impossible that such a large amount of heat could be liberated by such a small quantity of borings by a mere change in thermal capacity. This heat, he argued, could not also come from the water ; for water was only gaining heat. Rumford observed that the supply of heat produced by friction was unlimited, and he stated that anything which could furnish heat *without limitation* could not be a *material substance*. Heat was, according to him, not due to something material as the caloricists thought, but a **kind of motion.**

Davy's Experiment.—The final blow to the caloric theory was given by Sir Humphrey Davy who rubbed together in vacuum two pieces of ice, which melted to form water even when the initial temperature of the ice and its surroundings was 29°F., i.e. below the freezing point. Davy states : "From this Experiment it is evident that ice by friction is converted into water, and, according to the caloricists, its capacity is diminished, but it is a well-known fact that the capacity of water for heat is much greater than that of ice, and

ice must have an absolute quantity of heat added to it before it can melt. Friction consequently does not diminish the capacities for heat."

In spite of these experiments scientists continued to support the *caloric theory* until 1849, when the Dynamical Theory of Heat was established by the experiments of Dr. Joule of Manchester, who not only showed that heat is a form of energy, but also found the exact relation between heat and mechanical energy.

Since then it is now believed that heat is a form of energy possessed by a body due to the motion of its molecules. The more rapid the molecular motion, the hotter is the body.

159. Heat and Mechanical Work:—It is well known that when two bodies are rubbed against each other, heat is produced. It is produced at the expense of the work done. Similarly, when a body, falling from a height, strikes against the ground, it loses its kinetic energy acquired during the fall, which is converted into heat. Conversely, heat energy is transformed into work in the case of a steam engine or internal combustion engine. The heat is derived in the case of the steam engine from the combustion of coal and in the case of the internal combustion engine from the combustion of petrol, gas or oil mixed up with air.

Every cyclist knows that at the time of pumping the tubes the pump grows hot. This is due partly to the friction of the piston against the walls of the cylinder, but chiefly to the fact that the inward motion of the piston is transferred to the molecules of air coming into contact with it, which has the effect of increasing the velocity of these molecules. These molecules colliding with the advancing piston rebound with increased velocity which is so great that the temperature of the mass of gas at 0°C., when compressed to one-half of its former volume rises to about 87°C.

Shooting stars, meteorites are also other interesting examples. These are pieces of matter, cold to begin with, which are attracted by the earth. They run through the atmosphere with such enormous speed that there is rapid compression of gases in the atmosphere and, as a result of the work done, the rise of temperature is so high that these pieces of matter become luminous, and very often burn away altogether.

Again when a gas is made to expand suddenly, it cools down. This shows that the work is done at the expense of the heat drawn from the gas itself. When a liquid evaporates, it cools down. The work of expansion due to vaporisation is evidently done here at the cost of heat energy of the liquid.

The above facts indicate beyond all doubts that heat and work are intimately related to each other. The exact relation between them was established by Dr. Joule's experiment and is as follows:

Whenever work is converted into heat or heat into work, one is equivalent to the other. This principle of equivalence is otherwise known as the **First Law of Thermodynamics.**

160. Mechanical Equivalent of Heat :—According to the first law of Thermodynamics, as stated already, whenever heat is transformed into work or work into heat, one is equivalent to the other. *The amount of mechanical work equivalent to unit heat is known as the mechanical equivalent of heat* and represents only the rate of exchange between these two forms of energy. Thus if W and H represent respectively the mechanical work and heat when one is wholly transformed into the other, and J=mechanical equivalent of heat, *i.e.* mechanical work equivalent to unit heat, we have $W=JH$, from the first law of Thermodynamics, or $J=\frac{W}{H}$.

The mechanical equivalent of heat is represented by J in honour of Joule who first determined its value.

The Value of J in different Units.—

J=778 ft.-lbs. per B.Th.U.

$=778\times\frac{9}{5}=1400$ ft.-lbs. per C.H.U.

$=\frac{1400\times30{\cdot}48\times453{\cdot}6\times981}{453{\cdot}6}$ ergs per calorie [$\because$ 1 ft.=30·48 cms. and 1 lb.=453·6 gms.]

$=4{\cdot}186\times10^7$ ergs. per calorie

$=4{\cdot}186$ Joules per calorie [$\because$ 1 Joule$=10^7$ ergs]

The most approximate value of J is taken to be $4{\cdot}2\times10^7$ ergs per calorie for ordinary calculations.

161. Determination of J :—

(a) Joule's Experiment.—The first exact determination of the quantitative relation between heat and work, *i.e.* the *mechanical equivalent of heat*, was made by Dr. Joule in 1849.

In Joule's experiment work was expended in churning water contanined in a calorimeter and the heat produced was found from the resulting rise of temperature (Fig. 81).

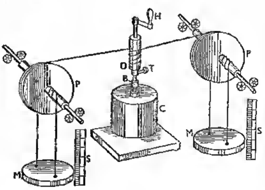
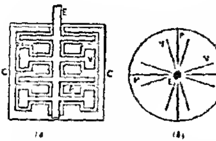


Fig. 81—Joule's Experiment.

A specially constructed copper calorimeter C containing water was taken [Fig. 82,(a)]. Four partitions P mutually at right angles to each other were fixed inside it [Fig. 82,(b)]. A paddle could be rotated in water in it about a vertical spindle. The spindle E carried a set of eight vanes v which passes through spaces cut in the stationary vanes and could turn inside in a way similar to a key turning in the wards of a lock. This



(a) (b)

Fig. 82.—Joule's Calorimeter

arrangement prevented the water from being rotated in the direction of the paddle and was as a result thoroughly churned. To prevent the conduction of heat along the metal spindle E it was interrupted at B [Fig. 81] by a piece of boxwood. A wooden drum D fitted with a turning handle H is fixed on the spindle E. The drum could be detached from the spindle by means of a removable pin T. A flexible cord passed round the wooden drum and its two ends were taken to opposite sides of the drum and wound over on two large pulleys P as shown in the figure. The axles of these pulleys were placed on friction wheels to diminish friction. The pulleys carried equal weights M hung by strings wound round the axles. The heights of these weights from the ground below could be read from vertical scale S. The motion of the paddle was produced by allowing the weights to fall. The pin T could be quickly removed and the weights M wound up again by turning the handle H without revolving the paddle. The fall-experiment was repeated a number of times and the temperature of the water recorded at intervals by means of a mercury thermometer.

Calculation.—In order to produce an appreciable rise of temperature, suppose the weights are raised and allowed to fall several times.

Let m = mass of water in the calorimeter, M = mass of each weight, h = height through which each weight falls, n = number of falls,

W = water equivalent of the calorimeter,

t = rise of temperature of water in the calorimeter,

v = velocity acquired by the weights on reaching the ground.

Potential energy in the raised position, for both the weights = $2Mgh$.

Kinetic energy just before striking the ground = $2 \times \frac{1}{2}Mv^2 = Mv^2$.

Total energy used $=n(2Mgh-Mv^2)$ ergs : and heat produced $=(m+w)t$ cal.

$$\therefore \quad J=\frac{W}{H}=\frac{n(2Mgh-Mv^2)}{(m+w)t}.$$

Errors and Corrections.—

Joule had to make various corrections in order to get a reliable result. He made an allowance for the energy converted into sound. Corrections were also made for the losses due to conduction, radiation, etc. and for the energy absorbed by friction.

The **defects** of his expt. were : (1) Joule, on the authority of Regnault, assumed the specific heat of water to be the same at all temps. ; (2) the mercury thermometer, he used, was not calibrated with reference to any standard thermometer, such as a gas thermometer ; (3) the rise of temperature attained in his experiment was very small.

The final mean result of the value of J given by Joule was 773·4 ft.-lb. per B.Th.U. But by later investigations, it was found that Joule's result was rather low, and the accepted result today is 778 ft.-lbs. per B.Th.U.

Importance of Joule's Experiment.—By finding that the value of J is constant, *i.e.* the rate of exchange between heat and work is constant when one is wholly transformed into the other, Joule established the equivalence between heat and work [First law of Thermodynamics]. This equivalence is independent of the way in which the work is derived or the means by which the transformation is effected. In a way thus he proved the law of conservation of energy as applied to the special case of heat and mechanical energy, and this proof forms one of the strong foundations on which the universal law of conservation of energy rests.

(b) Searle's (or Friction Cones) Method.—The apparatus (Fig. 83) essentially consists of two conical brass cups A and B, one of which fits closely into the other. The lower cup B is fixed on a non-conducting base which again is fixed on the top of a vertical spindle S. The spindle can be rotated by a hand wheel or a motor. A circular wooden disc CC is fixed to the inner cup A and a string wound round the circumference of the disc passes over a pulley and carries a suitable weight W at the other end. When the cup B is rotated, A is prevented from doing so by the tension of the string, and the speed of rotation, which is counted by a speed counter, attached to the spindle, is so adjusted that the weight W hangs stationary, the tension of the string acting as a tangent to the disc. Now, the work done against friction between the surfaces

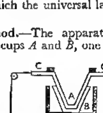


Fig. 83

of the cups, *i.e.* the mechanical energy, is converted into heat which rises the temperature of the cups and a known mass of water taken in A. A thermometer dipped in A records the rise in temperature.

Calculation.—Here for a steady suspension the moment of the average friction force F between the cups is equal to the moment due to the force $W (= Mg)$, where M is the mass of the weight. The former $= F \times a$, where a is the mean radius of the surfaces of the cups which are in contact. So, $Fa = Mg \times r$, where r is the radius of the disc. Now, the work done in ergs for each revolution $= 2\pi a \times F$. For n revolutions, the work $= 2\pi n a F = 2\pi n M g r$ ergs.

Again, if m be the mass of water in the cup, w the water equivalent of the cups, and $t°C$ the rise in temperature, the heat developed by rotation, $H = (m + w)t$ calories.

Hence, the mechanical equivalent, $J = \dfrac{2\pi n Mgr}{(m+w)t}$ ergs per calorie.

Otherwise thus.—The work can be calculated also thus. The work done for n turns of the spindle in overcoming the friction between the cups is the same as would have been spent if the spindle and the outer cup B had been kept stationary, and the inner cup A had been made to rotate by the wt., $W (= Mg)$, making n revolutions of the disc slowly.

The mechanical work done = (force × distance) $= Mg \times 2\pi r n$.

$$\therefore \quad J = \frac{\text{work done}}{\text{heat developed}} = \frac{2\pi n Mgr}{H}.$$

Radiation is the chief source of error in this experiment to reduce which the cups must be brightly polished.

Fig. 81

(c) A simple Laboratory Method for Determining J.—The following example illustrates a simple method for determining the value of J.

A cylindrical tube 15 cms. long made of a non-conducting material, closed at both ends, contains 500 gms. of lead shots, which when the tube is held vertically, occupy 6 cms. of the tube length. The tube is suddenly inverted so that the end originally above is now below, and the shots fall to the other end of the tube. The tube is then again suddenly inverted and the process is repeated 200 times. At the end of this process the temperature of the shots is found by means of a thermometer to be 1.4°C. higher than it was at the beginning of the experiment. Find the value of the mechanical equivalent of heat (Sp. ht. of lead is 0.03. It is assumed that no heat is lost by radiation or conduction). (C. U. 1910)

The lead shots fall through a height $= (15 - 6)$ cms. each time the tube is inverted. Hence the loss of potential energy for each time

$$= 500 \times 981 \times (15 - 6) \text{ ergs.}$$

$\therefore$ The total loss $= 200 \times 500 \times 981 (15 - 6)$ ergs.

Heat developed in lead shots $= 500 \times 0.03 \times 1.4$ cal.

$\therefore$ Mechanical equivalent $(J) = \dfrac{\text{work done}}{\text{heat developed}}$

$$=\frac{200\times500\times981\times(15-6)}{500\times0\cdot03\times1\cdot4} \text{ ergs per calorie}$$

$$=4\cdot2\times10^7 \text{ ergs per calorie (nearly).}$$

(d) Electrical Method.—*Vide* Current Electricity, Ch. V.

(e) Mayer's Method of determining J.—Mayer used the relation $C_p-C_v=R/J$ (vide Art. 82) in 1842 for the determination of the value of J before its first direct experimental determination in 1849 by Dr. Joule.

It is known that 1 c.c. of air at N.T.P. weighs 0·001293 gm. Therefore, the volume V_0 occupied by 1 gm. of air at N.T.P. is,

$$V_0=1/0\cdot001293 \text{ c.c.}$$

Since the normal pressure P_0 is $1\cdot013\times10^6$ dynes/cm.^{2}, the value of R for 1 gm. of air is given by,

$$R=\frac{P_0V_0}{273}=\frac{1\cdot013\times10^6}{273}\times\frac{1}{0\cdot001293}.$$

Taking the value of $C_p=0\cdot238$, and $C_v=0\cdot17$ for 1 gm. of air, we have,

$$J=\frac{R}{C_p-C_v}=\frac{1\cdot013\times10^6}{273\times0\cdot001293}\times\frac{1}{(0\cdot238-0\cdot17)}$$

$$=4\cdot2\times10^7 \text{ ergs per calorie.}$$

Another Relation.—The kinetic energy of a body of mass m moving with velocity $v=\frac{1}{2}mv^2$.

Heat developed H when the body meets an obstacle and stops suddenly$=mst$, where s is the specific heat of the body, and t the rise in temperature by the impact.

Then, $\quad H \propto$ kinetic energy $\propto \frac{1}{2}mv^2$;

or, $\quad JH=\frac{1}{2}mv^2$; or, $J\times mst=\frac{1}{2}mv^2$;

or, $\quad J=\dfrac{v^2}{2st}$.

(f) Determination of J by Continous Flow Calorimeter :—Callendar and Barnes' calorimeter consists of a wide cylindrical glass tube EF, the middle portion of which is drawn into a narrow tube, cc [Fig. 84(A).] In the two wider ends, E and F, are introduced two copper cylinders D_1 and D_2, between which is stretched a heating wire or a spiral of nichrome wire passing along the axis of the narrow tube, cc. Two platinum resistance thermometers PP, are inserted in D_1 and D_2 for the measurement of the steady temperatures of the incoming and outgoing water in E and F respectively. For flow of water through the tube $E\ cc\ F$, an inlet, a, is arranged in E and an outlet, b, in F. To reduce losses due to conduction and convection from the hot water in the central tube cc, the latter is jacketted by a vacuum tube, which in its turn is again surrounded by a constant temperature water bath. This water bath ensures a constant rate of

Then,

$$\frac{V_2 i_2 t}{J} = m_2 s\theta + h \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad (2)$$

From (1) and (2),

$$s = \frac{(V_1 i_1 - V_2 i_2)t}{J(m_1 - m_2)\theta} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (3)$$

Hence assuming the value of the specific heat of water, J may be determined.

The advantages of this method are : (1) As the temperatures are steady there is no question of thermometric lag ; (2) The water equivalent of the calorimeter is not involved in the calculations ; (3) The radiation correction is eliminated from two sets of observations.

Examples. *(1) How much work is done in supplying heat necessary to convert 10 gms. of ice at $-5°C.$ into steam, at $100°C.$ (sp. heat of ice$=0{\cdot}5$; $J=4{\cdot}2\times10^7$ ergs/calorie).* *(All. 1917 ; R. U. 1942)*

(a) Heat necessary to convert 10 gms. of ice at $-5°C.$ into ice at $0°C.=10\times0{\cdot}5\times5=25$ cals. (b) Heat necessary to convert 10 gms. of ice at $0°C.$ into water at $0°C.=10\times80=800$ cals. (c) Heat necessary to convert 10 gms. of water at $0°C.$ into water at $100°C.=10\times100=1000$ cals. (d) Heat necessary to convert 10 gms. of water at $100°C.$ into steam at $100°C=10\times536=5360$ cals.

$\therefore$ The total heat necessary$=7185$ cals.

Work done $J\times H=4{\cdot}2\times10^7\times7185$ ergs$=3{\cdot}0177\times10^{11}$ ergs.

(2) If a lead bullet be suddenly stopped and all its energy employed to heat it, with what velocity must the bullet be fixed in order to raise the temperature through $100°C.$, the specific heat of lead being $0{\cdot}0314$.

Let m be the mass of the bullet in grams and v its velocity in cms. per second ; then its kinetic energy$=mv^2/2$ ergs.

And heat produced H when the bullet is stopped$=m\times0{\cdot}0314\times100$ cals.

$\therefore \quad 4{\cdot}2\times10^7=\frac{mv^2}{2}\div(m\times0{\cdot}0314\times100)$; whence $v=162{\cdot}407\times10^2$ cms. per sec. nearly.

(3) A lead-ball dropped from an aeroplane at a temperature of $15°C.$, just melts on striking the ground. Supposing the whole of its kinetic energy is converted into heat, find the height of the aeroplane at the moment at which the ball is dropped (sp. ht. of lead$=0{\cdot}03$; melting point of lead$=350°C.$; latent heat of lead$=35$ calories). *(Pat. 1932 ; C. U. 1951)*

If h be the height of the aeroplane, the loss of potential energy of the ball $=m\times981\times h$ ergs. This is converted into heat, which first raises the temperature of the ball through $(350-15)°C.$, and then melts it.

$\therefore$ The total heat developed$=m\times0{\cdot}03\times335+m\times35=m\times45{\cdot}05$ cals.

This is equal to $(m\times981\times h)\div J$; so $4{\cdot}2\times10^7=981\times m\times h\div(m\times45{\cdot}05)$; whence $h=19287{\cdot}4616$ metres.

(4) If the mechanical equivalent of heat be 779 Foot-Pound-Fahrenheit units, from what height must 10 lbs. of water fall to raise its temperature by $1°C$? *(Pat. 1940)*

Let h ft. be the required height. Rise in temperature$=1°C.=\frac{9°}{5}F.$

$\therefore$ Heat produced $H=10\times1\times\frac{9}{5}=18$ B.Th.U.

Thus, the *work done by a gas during expansion is equal to the product of the pressure and the increase in volume.* Similarly, the work done on a gas can be shown to be equal to the product of the pressure and the decrease in volume.

Example. *How much work is done against uniform pressure when 1 gm. of water at 100°C. is converted into steam? Express your result in calories.* (*All. 1918*)

The pressure at which 1 gm. of water at 100°C. changes into steam is 76 cms. of mercury. The pressure $=76\times13{\cdot}6\times981$ dynes per sq. cm.

When water is changed into steam, its volume is increased 1670 times. So the volume of steam formed out of 1 c.c. of water is 1671 c.c. Hence the work done $=76\times13{\cdot}6\times981\times1670$ ergs.

This is equivalent to $\dfrac{76\times13{\cdot}6\times981\times1670}{4{\cdot}2\times10^7}$ calories $=40{\cdot}52$ cals.

163. Energy given out by Steam :—The high value of the latent heat of steam shows that when steam condenses, a tremendous amount of heat is given out, some of which is converted into work as in the case of a steam engine (*vide* Art. 165).

We have already seen that 1 lb. of steam in condensing at 100°*C.* would liberate about 965 B.Th.U. of heat, which would raise the temperature of 965 lbs. of water through 1°*F.* Each B.Th.U. is equivalent to 778 ft.-lbs. of work. So the energy given out $=778\times965$ $=750{,}770$ ft.-lbs.

This means that the above amount of energy which is liberated by 1 lb. of steam is also derived by a mass $=\dfrac{750{,}770}{2240}$ tons $=353$ tons (nearly), in falling through 1 foot. So the same amount of energy must be necessary in boiling 1 lb. of water into steam.

We have already seen also that 144 B.Th.U. will be necessary in melting 1 lb. of ice which is equivalent to 112,032 ft.-lbs. This energy will be liberated by a mass $=\dfrac{112{,}032}{2240}=50$ tons (nearly), in falling through 1 foot.

HEAT ENGINES

164. Conversion of Heat into Mechanical Energy :—The transformation of mechanical energy into heat has already been explained. Now we shall deal with the reverse process, that is, the conversion of heat into mechanical energy. The machines by which this is done are called *Heat Engines*, which include the Steam Engines, Steam Turbines, Internal Combustion Engines, such as Oil or Gas Engines, Petrol Engines, etc. These engines are often referred to as **prime movers** because they develop their motive power directly from fuel. Obviously, an electric motor is not a *prime mover.*

Boilers.—The steam engine is an external combustion engine, for the combustion of the fuel from which ultimately the motive power

is derived takes place in a separate unit, namely the *boiler* which is outside the steam cylinder. By the heat of combustion of a fuel, such as coal which is the most common form of fuel used for boilers, steam is raised in the boiler. In modern boilers there is arrangement [illegible] with safety valves which protect the boiler from development of high internal pressures detrimental to it.

Safety Valve.—An ordinary safety valve for a boiler is only a class III type of lever [*vide* Art. 180 *a*), Part I]. It consists of a straight lever *FB* pivoted at one end *F* [Fig. 85]. The valve *V* is attached to the lever at some intermediate point *A* close to *F*. The valve is held down on its seat against the upward steam pressure by a relatively small weight *W* hung on the lever at the distant end *B*. The weight *W* and the distance *FB* are so adjusted that if the steam pressure acting upwards exceeds a certain value, it overcomes the downward force exerted on the valve by the weight *W* and the valve opens up whereon steam continues to escape into the atmosphere until the pressure within the boiler falls to the normal value when the value closes again.

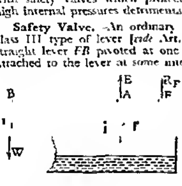


Fig. 85—Safety Valve

Regulation of Speed.—The speed of an engine is liable to change on change of load. To ensure a smooth running at constant speed, a device called a *governor* is employed. It is a self-acting machinery driven by the main-shaft of an engine and controls the supply of power to the piston. In the steam engine it regulates the supply of steam from the boiler to the cylinder.

Watt's Governor.—The [illegible] Fig. 86 illustrates such a governor communicating with a throttle valve. A throttle valve is usually fitted in the steam-supply tube between the stop-valve of the boiler and the steam-chest of the engine.

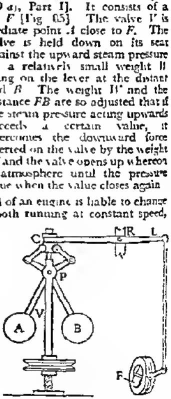


Fig. 86—Watt's Governor and Throttle-valve.

In the figure, a vertical spindle V has been shown to revolve by gearing with the engine shaft. So its speed rises or falls with that of the engine. It carries a pair of heavy balls A & B which are fastened to the spindle V by links pivoted at P. When the engine speed increases and the balls rise, they pull down the collar C which sides on the spindle V. The forked end of a lever L pivoted at R is fitted on this collar, the other end being ultimately connected to a tap F, in the steam pipe, called a *throttle-valve*. As the collar is pulled down, the tap tends to reduce the opening for steam whereby the steam supply to the engine falls and the engine slows down. If the engine speed goes down too much, the balls fall and so the collar is raised and the throttle opens up allowing more steam to pass into the engine and the engine speeds up.

The Crank and the Fly Wheel.—It was again Watt who first converted the to-and-fro motion of the piston of an engine into circular motion by means of a connecting rod and crank. Fig. 87 shows a crank fitted to a piston by means of a connecting rod which takes up the motion of the piston and converts it into the circular motion of a shaft. The crank C is a short arm between the connecting rod R and the shaft S. At the forward stroke

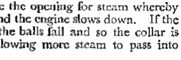
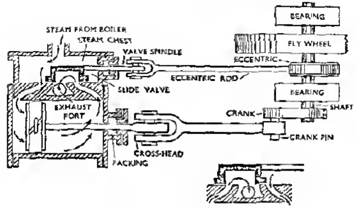


Fig. 87—A Crank.

of the piston the connecting rod pushes the crank while at the return stroke it pulls the latter resulting in a complete circular motion of the shaft. During each revolution of the shaft there are two points when the connecting rod and the crank come into in the same line and no turning moment is exerted on the shaft. These points are called the **dead centre positions.** Again at two points the crank and the connecting rod are mutually at right angles when the torque is maximum. The torque on the shaft being thus variable, the speed of rotation of the shaft tends to vary in course of each revolution. A big **fly wheel** is usually mounted on the shaft, which by virtue of its large moment of inertia (*vide* Art. 70, Part I) carries the shaft across the dead centre positions and by absorbing energy when the speed is greater due to greater torque during one-half of the revolution and releasing the same when the speed tends to fall owing to smaller torque and the next half revolution, serves to keep the speed of the shaft uniform. Thus it acts as a reservoir of energy or stabiliser and seeks to smoothen out any variation of speed during a revolution. So itmay be noted in this connection that the function of a governor is to prevent any variation of speed on change over from one load to another.

165. The Steam Engine :—In 1768 James Watt of England invented the steam Engine (*vide* life of James Watt, Art. 170). The following must be the essential parts of a Steam Engine, though engines of today may differ considerably in details of construction.



(a)

Fig 88—The Steam Engine

A Boiler. (1) —Steam is raised in this plant (*vide* Art. 164) which may be either of the *smoke-tube* type or of the *water-tube* type. For engines of large horse-power, superheated steam at high pressure is produced by the boiler. The steam from the boiler is led through a tube into a chest, called the *steam-chest* or *valve-chest* This supply-tube is provided with a valve, near the boiler-end, called the stop-valve for regulation of steam Further down towards the steam-chest a *throttle-valve* is situated

Fig 89

(2) *The Steam* or *Valve-chest* —It is a rectangular stout box (Fig. 88) mounted on the *cylinder* of the engine. It has three openings or *ports* The middle one is connected with the *exhaust pipe* while through the two side ones the steam-chest communicates with the cylinder. These two communicating ports are alternately closed and opened by means of what is called a *slide valve*.

(3) *The slide valve.*—It has a variety of forms, *D*-valve (shown in the figure), piston-valve, drop-valve, etc. Its function is to direct the steam into the cylinder through the two communicating ports alternately so that the *piston* which works in the cylinder is acted on from either side in turn, producing a to-and-fro (reciprocating) motion. It is provided with a spindle driven by a connecting rod joined to an *eccentric disc* mounted on the main shaft of the engine.

(4) *The cylinder and the piston.*—A steam-tight *piston* usually of cast steel, works inside the *cylinder* which is a cylindrical vessel of high strength and which communicates with the steam-chest through the two communicating ports. Its spindle called the piston rod works through a *packing* or *stuffing* box with which the front end of the cylinder is provided and is joined to the *driving rod* otherwise called the *connecting rod* at the *cross-head* by means of a pin called the *gudeon pin*. The cross-head moves along a fixed groove in a *guide* producing a straight line motion. The driving rod is connected to the *crank* by the crank pin. The crank which is mounted on the shaft is a contrivance for converting the to-and-fro motion of the piston rod into circular motion of the shaft.

(5) *The fly wheel.*—It is a large and massive wheel (*vide* Art. 164) mounted on the main shaft. The turning efforts on the shaft produced by the crank is no' constant during a revolution. It is the fly-wheel which keeps the speed of the shaft constant by smoothening down the variation by means of its large moment of inertia. It also helps the crank to move across the dead-centre positions.

(6) *The governor.*—On change of load, the speed of the engine varies. To keep the speed approximately constant on all loads, a self-acting machinery, called the *governor*, driven by the main shaft, is used (*vide* Art. 164). It is connected by a system of livers to a regulating valve, one form of which is called the *throttle-valve*. The revolving balls, with which the governor is provided, rise or fall according as the speed increases or decreases. This rise or fall of the balls operates a sleeve which communicates through a lever system with the throttle-valve and accordingly steam-supply is so reduced or increased as to keep the speed constant.

Principle of Action.—Here the heat energy of steam is transformed into mechanical work through expansive action.

Steam from the boiler is led into the steam-chest whence it enters into the cylinder. When steam enters the cylinder through the lower steam port shown in Fig. 89, the slide valve covers the *exhaust* and the *upper port* so that these two are put into communication. The pressure of the steam due to its expansive action pushes the piston forward and forces out the cushion steam on the other side through the exhaust. The movement of the piston rotates the crank shaft whereby the motion is also communicated through an eccentric disc to the slide valve which moves *opposite to the piston*. The slide valve then covers up the lower port and the exhaust by the time the

piston reaches the forward end. The steam now enters through the upper port and the same action as in the previous stroke occurs but the motions are all reversed. These two strokes, forward and backward, form a cycle of operations which is repeated successively. The to-and-fro motion of the piston is transformed into a rotatory motion of the shaft by means of the crank. Twice during each revolution of the shaft, the crank and the connecting rod come into the same straight line, when there is no turning effect on the shaft. This positions are called the *dead-centres* or *dead points*. Again, at two positions during each revolution, the crank is at right angles to the connecting rod when the turning effect is maximum. The heavy fly-wheel carries the shaft through the dead-centre positions and smoothens out the
during a cycle of e
load on the engine
by the main shaft. Its balls rise or fall as speed increases or decreases. This rise or fall of the balls operates a sleeve which communicates with a throttle-valve and accordingly the steam-supply is reduced or increased so as to keep the rated speed constant.

Condensing and Non-condensing Engines.—The engine in which the steam passing through the exhaust-pipe escapes into the atmosphere is called a *non-condensing* engine, and the engine in which the exhaust steam is led into a vessel, called a *condenser*, where it is condensed at a low temperature and pressure into water again, is called a *condensing engine*. When the steam exhausts into such a condenser where the pressure is kept low (which usually is not more than a pound per square inch), the back pressure against that end of the piston which is open to the atmosphere is reduced from about 15 lbs. to 1 lb. and in that case the effective pressure, which the steam on the other side of the piston can exert, is increased. The condensed water (condensate) is again used in the boiler to raise steam.

Single and Double-acting Engines.—The engine, we have already considered is a double-acting one, as here the steam pressure acts on the two sides of the piston alternately. In a single-acting engine, steam pressure acts on one side and the atmospheric pressure acts on the other side of the piston. The power developed in latter engines is half of that in a double-acting engine of the same size. Excepting in very small engines, single-acting engines are now-a-days seldom used.

166. The Internal Combustion Engine :—The engine used in air crafts, motor-cars, oil engines, etc. are known as *Internal Combustion Engines*, so named because the combustion of the fuel is carried out inside the cylinder of the engine, and not outside the cylinder as in the boilers of steam engines. So internal combustion engines occupy less room and are specially suited for small power purposes. Their thermal efficiency is higher than that of the steam engines. Compared to that of steam engines, their speed is also much greater.

The general arrangement of the cylinder and piston in the case of an internal combustion engine is almost the same as in the steam engine, but whereas in the steam engine the piston moves by the force of expanding steam, in the internal combustion engine the movement of the piston is produced by the explosive force generated by the combustion of a fuel, supplied in the vapour form mixed with air. The fuel used is either a gas—such as *coal-gas, town-gas,* etc. or a liquid such as *petrol, benzene, alcohol,* etc. which are readily vaporised, or a heavy oil, like Diesel oil, etc. and every one of these, when vaporised, forms an explosive mixture with air.

A *gun firing a bullet* is an example of a simple internal combustion engine. Here the spark produced by striking the trigger against the cap explodes the powder and converts it into hot gases which drive the bullet forward with a great force.

Principle of Action.—Internal combustion engines are generally four-stroke engines, *i.e.* they require four strokes of the piston to complete a cycle of operations within the cylinder. There are also *two-stroke* engines.

The four-stroke cycle is simple and is of proved economy and is generally used in stationary engines of small and medium power. It is also not unoften used for stationary engines of large power. A two-stroke cycle engine has advantages of lighter weight and smaller space requirements and are, therefore, almost always preferred for marine purposes.

The engines commonly used in motor-cars, aeroplanes, etc. are all four-stroke engines working on the Otto-cycle. The operations of a four-stroke internal combustion engine of the Otto-type may be explained as follows (Fig. 90).

OTTO-CYCLE

(1) **First Stroke** (*Charging Stroke*).—The piston moves outwards and draws into the cylinder an explosive mixture of air and gaseous fuel through the inlet valve E which then opens up.

(2) **Second Stroke** (*Compression Stroke*).—The piston makes its return stroke, *i.e.* moves inwards and compresses the explosive mixture,

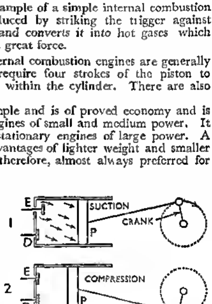


Fig. 90—Four-stroke Otto-cycle.

the values (admission valve E and exhaust valve D) being closed.

(3) **Third Stroke** (*Working Stroke*).—At the beginning of this stroke, the mixture is ignited by electric spark and explosion occurs whereby the contents rise in pressure and temperature almost at constant volume The piston is driven outwards by the expansive action of the gases, all the valves being closed and energy is communicated to the fly-wheel enabling the engine to do work.

(4) **Fourth Stroke** (*Exhaust Stroke*).—The piston moves inwards and the spent gases are forced out of the cylinder through the exhaust valve D which is then opened.

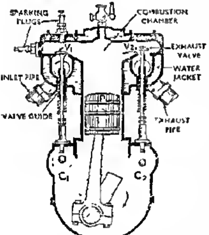


Fig 91—The Petrol Engine

In the above sequence of operations, which is called a *cycle*, the engine is fired only once and work is also done in one stroke in the course of two forward and two backward strokes of the piston. For this reason, an engine working on this plan is called a *four-stroke engine*. In a two-stroke cycle engine, the operations of *charging* compressing, working and exhaust are all done in two strokes of the piston. Therefore, for the same speed of engine, a two-stroke cycle engine does twice as much work as a four-stroke cycle engine does.

167. Internal Combustion Engines of different types :—

(a) The Petrol Engine.—There is no difference in principle between a petrol engine and any gas engine , the former is only more compact and light The petrol engines are commonly used in motor-cars and aeroplanes

In Fig 91 is shown the diagram of a petrol engine where there is a *piston* working in the *cylinder P* as in a steam engine. Above the cylinder there is a chamber, **called the combustion chamber,** where the mixture of air and petrol vapour is ignited by means of electric sparks from **sparking** p... y of the fuel into the chamb... by the *exhaust pipe* are ... of

mushroom type held down on their seats by springs and lifted at proper moments by the action of **cams,** C_1 and C_2, fixed on a rotating shaft driven by the engine itself. The cylinder is water-jacketed in order to prevent the temperature from rising too much, usually not greater than about 180°*F*.

The explosive mixture of petrol vapour with correct proportion of air is formed in an arrangement known as the **carburettor,** and the air so charged with petrol vapour is said to be *carburetted*.

The current for the ignition of the charge is supplied by a *magneto* which is a magneto-electric machinery driven by the engine itself.

A petrol engine, as in a motor-car or aeroplane, is provided with a bank of cylinders, usually a multiple of two. The pistons of all the cylinders contribute their efforts to the same main shaft through their individual cranks which are fitted on the main shaft at equal angular spacings and the total power developed is the sum of the powers developed in the different cylinders.

(b) The Gas Engine.—A *Gas Engine* employing about one part by volume of coal gas and eight parts of air works like a petrol engine and is driven by properly-timed explosions of the mixture of gas and air occurring within the cylinder. The ignition of the explosive mixture is effected by contact with the hot walls of a metal tube or by means of electric spark.

A Gas Engine and a Steam Engine Compared.—Though the fuel used in a gas engine is comparatively expensive, still a gas engine is better for the following reasons :—(*a*) its efficiency is much higher than that of a steam engine ; (*b*) it occupies smaller space and it is more free from smoke.

(c) The Oil Engine.—In an *Oil Engine*, the oil which is used as fule is supplied in the form of spray into a *vaporiser* tube—a red-hot metal tube, and at the same time air is also admitted there. The oil is converted into vapour, and the mixture of vapour and hot air explodes either with or without the help of spark. Hot gases are produced in a small space due to which the pressure and temperature becomes high, and so the piston is driven with a considerable force.

In a **Diesel Oil Engine,** so named after the inventor, the cycle of operations works in the following way :—

At the *first stroke* only air is sucked in at a pressure less than the atmospheric pressure, and at the *second stroke*, the air is very strongly compressed, keeping all the valves closed, so that much heat is developed within the cylinder. At the beginning of the *third stroke*, oil is injected at very high pressure into the cylinder whereby it vaporises, which coming in contact with the intensely heated air of the cylinder takes fire spontaneously almost at constant pressure. After this, the volume expands and work is done. During the *fourth stroke*, the exhaust opens and the burnt gases escape.

A Diesel Engine has only air in the cylinder during compression and so the compression pressure may be raised as high as necessary consistent with the stoutness of the engine without any change of pre-ignition and thereby the efficiency may be increased. The Otto Engine, of course, is *inherently* more efficient

(d) Aero-engines.—These are also internal combustion engines. An aero-engine should be as light as possible and in this respect it differs from other I. C engines. The weight-reduction has been today carried to such an extent that a modern engine of this class has a weight of even less than one pound per horse-power, whereas in other types of engines approximately a weight of 10 lbs per horse-power is considered necessary. Besides its low weight it has the advantage that it produces its power from the minimum quantity of fuel. Aero-engines are all four-stroke Otto engines.

168. Thermal Efficiency of an Engine :—

$$\text{Thermal efficiency} = \frac{\text{Heat converted into work}}{\text{Heat taken in}}.$$

The heat converted into work can be known from the horse-power developed by the engine and the heat taken in can be determined, (*a*) in the case of the steam engine from the quantity of steam used and the initial and final conditions of the steam, and (*b*) in the case of an internal combustion engine, from the quantity of fuel consumed and the calorific value of the fuel

The thermal efficiency of a steam engine is not even more than 20% , that of an ordinary locomotive is seldom greater than 10%. An internal combustion engine has a thermal efficiency of about 30%.

Indicated Horse-power [I.H.P.).—It is the actual power developed in the engine cylinder by the steam in the case of a steam engine, by the combustion of a gas in a gas engine, and by the combustion of the liquid fuel in an oil or petrol engine. It depends on the following : (1) the mean effective pressure on the engine piston during the stroke (p_m), (2) the cross-sectional area of the piston A), (3) the length of stroke of the piston (L), and (4) the number of working strokes per minute (N). For a single cylinder engine,

$$\text{I.H.P.} = \frac{p_m.L.A.N.}{33000},$$ where p_m is in lbs/in^2, L in feet, A in sq inches and N depends on whether the engine is single or double acting and also on whether it is a two-stroke or four-stroke cycle engine. The horse-power so obtained is called the indicated horse-power, because it is indicated by the action of the working substance in the cylinder and determined from the mean effective pressure of the working substance on the piston, which is usually found by means of an instrument known as the *indicator*.

[N.B.—In the case of the steam engine, for each revolution of the main shaft of the engine, which produces two strokes of the piston —one forward and the next backward, there are two working strokes when the steam is double-acting. Therefore, for a double-acting steam engine $N = 2 \times$ Revolutions Per Minute (R.P.M.) of the engine, while for a single-acting steam engine (which is rare) $N = R.P.M.$ of the engine.

In the case of internal combustion engines (which are almost always single-acting), $N = R.P.M.$, when it is a two-stroke cycle engine, and $N = \frac{R.P.M.}{2}$ when it is a four-stroke cycle engine.]

Brake Horse-power (B.H.P.).—All the power developed within an engine cylinder as represented by its *I.H.P.* is not available for useful purposes, for a part of it is used up by way of *mechanical losses* in the driving of the engine itself. So the effective horse-power, which remains available for driving outside machinery is always less than the *I.H.P.* and is known as the Brake Horse-power of the engine, and is so named as it is commonly determined by making the engine operate with a brake on the fly-wheel, the test being known as a brake test.

Mechanical efficiency $= \frac{\textbf{B.H.P.}}{\textbf{I.H.P.}}$ and it varies according to the load on the engine. In modern engines the mechanical efficiency is often greater than 80% at full load.

169. James Prescott Joule (1818—1889) :—An English Physicist born at Salford near Manchester. He had a delicate health and so he was educated at home. While quite young he felt an urge for scientific work as a result of his contact with John Dalton who was his private tutor. His father had a large brewery where he started his researches in electricity. At the age of twenty-two he discovered the law for electric heating. His attention then turned to engines. He noticed that in all engines the mechanical work is obtained at the cost of some heat. He investigated on the relation between the two and discovered the equivalence between them. known now-a-days as the first

James Prescott Joule

law of Thermodynamics. In his honour the mechanical equivalent of heat is expressed by the first letter J of his name. In 1849 he experimentally determined J by converting work into heat and found it to be a constant irrespective of the magnitudes of the work. Invention of some kinds of electric metres, speed counters, etc. also stand to his credit.

170. James Watt (1736—1819) :—A British inventor born at Greenock, Scotland. He showed early signs of skill at craftsmanship and began his life as a mechanic in the University of Glasgow at the age of twenty-one, where his skill, versatility and simple nature attracted the notice of some of the University Professors of whom Prof. Black's name must be mentioned.

In popular writings it is often found mentioned that the expansive force of steam issuing from of a kettle struck Watt's imagination so much that he hit upon a plan for a heat engine from it. Such a story seems not to be true for Savery and Newcomen Engines were in use for more than seventy-five years previous to James Watt. It is rather said that Newcomen engine belonging to the University lying unused for long was placed at his workshop for repairs. Watt had an inquisitive mind and an inventive temperament. He noticed that the engine was extremely wasteful of fuel. He seriously devoted himself to its improvement and this ultimately gave the world the modern steam engine. In perfecting his design he got the assistance of Mathew Boulton, who possessed a first class workshop then at Soho near Birmingham. The locomotive engines were constructed afterwards of Trevithick, Stephenson and others.

James Watt

He for the first time used the term *horse-power* for rating mechanical work. He found that a horse could raise 150 lbs of coal through an effective height of 220 ft. in one minute on the average, *i. e.* 33000 ft.-lbs of work per minute is the average *power* of a horse. Thus rate of work he named one *horse-power*. Watt, which is the electrical unit of power and is equivalent to 1/746 H.P., is named after him.

Examples. (1) ... *horse-power per hour. The* ... *15 lbs of water at 100°C.* ... *s wasted [Latent heat of*

Heat of combustion of 4 lbs. of coal $= 4 \times 15 \times 964 \cdot 8$ B.Th.U.
$= 4 \times 15 \times 964 \cdot 8 \times 778$ ft.-lbs.

The work done by the engine per H. P. hour $= 33000 \times 60$ ft.-lbs.

for 1 H.P. $= 33000$ ft.-lbs. per minute. $\therefore$ The efficiency of the engine $= \frac{33000 \times 60}{4 \times 15 \times 964 \cdot 8 \times 778} = 0 \cdot 043$ or $4 \cdot 3$ per cent. That is, 4·3 per cent. of heat produced is converted into work. $\therefore$ $100 - 4 \cdot 3 = 95 \cdot 7$ per cent. heat is wasted.

(2) *What would be the horse-power of a steam engine which consumes 200 lbs. of coal per hour, assuming that all the heat supplied is turned into useful work. (1 lb. of coal gives 12500 B.Th.U. ; J is equivalent to 770 ft.-lbs. per B.Th.U.)*

Amount of heat available per hour $= (12500 \times 200)$ B.Th.U.

Equivalent amount of work $= 12500 \times 200 \times 770$ ft.-lbs. per hour.

$\therefore$ Work done per minute $= \frac{12500 \times 200 \times 770}{60}$ ft.-lbs.

$\therefore$ Horse-power $= \frac{12500 \times 200 \times 770}{60 \times 33000} = 909 \cdot 1$ (approx.).

Questions

1. Explain what is meant by saying that heat is a form of energy. (Pat. 1926 ; Dac. 1928, '30 ; C. U. 1941)

2. Give an outline of the arguments which led to the conclusion that heat is a form of energy. (*cf.* C. U. 1937 ; All. 1918, '32 ; *cf.* Bihar, 1956)

3. Explain why does a falling body become hotter when it strikes the ground. (Dac. 1927)

4. Explain why does a bicyle pump get heated when the tyre is pumped. (Dac. 1932)

5. Describe experiments to establish the connection between heat and work and deduce from them the idea of mechanical equivalent of heat. (R. U. 1941 ; Dac. 1927, '41 ; Pat. 1930, '42)

6. State the First Law of Thermodynamics. What experiments would you perform to demonstrate the truth of the Law ? (Pat. 1932, '42)

7. A mass weighing 2000 grammes falls from a height of 300 cms. If all the energy is converted into heat, find the amount of heat developed (Mechanical equivalent of heat $= 4 \cdot 2 \times 10^7$). (C. U. 1920)

[*Ans.* 14 calories]

8. What experiment would you perform to establish accurately the equivalence between work and heat ? (Utkal, 1950)

9. Define mechanical equivalent of heat. Describe a method of finding it experimentally. (C. U. 1951, '53 ; Dac. 1927 ; Nagpur, 1954 , C. U. '47, '49, '50 ; U. P. B. 1948 ; Pat. 1942, '44, '52 ; Del. 1942, '51 ; R. U. 1946, '49 ; Del. H. S. 1951)

10. What is meant by the 'mechanical equivalent of heat' ? Write down it value, and describe a method of determining it. (G. U. 1949)

11. How long will it take for an electrical heating rod of 420 watts to heat 100 c.c. of water by 10°C., if no heat is lost ? ($J = 4 \cdot 2 \times 10^7$ ergs/calorie) (Benares, 1953)

[*Ans.* 10 secs.]

12. Mention clearly the units in which the mechanical equivalent of heat is measured. (C. U. 1939, '41 ; Pat. 1930)

13. Calculate the difference in temperature of the water at the top and at the bottom of a waterfall where the height is 200 metres. (Bihar, 1956)
[*Ans.* 0 467°C]

14 An engine of one horse-power is used in boring a block of iron of mass 1000 lbs [illegible]

[illegible] horse-power = 550 ft-lbs. per sec.) (C. U. 1909)
[*Ans* 8 55°F]

15 (*a*) Calculate the work done by a gas in expanding against uniform pressure

(*b*) A ball of iron has its temperature raised through 0 6°C. through a fall of 25 metres Calculate the value of J (All 1910)
[*Ans* 4.09×10^7 ergs per cal]

16 How much work is done in supplying heat necessary to convert 40 gms of ice at $-10°C$ into steam at $100°C$. ? Sp heat of ice = 0 5 (U P. B. 1948)
[*Ans* 1.2×10^{12} ergs]

17 Describe how the mechanical equivalent of heat is determined by the frictional cones method (R U 1951)

18. Calculate the difference in temperatures between the water at the top and that at the bottom of a waterfall which is 50 metres high, given $J = 4.2 \times 10^7$ ergs/calorie (G. U 1953)
[*Ans* 0 12°C]

19. Describe Joule's method for determining the mechanical equivalent of heat (Del U 1939)

20 A tube 6 ft long containing a little mercury, and closed at both ends, is rapidly inverted fifty times What is the maximum rise in temperature that can be expected ? (Sp. ht of mercury = $\frac{1}{30}$, 1 B Th U is equivalent to 778 ft-lbs)
[*Ans* 13 88°F]

[illegible] 300 [illegible] of lead shot were placed in a [illegible] ch that on reversing [illegible] erature of the shots [illegible] was found to have risen to 28 81°C Find the value of J in ergs-calorie (sp heat of lead = 0 031) (C U, 1950)
[*Ans.* 4.12×10^7 ergs/calorie]

22 A block of ice is dropped into a well of water, both ice and water being at 0°C. From what height must be ice fall in order that one-fifteenth of it may be melted ?
[*Ans* 2283 7 metres approx]

23 Two balls of equal weight, one of India-rubber, and the other of soft clay, are dropped on to a hard floor from the same height. Which would develop the greater amount of heat by impact on the floor ?

[**Hints.**—Though K E of both on reaching the floor would be the same, the amount of heat developed by soft clay would be greater as it would remain on the floor when the energy would be converted into heat The rubber ball would at once rebound and so a large amount of its K E would be used up in overcoming g when going up.]

24 From what height would a piece of ice at $-10°C$ have to fall so that the energy to bring it to rest would generate enough heat to melt just one-tenth part of it. Given sp heat of ice = 0 5. (G U 1954, '55)
[*Ans* 5571 metres approx]

25. Calculate the velocity of a lead bullet on striking an unyielding target, if the temperature rises 200°C. and the whole of the heat generated by the impact remains in the lead. (sp. ht. of lead is 0·03). (C. U. 1937, '41, '44)
[*Ans.* $22{\cdot}45 \times 10^3$ cms. per sec.]

26. Explain why it is that while the value of the latent heat of water is less when expressed in terms of the centigrade scale, than when expressed in terms of the Fahrenheit scale, just the opposite holds in the case of numerical values of the mechanical equivalent of heat. (C. U. 1937)

27. Describe a laboratory method of determining the mechanical equivalent of heat. (R. U. 1946, '48)

28. If the two specific heat of gases C_p and C_v are respectively 0·2375 and 0·1690 calories, calculate the value of the mechanical equivalent of heat. (1 c.c. of dry air at N.T.P. weighs 0·00129 gm. and the value of atmospheric pressure $1{\cdot}013 \times 10^6$ dynes per sq. cm.) (R. U. 1944)
[*Ans.* $4{\cdot}19 \times 10^7$ ergs per calorie.]

29. Specific heat of argon at constant pressure is 0·125 calorie/gm. and at constant volume 0·075 calorie/gm. Calculate the density of argon at N.T.P. ($J = 4{\cdot}18 \times 10^7$ ergs/calorie ; normal pressure $= 1{\cdot}01 \times 10^6$ dynes/cm.^{2}) (Rajputana, 1949)
[*Ans.* $1{\cdot}8 \times 10^{-3}$ gm./c.c.]

30. Explain how the difference of sp. heats of a gas enables you to evaluate the mechanical equivalent of heat. (Rajputana, 1949 ; U. P. B. 1952)

31. Describe the principle and action of a steam engine giving a sectional diagram.

(Vis. U. 1954 ; Del. U. 1932 ; East Punjab, 1952 ; Pat. 1954 ; C. U. 1947 ; Dac. 1930, '41 ; U. P. B. 1941, '50, '53)

32. Describe with a neat diagram any from of a modern petrol engine. How does it act ?

(U. P. B. 1954 ; East Punjab, 1953 ; C. U. 1948, '53 ; G. U. 1950, 52)

33. What is the essential difference between a steam engine and an oil engine ? (R. U. 1955 ; *cf.* East Punjab, 1950 ; C. U. 1948)

34. A petrol engine uses every hour 1 lb. of petrol which produces 22000 B.Th.U. of heat, and has an efficiency of 30 per cent. What is its H. P. ? (G. U. 1950)
(1 H.P. = 33000 ft.-lbs. per min. and 1 B.Th.U. = 778 ft.-lbs.)
[*Ans.* 2·593 H.P.]

35. Write a note on 'Petrol Engines'. (*cf.* Dac. 1942 ; U. P. B. 1947)

36. Describe the essential parts of any heat engine, and describe its working during a complete cycle. (C. U. 1951)

37. A gas engine having an overall efficiency of 20% burns 1500 cu. ft. of gas per hour. If the calorific value of the fuel is 800 B.Th.U./cu. ft. find the H.P. of the engine.
[*Ans.* 94]

38. A motor car uses petrol whose calorific value is 11×10^4 B.Th.U. per gallon. The car covers on average 20 miles to the gallon running at 24 miles per hour during which the average output is 10 H.P. Find the overall efficiency of this car.
[*Ans.* 19·3%]

PART III

SOUND

CHAPTER I

PRODUCTION AND TRANSMISSION OF SOUND

1. Definition of Sound :—Sound is a kind of sensation received by means of the ears and carried to the brain which is responsible for the perception The external cause which produces such sensation is a form of energy.

Acoustics is that branch of Physics which deals with the study of the nature and propagation of sound

1(a). Sound is produced by the vibratory motion of a material body :—

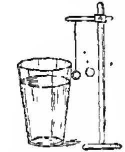


Fig. 1

Whenever any sound is produced, on tracing its origin it will be found that it is due to the vibratory movement of a material body The vibrations may in some cases be too rapid to be seen by our naked eyes but we can feel their existence by touching the source When air is blown through a whistle, a nail is struck by a hammer, or ammunition explodes in a gun, we have instances where sounds are produced by matter in motion.

Expt.—When we strike a metal vessel with a piece of matter we hear a sound, and the indistinctness of the outline of the vessel shows that it is vibrating. By touching the body the vibrations are stopped, and sound also is stopped at the same time.

Pour water in a wide-mouthed thin-walled glass-tumbler until it is almost full and keep a pith-ball suspended by a fine thread in touch with the rim of the vessel (Fig 1). On bowing the edge of the tumbler with a violin bow, ripples will be produced in the water, and the pith-ball will be observed to jump forward by receiving a series of shocks from the rim on coming in contact with it, proving that the vessel is in a state of vibration.

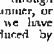


Fig. 2—Tuning-fork

1(b). A Tuning-Fork.—It is a *U*-shaped steel bar provided with a handle at the bend of the *U* (Fig 2) and is made to vibrate by striking one of its prongs on the knee or on a hard cushion. Its special quality is that it produces a sound of single frequency.

If a sounding tuning-fork is brought into contact with a pith-ball suspended by a thread, the pith-ball will be thrown into vibration.

On examining the string of a sounding violin it will be found to have a blurred outline due to its to-and-fro vibratory motion, which can be detected by placing a *V*-shaped paper rider on the string.

Thus a body must be made to vibrate in order to emit a sound, but, even when it is vibrating, the sound cannot be received or heard unless the mechanism of the ear also vibrates. We receive sound by the vibrations of a membrane in the ear, called the *eardrum*, and these vibrations are transmitted to the brain and interpreted as sound.

It should, however, be noted that the rate of vibration must lie within a limited range in order to produce an audible sound. If the rate falls below about 30 per second, or goes above 30,000 per second, the sound becomes inaudible. The above limits are only rough values, and may vary from one person to another.

2. Propagation of Sound (a material medium necessary) : —In order that sound may be heard, the disturbance from the source must be carried to the ear through a space. This space is spoken of as the *medium*. Air is the usual medium through which sound travels, but it can also pass through any other material medium provided it is elastic and continuous. Thus an observer placing his ear against a continuous iron rail can hear distinctly even slight taps, given on the metal, several hundred yards away. The ticking sound of a watch placed at one end of a table is heard clearly by applying the ear to the other end of it. Again a diver inside water distinctly hears any sound produced in the water. *Sound cannot, however, travel through a vacuum* and in this respect, *it differs from light which can easily pass through a vacuum*. Sound requires a *material* medium for its propagation.

That sound requires a material medium for its propagation and cannot travel through a vacuum may be demonstrated by the following experiment :—

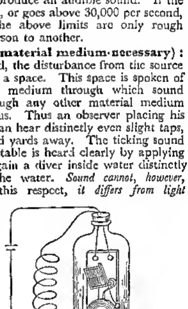


Fig. 3

Expt.—An electric bell (Fig. 3) is placed inside the receiver of an air-pump and worked by a cell placed outside the jar. The bell is suspended inside the receiver by means of a hook passing through a rubber stopper fitted tightly into the neck. The sound of the bell is distinctly heard as long as there is air inside the receiver ; if the air is gradually pumped out, the sound grows

fainter and fainter and finally becomes quite inaudible. On re-admission of air, the loudness of the sound increases again.

It must also be noted that for the propagation of sound, not only the medium must be a material one but also it should be elastic and continuous. Inelastic substances are not able to transmit sound to a great distance as the energy is dissipated very quickly. Again, non-continuous substances, such as saw-dust, felt, etc. are bad conductors of sound.

3. Essential Requirements for Propagation of Sound :—

(i) A vibrating source to emit sound. (ii) A medium to transmit the sound, the medium must be *material*, *elastic*, and *continuous*. (iii) A receiver capable of vibration to receive the sound.

4. Propagation of Sound :— Let us examine the method by which sound is actually propagated through air. Suppose a body is struck. As a result of this, every particle constituting the body begins to vibrate—that is, to move to-and-fro to a nearly equal distance on both sides of its mean position of rest. During this state of vibration, each of the extreme particles of the vibrating body in contact with air, at the time of moving to-and-fro between its extreme positions, strikes the line of air-particles in contact with it, and starts them moving to-and-fro These air-particles in their turn strike the particles beyond them, and set up similar vibrations in them, and this goes on from particle to particle. In this way a chain of vibrations is set up from the sounding body, each particle on the way begins to vibrate when it is struck by its neighbour, and in its turn strikes its next neighbour, until the vibrations reach the membrane of the ear of the listener. The motion of the membrane is communicated to the brain by the mechanism of the ear and perception of the sound is caused.

[illegible]

The time taken by the prong to move from one extreme position to the other and back again to the first position, i.e. from a to c and back, is called the **period of vibration.**

Let us imagine that the air in front of the fork is divided into layers of equal thickness. Fig 4(i) depicts the layers in front of the undisturbed position *b* of the right-hand prong of the fork.

Now, as the prong moves from *a* towards *c*, it presses the air-particles in front of it, which in turn press the particles next to them, and this pressure is passed on to the successive layers of the medium. So, considering the effect of the movement of the prong upon a column of air on the right-hand side, it will be seen that, by the time the prong reaches *c*, the air-particles between *A* and some point *C'* [Fig. 4(ii)] will be compressed, and a pulse of *compression*

will move forward (with the velocity of sound). During the return movement when the prong moves back from c to a, it tends to leave a partial vacuum behind it, due to which the layer in contact,

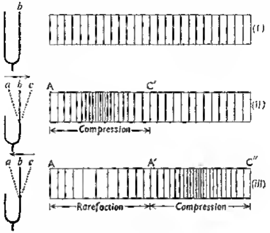


Fig. 4—Propagation of Sound-waves.

being relieved of pressure, expands on the side of the prong and the pressure is consequently diminished. Each succeeding layer acts in the same way and a *rarefaction* pulse is handed on from layer to layer, travelling in the *forward* direction and with the same velocity as that of the compression pulse. This goes on up to the time the prong takes to reach a. During the time taken by the prong to travel from c to a, the compressed pulse also travelled ownwards, and occupied a region $A'C''$ [Fig. 4(*iii*)] equal to AC' in [Fig. 4(*ii*)], which is now occupied by the rarefied pulse as given by AA' [Fig. 4(*iii*)]. So in a complete period of vibration of the prong, the disturbance travels up to C'', one-half of which $A'C''$, is occupied by a *compressed pulse* and the other half AA' by a *rarefied pulse. A compressed pulse followed by a rarefied pulse together forms a complete sound-wave.*

The amount of compression or rarefaction is not, however, equal at all points in the complete wave. The reason is that the energy communicated by the prong to the air at any instant depends on the velocity of the prong which varies from instant to instant in course of period of vibration. The velocity of the prong being maximum at the mean position and zero at the extreme positions, the compression or the rarefaction is also maximum in the middle and zero at the ends of a zone of compressions or rarefaction, as shown in (*ii*) and (*iii*) of Fig. 4.

If the displacements of the particles lying along the line of propagation at any given instant of time be plotted in the ordinate against

their distances as abscissa, the graph assumes the form of a wave. *The wavelength is the distance covered by one compression pulse and a rarefaction pulse together, i.e. the distance through which the disturbance travels in one period of vibration of the source.*

It is, however, to be noted that each particle in the medium of propagation during a periodic time of its vibration passes through all the phases of displacement as depicted in one wavelength in a sound-wave, while if the particles are considered at the same instant of time, they only successively differ in phase from one to the next.

When we say that 'sound travels in the form of waves', it is thus *not that the sound travels in a wavy path* but that if the displacement of any particle in the medium is plotted for a periodic time, or the displacements of the various particles in the path of propagation considered at the same instant of time are plotted against their distances, the curve obtained assumes a wavy form.

When a body is sounded in a homogeneous medium, alternate pulses of compression and rarefaction start out in succession in all directions travelling with the same velocity. These pulses are like so many spherical shells of equal thickness spreading out with an expanding radius with the passing of time (Fig. 5). They are analogous to the circular waves caused around a stone thrown into a calm sheet of water. Here a series of circular waves having alternate depressions and elevations are generated. They appear and disappear in succession during a periodic time. The depressions are called the *troughs* and the elevations the *crests*. They also spread out with an expanding radius till they reach the shore. The trough and the crest respectively correspond to the maximum rarefaction and maximum compression state of a medium when a sound travels through it.

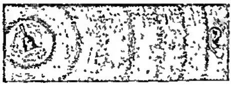


Fig. 5—Sound-waves caused by a vibrating Bell.

The difference between the two cases is that a particle on a sheet of water is displaced up and down at right angles to the path of propagation of the disturbance which travels along the surface towards the shore, whereas a particle in the medium of propagation of sound is displaced to-and-fro along the same path in which the sound travels. That is, water-waves are **transverse**, whereas sound-waves are **longitudinal** (*vide* Art. 6).

5. Representation of a Sound-wave:—Let a series of dots [Fig. 6(*a*)] represent a row of undisturbed particles of air. When a

sound-wave passes along this row, the particles in certain portions of the row will, at a given instant, come closer (*i.e.* compressed), and in certain other portions be drawn apart (*i.e.* rarefied) as represented in Fig. 6(*b*).

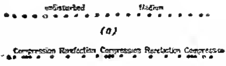


Fig. 6

Questions

1. Describe experiments which prove that sound is due to vibrations. (Pat. 1921, '32, '33)

2. Explain why a medium is necessary for the propagation of sound and describe an experiment to prove the statement. (C. U. 1934, '53)

3. Describe an experiment showing that sound cannot pass through empty space. (C. U. 1952)

4. Describe an experiment showing that air or some other medium is necessary for transmission of sound. What practical difficulty arises in such an experiment?

A metal pointer attached to the prong of a tuning-fork of frequency 256 makes a wave-trace consisting of 96 complete waves round exactly half the circumference of a smoked rotating cylinder. Find the speed of rotation of the cylinder in revs. per min. (Bihar, 1954)

[*Ans.* 80 revs. per min.]

5. Explain, as far as you can, the mode of propagation of sound through air. (Utkal, 1948; C. U. 1924, '26; *cf.* Pat. 1931, '39, '46, '53; Dac. 1928)

CHAPTER II

WAVE-MOTION: SIMPLE HARMONIC MOTION

6. Wave-motion :— Every one is familiar with the circular waves which are produced when a stone is thrown into still water. The waves consisting of a series of *crests* and *troughs* travel outwards from the centre of disturbance in gradually widening circles. But if some pieces of cork, or bits of paper, floating on the water, are carefully watched, it will be found that the floating objects, and therefore, the particles of water, are only moving up and down, but they do not travel outwards with the waves. It should be noted also that they rise and fall, not together but in succession, one after the other, showing that when the waves pass over water each separate particle of the medium must perform the same movement, not simultaneously, but each one a little later than the one preceding it. It is the wave-form which travels forward, while every particle of the water moves up and down about its own mean position of rest. Similarly, when a wave crosses a cornfield the tips of the corn-blades are not carried away

forward, the form of the wave only moves forward. The vibratory motion of a series of particles in a medium as referred to above gives rise to a *wave-motion*.

(a) Transverse and Longitudinal Waves.—

In the case of water-waves, the motion of the water particles is at right angles to the direction of propagation of the waves. Such a wave is called a **transverse wave.**

When a wave-motion passes through a medium in such a way that the vibratory motion of the particles of the transmitting medium is along the same line as the line of propagation of the sound, it is called a **longitudinal wave-motion.**

Sound-waves in air or in any other medium, which comprise pulses of compression and rarefaction, are *longitudinal* while radiant waves in ether, such as heat-waves and light-waves, are *transverse*. The electric waves used in wireless telegraphy and telephony are also instances of transverse wave-motion.

N.B. It should be noted that *gases can transmit only longitudinal types of wave-motion*, because there being very little cohesion between the molecules of a gas, transverse waves cannot be formed at all in gases, but solids and liquids can transmit both longitudinal and transverse waves.

(b) Progressive Waves.—

The longitudinal sound-waves in air or in any other medium as well as the transverse waves like the water-waves, or heat (or light)-waves are characterised by the fact that a particular state of motion in each case is handed on from one part of the medium to the other with the passing of time and the wave-form travels outwards with a definite velocity. That is why general name for these waves is **progressive waves.**

(c) Representation of transverse and longitudinal Wave-motion.—

In Fig. 7(*a*) *AB* represents a row of particles transmitting a transverse wave. As the wave passes, each individual particle of the medium will move up and down one after another at right angles to the line *AB* (as shown by the double-headed arrows) along which the wave is propagated.

Fig. 7—Illustration of Transverse and Longitudinal Wave-motion

When a longitudinal wave passes along such a row of particles, each particle will vibrate to-and-fro about a mean position along the line of propagation *CD* [Fig. 7(*b*)] and such motion of the particles will take place one after another in succession. The dot represents the mean position and the two arrows on either side the to-and-fro motion.

(d) Demonstration of wave-motions.—

(i) **Longitudinal Waves.**—The propagation of longitudinal waves can be conveniently illustrated by a spiral spring suspended hori-

zontally by threads from two parallel bars AB and $A'B'$, as shown (Fig. 8). On slightly pushing the end A of the spring suddenly forward, the nearest turns are compressed and the compression is seen to move forward along the coil with a certain velocity towards the other end, each turn moving *forward* a little when the compression reaches it. This represents the state of the layer of air-particles when a wave of compression travels through it. Again, if the end A be suddenly pulled outwards, the end turns will be separated from each other and this state of rarefaction as we call it, will be seen to be travelling along the coil to the further end, each turn of the spiral moving *backward* a little when the extension reaches it. This represents the state of rarefaction travelling through air.

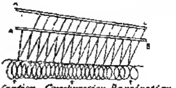


Fig. 8

Thus, if one end of the coil be alternately pushed forward and pulled outwards in a periodic manner, longitudinal wave-motion of compression and rarefaction will be seen to travel along the spiral with a constant velocity. Each turn of the spring executes a to-and-fro movement in the line of propagation of the pulse, but it is not bodily transferred from one position to another. In the same way, at the time of propagation of sound through air, the *particles* of the air only move about their mean positions of rest, and are not *bodily transferred from one place to another*. **It is the wave-form, or a succession of compressed and rarefied pulses, that travels forward.** For this reason a blast of air is never felt to spread outwards even in the case of the loud report of a cannon.

Fig. 9—Demonstration of Transverse Waves.

(ii) **Transverse Waves.**—Fig. 9 illustrates a popular model for demonstrating transverse waves. A number of straight and parallel rods, each of which carries a small ball at the top, are placed at equal

spaces apart in the same vertical plane in a stand. Each rod rests on an eccentric wheel and passes through a hole provided for it in a cross-piece held horizontally by the stand All the eccentric wheels have a common spindle which can be rotated by a handle. When the handle is turned continuously, each ball undergoes a periodic up-and-down motion while a wave-form travels from one ball to the next onwards from one side of the frame to the other as shown in the figure. The motion of each ball being transverse to the line of propagation of the wave-form, the waves produced are known as transverse waves

7. Graphical Representation of a Sound-wave:—In a transverse wave, the movements of the particle and the line of motion of the wave are mutually at right angles to each other and they can be represented in the ordinate and abscissa respectively. But in a longitudinal wave, as in the case of sound, the displacements of the particles take place along the path of propagation of the wave and so a similar representation, as in the case of a transverse wave, showing the *actual positions* of displacements along the line of propagation will result in tracing out a straight line along the line of propagation.

But if the displacements of particles are shown in the ordinate against their *mean positions* in the line of propagation represented as abscissa, for the same instant of time, a very valuable graph is obtained, which is known as the *displacement curve* for the longitudinal wave For each particle, at its mean position of vibration, a perpendicular is to be drawn to the line of propagation proportional to its displacement at the same instant of time The perpendicular is to be drawn *above* the line of propagation if the particle moves to the *right* at the instant considered and *below* the line if it moves to the *left* at that instant The displacement curve traced out in this way reveals all the properties, *e.g* velocity, acceleration, state of compression or rarefaction, etc. of the particles in the medium

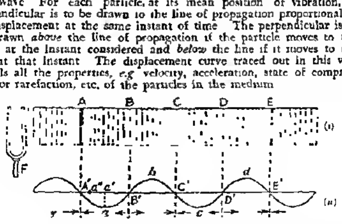


Fig 10

In Fig 10(*i*), a vibrating tuning-fork *F* (which, it should be noted, always emits a simple harmonic type of sound-wave) and a horizontal column of air in front transmitting the emitted sound are shown, where

Phase.—The phase of a vibrating particle at any instant is the state of the particle in regard to its position and direction of motion in the path of vibration at that instant *Two particles moving exactly in the same way are said to be in the same phase*; that is, particles which are at the same distance from their positions of rest and are moving in the same direction, are said to be in the same *phase*. Thus anything by which the direction of motion and displacement of a vibrating particle can be specified, will be a measure of its *phase* at that time.

A water-particle A (Fig 11) at the highest point of the crest of a water-wave is in the same phase with the particle B at the highest point of the next crest, and no other particle between these two positions are in the same phase.

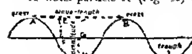


Fig 11

Phase may be expressed in three ways· (i) *By the fraction of the period that elapses after the vibrating body passes through some standard position, say, the mean position of rest, in a given direction*

Thus, in Fig 83, Part I, the phase of the oscillating bob at C in the direction BC is expressed by $\frac{3}{4}T$, at D in the direction BD by $\frac{1}{4}T$, when B is its mean position of rest

(ii) *By the angle* (as θ in Fig 12) *traced out by the generating point with reference to either of the co-ordinate axes* (*vide* Art 10) Thus, the phase of the vibrating particle M is denoted by the angle θ (Fig 12) traced out by the generating point P rotating along the circumference of the circle Again, it will be observed that the phases 90° and 450° are the same, while phases 90° and 270° are opposite to each other

(iii) *The difference of phase, of two points on a wave are also expressed by their path difference, i.e. by the fraction of a wavelength.* In Fig 11, A and B are in the same phase, the path difference being one wavelength and A, C are in opposite phases, their difference in phase being half the wavelength

Wavelength.—It is the distance through which the wave-motion travels in the time taken by the vibrating body or any of the particles of the medium of propagation, to make one complete vibration It can also be defined as the *least distance* between two particles in the same phase of vibration

In the case of a *transverse wave* the wavelength is the distance between one *crest* (or *trough*) and the next *crest* (or *trough*), as AB or CD in Fig 11 In the case of a *longitudinal wave* it is the length occupied by a pulse of *compression* together with a pulse of *rarefaction*, as AC or BD in Fig. 10.

Wave-front.—It is defined as the trace drawn through all the points on a wave which are exactly in the same condition as regards displacement and direction of motion, *i.e.* in the same phase. Thus, a surface drawn along the crests of a water-wave is a wave-front and also a surface drawn along the troughs would be another wave-front.

In a homogeneous medium a wave generated at a point travels out in all directions around the point with the same velocity. At any instant of time, the wave-motion lies upon the surface of a sphere whose centre is the generating point and radius equal to the product of the velocity and time. On this sphere the particles are all in the same phase of motion. This equal-phase surface is the wave-front at the time. At a very long distance from the source of disturbance, the spherical surface, over a limited region, may be treated as plane. So the wave-front may be taken as plane, if the source of disturbance is at a very long distance.

Period.—The period of vibration is the time taken by a vibrating body to execute one complete vibration.

9. Velocity of Sound-waves:—It is measured by the distance travelled over by a sound-wave in one second. If the Greek letter λ (pronounced 'lamda') denotes the wavelength of a sound-wave and n the frequency of vibration, then in one second there will be n complete vibrations and for each vibration the wave travels forward through a distance λ. Therefore the total distance travelled in one second $= n\lambda$. Hence, if V be the velocity of propagation of the wave, we have, $V = n\lambda$.

Otherwise, velocity $= \dfrac{\text{distance travelled}}{\text{time taken}}$, i.e. $V = \dfrac{\lambda}{T} = \dfrac{1}{T}\lambda = n\lambda \; (\because nT = 1)$

Examples. (*1*) *A body vibrating with a constant frequency sends waves 10 cms. long through a medium A and 15 cms. long through another medium B. The velocity of the waves in A is 90 cms. per sec. Find the velocity of the waves in B.* (*C. U. 1931*)

Let V be the velocity of the wave in B. Since velocity = frequency × wavelength, we have $90 = n \times 10$, where n is the frequency of vibration. $\therefore$ $n = 9$ per second. Again, for the medium B, $V = n \times 15$ (n being constant in both the cases) $= 9 \times 15 = 135$ cms. per sec.

(*2*) *If the frequency of a tuning-fork is 400 and the velocity of sound in air is 320 metres per second, find how far sound travels when the fork executes 30 vibrations.* (*C. U. 1913*)

In one second the sound travels 320 metres when the fork executes 400 vibrations. $\therefore$ In the time taken by the fork to execute 30 vibrations, the sound travels $\frac{320}{400} \times 30 = 24$ metres.

10. Simple Harmonic Motion:—If a motion is repeated at regular intervals of time, the motion is said to be periodic. Thus the motion of a particle, continuously moving round a circle, or an ellipse, in a constant time, is said to be *periodic* and, in this sense, the motion of the earth is periodic.

A vibratory or oscillatory motion is a periodic motion that reverses in direction. It has a position of rest at which the reversal in direction takes place. The motion of a pendulum is oscillatory.

The simplest type of vibratory motion is that executed along a straight line by a particle moving to-and-fro. If this vibratory linear motion be such that the acceleration of the moving particle is always directed towards a fixed point in its path and is always proportional to the displacement of the particle from that fixed point, the motion is called *a Simple Harmonic Motion* (also written S.H.M.)

To understand the nature of a particle executing simple harmonic motion, let us imagine a particle P (Fig. 12) moving round a circle with uniform speed. The particle P is called the *generating point* and the circle $XYX'Y'$ round which it moves is known as the *circle of reference*.

Let PM be a perpendicular dropped from P on any fixed diameter XX' of the circle. Now as P moves once round the circle in the direction of the arrow and describes a complete revolution, the foot M of the perpendicular, PM, moves to-and-fro along the diameter XX' from the starting point M upto X, then back to X', and then back to the starting point M again.

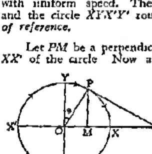


Fig. 12

This to-and-fro movement of M about O along XX' continues as P moves round the circle with uniform speed. It can be proved that the acceleration of M is always directed to the mean position O and is proportional to its displacement measured from O. The motion of M is thus a simple harmonic motion. **So, if a point moves with constant speed along the circumference of a circle, and if a second point moves along the fixed diameter of the circle so as always to be at the foot of the perpendicular drawn from the first point on the said diameter, then the motion of the second point is Simple Harmonic.**

[**Note.** The use of the term *harmonic* arose on account of the fact that the study of this was first made in connection with the study of musical vibrations.]

11. Equation of a Simple Harmonic Motion:—Let P be a point which is travelling in the direction of the arrow round the circumference of a circle $XX'P$ of radius $OP(=a)$ with uniform speed, and let XOX' and YOY' be two diameters of this circle at right angles to each other (Fig. 12). Let T be the period, *i.e.* the time for one complete revolution of P, and ω its angular velocity, *i.e.* the angle through which the radius OP revolves in 1 second. Then (*vide* Art. 36, Part I),

$$\omega T = 2\pi\text{; or, } T = 2\pi/\omega \qquad \ldots \quad (1)$$

As P moves round the circle, the point M, the foot of the perpendicular drawn from P on XOX', moves in S.H.M., and the frequency of vibration of M is the same as that of the point P. Hence the frequency of M, $n=1/T$.

Let the time be counted from the instant when M is passing through its mean position O in the positive direction (*i.e.* from left to right, when it is crossing the line YOY'). Let t be the time which has elapsed since M was last at O, *i.e.* the time taken by OP to make an angle θ with OY. The angle θ is called the phase of the vibrating particle M at that instant.

Then, $\theta=\omega t$. We have, $OM/OP=\cos\ POM=\cos(90°-\theta)=\sin\theta$

$\therefore$ The displacement x of P (i.e. OM) $=OP\sin\theta=a\sin\theta$

$$=a\sin\omega t=a\sin\frac{2\pi}{T}t \qquad \dots \quad (2)$$

$=a\sin 2\pi nt$, where n is the frequency.

N.B. If time is recorded from an instant when the generating point P is on the left of Y (i.e. M is also on the left of O) to such an extent that the generating line OP makes an angle α with OY, then $\theta=\omega t-\alpha$. That is, $x=a\sin(\omega t-\alpha)$. This $-\alpha$, which is the *phase at the commencement of time, is called the* **epoch**. The sign of the *epoch* may be positive or negative depending upon the *position* of the particle from which the time is measured.

The greatest value of $\sin\theta$ is unity; hence the maximum value of x is a, which is therefore, the amplitude of vibration. Thus, the displacement has a positive maximum value at X when $\theta=90°$ and a negative maximum value at X' when $\theta=270°$.

The displacement of a body executing a S.H.M. is always given by an equation like (2).

12. Velocity and Acceleration in S.H.M. :—

Velocity.—The velocity of M at any instant along XX' is the same as the component of the velocity of P parallel to XX' (Fig. 12). Let PD be the tangent at P, meeting $X'X$ at D. The linear velocity of P at any instant is equal to v and is along the tangent PD. The component of v parallel to XX', i.e. in the direction $OD=v\cos PDO$ $=v\sin POD=v\sin(90°-\theta)=v\cos\theta$. ... ... (3)

Thus, the velocity of M is zero at X, where $\theta=90°$ ($\cos 90°=0$), and also at X', where $\theta=270°$. The velocity is a maximum at O, where $\theta=0$, and $\cos\theta=1$ (the maximum value of $\cos\theta$), and also it is a maximum in the negative direction where $\theta=180°$; and, after a complete swing, when $\theta=360°$, the velocity is again a maximum in the positive direction. Thus, *in one complete oscillation* the velocities of M are *zero at the ends* of the swing, *i.e.* at X and X', and *maximum* when passing *through the origin* O. At O the velocity of M is parallel, and so equal, to that of P.

Acceleration.—The *generating point P* moving with constant speed round O has an acceleration v^2/a directed towards O, where a is the radius of the circle of reference (Fig 12). The acceleration of M is the component of the acceleration of P along OX. Hence the direction of the acceleration f of M is towards O and is given by,

$$f=\frac{v^2}{a}\cos POM=\frac{v^2}{a}\sin\theta \qquad (4)$$

But because v is the linear velocity of P, which describes the distance $2\pi a$ in time T, we have $2\pi a = vT$.

Or, from (1) $\quad v=\frac{2\pi}{T}.a=\omega a$; or, $v^2=\omega^2a^2$.

$\therefore$ From (4), acceleration f of $M=\omega^2 a\sin\theta=\omega^2\times$*displacement* (5)

Hence, $\frac{\text{acceleration of } M}{\text{displacement of } M}=\omega^2=\frac{4\pi^2}{T^2}=$ a constant.

Thus, *when a particle is describing a S.H.M., the ratio of the acceleration to the displacement is constant; that is, when a particle M executes a S.H.M., its acceleration is proportional to its displacement OM, and is directed towards a fixed point O in the line of vibration.*

The acceleration of M depends upon the sine of an angle just as displacement does, and so the maximum and minimum values of acceleration occur exactly at times as those of displacement.

13. Characteristics of Progressive Wave-motion :—

Regarding the characteristics of wave-motion *two points* are to be noted. (*i*) It is the *disturbance* which travels forward and not any particle of the medium.

(*ii*) The movement of each neighbouring particles begins a little later than that of its predecessor, or, in other words, there is a definite *difference in phase* between two neighbouring particles

14. Characteristics of S.H.M. :—(*i*) The motion is periodic (*ii*) It is a vibratory (to-and-fro) motion (*iii*) The motion takes place in a straight line (*iv*) The acceleration of the body executing a S.H.M. is proportional to its displacement and is directed towards a fixed point in the line of vibration

15. The Displacement Curve of a S.H.M. :— The displacement x of a particle executing a simple harmonic motion is given by the equation $x=a\sin\omega t$. If we plot a curve to show the relation between x and t, the curve will be a *sine-curve.* Fig. 13 represents the displacement curve of a

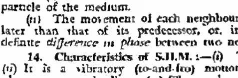


Fig 13

point M starting from O and moving with S.H.M. along YOY' due to the point P moving from X with uniform speed along the circumference of the circle having O as centre in the direction XY as shown by the arrow. Divide the circumference into any number of equal parts, say, eight, and draw straight lines through the points of divisions P, Y, P', etc. parallel to XOX'. If AB represents the period T, divide it into 8 equal parts. The time $(T/8)$ taken by P to move through each part of circumference will then be represented by each division of AB. Draw ordinates at the points 1, 2, 3, etc. that are equal to the displacements OM, OY, etc. In plotting the distances, the points below O should be taken as of opposite sign to those above O. Now, joining the tops of these ordinate lines, the displacement curve is obtained which is identical with the well-known *sine-curve.*

N.B.—Each particle in the medium transmitting a longitudinal sound-wave executes a S.H.M. with time. So the time-displacement curve for each particle in the medium will also be a sine-curve. The displacement, however, is in the line of propagation of the sound. The motion of the succeeding particles lying on the line of propagation reckoned at the same instant of time will differ in phase from particle to particle. If the displacements of the particles at the same instant are plotted in the ordinate against their distances as abscissa (though they are in the same straight line), the graph will also be a sine-curve.

16. Examples of S.H.M.:—The to-and-fro movement of one prong of a vibrating tuning-fork, the movement of a point in stretched string when the string is plucked sideways, and also the motion of the bob of a simple pendulum oscillating with a small amplitude, are some familiar examples of Simple Harmonic Motion.

17. Importance of Simple Harmonic Motion:—The Simple Harmonic Motion is of great importance in the study of sound as a vibration of this type only gives the sensation of a pure tone. Any other kind of vibration gives rise to a compound note which is composed of two or more simple tones. The importance of a tuning-fork in sound is in its unique property of giving a pure tone when sounded. All other known sources of sound give out, when sounded, complex notes which contain a number of tones. So when a sound of single frequency is required, a suitable tuning-fork is used.

18. Sound is a Wave-motion:—Sound is produced by the *vibration* of a sounding body, and the assumption that it is conveyed to the ear by means of waves is based on the consideration that the characteristics of wave propagation do also apply to the case of transmission of sound.

(1) A wave takes time to travel from one place to another.

Sound also takes time to travel from one place to another, *i.e.* it has a definite velocity.

(2) A wave requires a medium to pass through. Sound also requires an elastic medium to pass through.

The medium as a whole does not move but only allows the sound to pass through it.

(3) Waves are reflected or refracted obeying definite laws.

Sound is also reflected or refracted according to the same laws.

(4) Two sets of waves meeting each other at the same place of a medium at the same time may destroy the effect of each other under certain conditions. This is the phenomenon of *interference*.

Sound also shows interference, as in the phenomena of beats (*vide* Art. 45), stationary vibrations (*vide* Art. 50), etc.

(5) Sound can bend round an obstacle. Moreover, sounds of different acuteness or pitch (*vide* Art. 54) show this effect by different amounts. The phenomenon is known as diffraction. Diffraction is possible owing to the wave character of sound. Since sounds of different acuteness have different wavelengths, the amount of diffraction caused by them should be different.

(6) A wave of condensation started from a source has actually been photographically detected by R. W. Wood. The reality of *secondary wavelets*, first conceived by Huygens in his wave-theory, has been thus proved.

(7) The phenomenon of polarisation is shown by transverse waves only. Light-waves being transverse show the phenomenon of polarisation but the fact that sound-waves fail to show the phenomenon of polarisation prove that the vibration in this case is longitudinal and not transverse.

19. Expression for Progressive Wave-Motion:—Assuming the motion of any particle in the case of a progressive wave to be simple harmonic, the displacement of the particle at any instant is given by,

$$x=a\sin(\omega t-\epsilon)$$

$$=a\sin\left(\frac{2\pi}{T}t-\epsilon\right)=a\sin\left(\frac{2\pi}{\lambda/v}t-\epsilon\right)=a\sin\left(\frac{2\pi vt}{\lambda}-\epsilon\right)$$

where v = velocity of the wave, λ = wavelength, T = time-period, ϵ = epoch, a = amplitude, and ω = angular velocity. The wave lags a phase-angle ϵ behind the origin, i.e. a distance r given by, $r=\frac{\lambda}{2\pi}\epsilon$, since a distance λ corresponds to a phase-angle 2π, where r = distance of the particle from the origin; that is, $\epsilon=\frac{2\pi r}{\lambda}$.

$$\therefore\quad x=a\sin\left(\frac{2\pi vt}{\lambda}-\frac{2\pi r}{\lambda}\right)$$

$$=a\sin\frac{2\pi}{\lambda}(vt-r)$$

Questions

1. Explain, with the aid of a diagram, what you understand by 'wave-motion' and mention its characteristics. How do sound waves differ from light waves ? (G. U. 1957 ; C. U. 1953)

2. Distinguish between longitudinal and transverse waves. (Del. H. S. 1951, '53 ; Pat. 1941, '47 ; And. U. 1950, '51 ; Vis. U. 1955 ; C. U. 1956)

3. Establish the relation $v = n\lambda$ for a wave-motion. (And. U. 1951 ; Pat. 1948, '50, '51)

4. Define 'amplitude', 'frequency' and 'wavelength'. What is the relation between velocity and wavelength ?

Compare the wavelengths in air of the sounds given by two tuning forks of frequencies 128 and 384 respectively. (C. U. 1950)

[*Ans.* 3 : 1]

5. State what is meant by transverse and longitudinal waves.

Define wavelength, frequency and amplitude of a wave. What is the relation between wavelength, frequency and velocity of propagation ?

If the frequency of a tuning fork is 560, find how far the sound will travel at the instant when the fork just completes 100 vibrations. Velocity of sound is 1120 ft./sec.

[*Ans.* 200 ft.] (C. U. 1956)

6. When are two particles said to have the same phase ? (C. U. 1910 ; Pat. 1918)

7. Describe and explain the terms 'frequency', 'amplitude' and 'wavelength' as applied to sound waves in air. What are the differences in sensation perceived which correspond to differences in these quantities ? (All. 1923)

8. Describe the motion of a sounding body. How would you demonstrate the nature of this experimentally ?

[**Hints.**—For the first part, *see* Art. 4. For the second part, *see* Ch. VI. The nature of the motion of the vibration of the body will be represented by the wave-line on the smoked paper.]

9. A given tuning fork produces sound waves of wavelength 30 inches. If the velocity of the wave is 1100 ft./sec., what is the frequency of the fork ?

[*Ans.* 440 per sec.] (Guj. U. 1951)

10. Sound travels in air with a velocity of 330 metres/sec. at 0°C. What are the wavelengths of notes of frequencies 20,000 and 20 per second ?

[*Ans.* 1·65 cms. ; 1650 cms.] (Benares, 1950)

11. A tuning fork vibrating in air sends waves of length 100·6 cms. The same tuning fork sends waves of length 382·4 cms. in hydrogen. If the velocity of sound waves in air be 332 metres/sec., calculate the velocity of sound in hydrogen.

[*Ans.* 1261·6 metres/sec.] (Vis. U. 1955)

12. Define the angular velocity of a body moving uniformly in a circle. Find its periodic time. Show that the foot of the perpendicular drawn from the body to a fixed diameter of the circle describes Simple Harmonic Motion and hence define such a motion. (C. U. 1933)

13. Define Simple Harmonic Motion, and explain it with reference to any familiar example. (C. U. 1921, '35 ; Pat. 1941)

14. Explain Simple Harmonic Motion and state its characteristics. Show that the motion of a simple pendulum is simple harmonic. What part does S.H.M. play in sound ? (U. P. B. 1948 ; Nagpur, 1952)

15. What are the principal characteristics of a simple harmonic vibration as illustrated by the motion of a pendulum ? In what respects is the motion of a pendulum similar to the vibration of a tuning fork ?

16. Describe experiments to demonstrate that sound consists of a wave-motion in air. What is the nature of the wave constituting a sound? (Pat. 1927)

17. What reasons are there for believing that sound is conveyed by wave-motion? (Rajputana, 1951, C U 1929, '53; Dac 1932, '40; Utkal, 1951)

18. What are the evidences in support of the view that sound is propagated by means of wave-motion, and that some matter is essential for its propagation? (Pat 1933, '40, G U 1949, '57, C. U. 1953)

How do sound waves differ from light-waves? (C. U 1919; C U 1953)

19. What are the main characteristics of wave-motion? Point out the chief resemblances and differences between waves of sound and waves of light (cf C. U. 1953)

20. What is the importance of S H M in sound? Deduce an expression for the Motion of particle under S H M

CHAPTER III

VELOCITY OF SOUND

20. Velocity of Sound in Air:— Numerous examples can be cited to show that sound takes an appreciable time to travel from one place to another Thus, though lightning and thunder are produced together, the flash of the lightning is seen much before the report of the thunder is heard When a gun is fired at some distance, the flash is seen before the sound is heard, the puff of steam issuing from the whistle of a distant locomotive engine is seen before the sound is heard, so also the striking of a cricket ball with the bat is seen before the hearing of the sound In each of these cases the time-interval between seeing and hearing is due to the difference between the times taken by light and sound to travel from the source to the observer. As light-waves travel almost instantaneously (186,000 miles per sec) the time taken by light can be neglected in the determination of the velocity of sound The velocity of sound in air at $0°C$ is generally accepted as 332 metres per sec

21. Experimental Determination of the Velocity of Sound in Air:—

(a) Open-air method.— Some members of the Paris Academy first determined the velocity of sound in open air in 1738 Their findings show that the velocity of sound (i) does not depend upon any changes of the atmospheric pressure; (ii) increases with temperature and humidity, (iii) increases in the direction of the wind and decreases against it According to the Dutch physicists Moll, Van Beek, and others, the velocity of sound at $0°C$ is 332·26 metres per sec Bravais and Martins determined the velocity of sound along a slope, the difference of altitude between Faulhorn, the upper station

and the Lake of Briez, the lower station, being 2079 metres while their distance was 9560 metres. They found velocity corrected to $0°C$. to be 332·37 metres per sec. During the Arctic expedition of Parry and Greely, the experiments were done at very low temperatures and almost the same result was found.

Arago did the following experiment in 1829. Two observers were stationed on the tops of two hills several miles apart. One of them was provided with a gun while the other had an accurate stop-watch. The first man fired his gun, the second man started his watch on seeing the flash and kept a continuous record of the time until the sound of the firing was heard. A large number of observations under similar atmospheric conditions were taken and the mean value (t secs.) of the recorded times was taken. If x ft. is the distance between the two stations, the velocity of sound v is given by,

$$v = x/t \text{ ft. per sec.}$$

Such determinations are liable to two principal errors, *viz.* (1) *the error due to the wind velocity*, and (2) *the personal equation of the observer.*

The first error is that the velocity of sound is affected, though slightly, by the velocity of the wind, it being greater in the direction of the wind and smaller against it. It is corrected by the *method of reciprocal observations* in which both the observers are provided with a gun as well as a stop-watch. When one fires, the other records the time and *vice versa.* Suppose t_1 and t_2 are the mean values of the time recorded by the first and second observer respectively. If the wind is blowing in the direction of the second station from the first at the rate of e ft./sec.,

$$v + e = x/t_1, \text{ and } v - e = x/t_2$$

$$\therefore \quad v = \tfrac{1}{2}\left(\frac{x}{t_1} + \frac{x}{t_2}\right) \text{ ft. sec.}$$

Thus the effect of the wind is eliminated.

The second error is that every man is apt to delay some fraction of a second to start the watch after he actually sees the flash of the firing, and this delay-period varies from person to person and is a personal factor of the person making the experiment. This error can be avoided by making electrical arrangements for the recording of the exact moment of the gun-fire at one station and the report of the second at the other.

Regnault took both of these precautionary measures in the determination of the velocity of sound in open air in 1864 at Versailles. He found the velocity greater in the case of *sounds having great loudness*, such as explosions of bombshell, etc. *Sound-ranging* methods (*vide* Art. 30) used during the Great War of 1914-18 for the location of enemy guns, etc. give the most recent and modern means of determining the velocity of sound in open air.

(b) Laboratory Method.—

By resonance of an air column (*vide* Chapter VIII).

21. (A) Velocity of Propagation of Sound through Rare Gases :— *Kundt's Tube Method* (*vide* Chapter VIII).

22. Newton's Formula for the Velocity of Sound :— Sir Isaac Newton was the first to formulate a law that the velocity of transmission of a compression or rarefaction wave in an elastic medium is equal to the square root of its bulk-elasticity [*vide* Art. 216 (*ii*) Part I] divided by the density, that is,

$$\text{Velocity} = \sqrt{\frac{\text{Elasticity}}{\text{Density}}}\,;\ \text{i.e. } V = \sqrt{\frac{E}{D}},$$

where E is the *modulus of bulk elasticity* of the medium, and D, the *density of the medium.*

Now, the modulus of bulk-elasticity, $E = \dfrac{\text{stress}}{\text{strain}}$.

In the case of gases, *stress* is the change in *pressure per unit area* and *strain* is the corresponding change in *volume* produced *per unit volume* [*vide* Art 211, Part I].

Consider a gas of volume V c.c. under a pressure P dynes per unit area. Let the pressure be now increased by a very small amount p per unit area, and consequently let the volume be decreased by a small amount v, the temperature remaining constant.

Then, the isothermal bulk-elasticity,

$$E = \frac{\text{stress}}{\text{strain}} = \frac{\text{increase of pressure per unit area}}{\text{consequent decrease of volume per unit volume}}$$

$$= \frac{p}{v/V} = V\frac{p}{v} \qquad \dots (1)$$

Newton assumed that when sound travels through a gas, the change of pressure takes place under isothermal condition, i.e. it takes place so slowly that there is no change of temperature of the medium. So, we have, according to Boyle's law,

$$PV = (P+p)(V-v) = PV + pV - vP - pv.$$

Since in the case of sound-waves the changes in pressure and volume are very small, p and v are very small, and so the product pv is negligible.

$$\therefore\ pV = vP\,;\ \text{or } pV/v = P \qquad \therefore\ E = P \qquad \text{from (1).}$$

Thus *the isothermal elasticity of a gas is equal to its pressure.*

Hence, by Newton's law, the velocity V of sound in a gas is given by,

$$V = \sqrt{\frac{P}{D}}\,;$$

23. Calculation of the Velocity of Sound in Air at N.T.P. :—Normal pressure is the pressure exerted by a column of mercury 76 cms. in height at $0°C$. at the sea-level at 45° latitude, *i.e.* $P = 76 \times 13.596 \times 980.6$ dynes/cm.$^2 = 1.013 \times 10^6$ dynes/cm.^{2}
Again density of air at $0°C. = 0.001293$ gm./c.c.

$$\therefore \text{ Velocity of sound at N.T.P.} = \sqrt{\frac{P}{D}} = \sqrt{\frac{76 \times 13.596 \times 980.6}{0.001293}} = 280$$

metres/sec. (approximately).

But this value of the velocity of sound at $0°C$. is not in agreement with the value obtained by actual experiment, which is 332 metres per second.

24. Laplace's Correction (Isothermal and Adiabatic Elasticities) :—The calculation of the elasticity of a gas, according to Newton, involved Boyle's law according to which changes of pressure and volume of a given mass of gas take place at a constant temperature. Newton assumed that the changes in the air taking place in wave-motion had no effect on the temperature, *i.e.* the changes were *isothermal*. About 20 years later Laplace pointed out in 1817 that the changes of pressure, when sound-waves travel through a gas, are so *rapid, and the radiating and conducting powers of a gas are so poor,* that equalisation of temperature is improbable. So Newton's assumption that the temperature remains constant is not correct. According to him the changes that take place in a gas when sound-waves travel through them are *adiabatic* (*vide* Art. 65, Part II), *i.e.* no heat enters the gas from outside, or leaves it from inside. That is, Laplace held that the alternate compressions and rarefactions take place so rapidly that the heat developed in the compressed layer remains fully confined to the compressed layer and has no time to be dissipated into the entire body of the gas, and similarly the cold caused in the rarefied layer cannot be compensated for by flow of heat into it from other layers. So Boyle's law does not apply to this case.

[When sound travels in air, or in any other gas, the particles of the gas are suddenly compressed at the condensed part of the wave, and suddenly separated at the rarefied part of the wave. If a gas is compressed, or allowed to expand, suddenly, its temperature rises or falls momentarily, and, with the rise or fall of temperature, the gas expands or contracts. Now consider the effects of changes of temperature on the elasticity of a gas. During compression the temperature of the gas rises owing to which the volume of it tends to increase and so a greater increase of pressure is necessary to produce a given diminution of volume than what is necessary if the temperature of the gas remained constant (*i.e.* Boyle's law held good) during the compression. So the *elasticity* in the first case (when temperature increases) is *greater* than that in the second (when temperature is constant). Similarly, during rarefaction the temperature of a gas

falls owing to which the volume of it tends to diminish, and so a greater diminution of pressure is necessary to produce a given increase in volume than what is necessary if the temperature of the gas remained constant. So here also the *elasticity is greater* than that in the isothermal case. Considering the above, Laplace said that the value for the elasticity E under *adiabatic conditions* should be used in the Newton's formula for the velocity of sound.]

It is known that the relation between the pressure P and volume V of a certain mass of gas under adiabatic conditions is given by PV^γ = constant (*vide* Art. 65, Part II),

where $\gamma = \frac{C_p}{C_v} = \frac{\text{sp. ht. of the gas at constant pressure}}{\text{sp. ht. of the gas at constant volume}}$.

The value of γ for a *di-atomic* gas like oxygen, nitrogen, or air is 1·41 (for a tri-atomic gas like CO_2 it is 1·33). Now suppose the pressure of any particular layer of air is increased adiabatically by a small amount p by which the volume is decreased by a small amount v; then, we have,

$$PV^\gamma = (P+p)(V-v)^\gamma = V^\gamma (P+p)\left(1-\frac{v}{V}\right)^\gamma$$

$$= V^\gamma (P+p)\left[1-\gamma\frac{v}{V}+\frac{\gamma(\gamma-1)}{2}\left(\frac{v}{V}\right)^2+\cdots\right]$$

by the Binomial theorem.

But as (v/V) is very small, higher powers of it are still smaller and can be neglected. So we have,

$$P = (P+p)\left(1-\gamma\frac{v}{V}\right)$$

$$= P - \gamma\frac{Pv}{V} - \gamma\frac{pv}{V} + p, \qquad \text{whence } p = \gamma\frac{Pv}{V} + \gamma\frac{pv}{V}.$$

But since p and v are each small, pv is still smaller and can be neglected.

So, $p = \gamma\frac{Pv}{V}$; or, $\frac{pV}{v} = \gamma P$.

So the adiabatic elasticity $= \frac{pV}{v} = \gamma P$, (*vide* Art. 22)

This shows that the adiabatic elasticity of a gas is γ times the isothermal elasticity P.

Therefore, the formula for the velocity of sound in air with Laplace's correction becomes, $V=\sqrt{\dfrac{\gamma \times P}{D}}$.

Hence the value of the velocity of sound in air

$$=\sqrt{\frac{1{\cdot}41 \times P}{D}} \text{ metres per sec.}$$

$$=280 \times \sqrt{1{\cdot}41}=332{\cdot}5 \text{ metres per sec.}$$

25. Effect of Pressure, Temperature, and Humidity on the Velocity of Sound in a Gas :—

(I) *Effect of Pressure.*—If temperature remains constant, a change of pressure does not affect the velocity of sound through a gas. Let P_1 and P_2 be the pressures of a given mass of gas, v_1 and v_2 the volumes, and D_1 and D_2 the corresponding densities. Temperature being constant, we have, by Boyle's law, $P_1v_1=P_2v_2$;

$$\text{or, } \frac{P_1}{P_2}=\frac{v_2}{v_1}.$$

But volume varies *inversely* as density, *i.e.* $\dfrac{v_2}{v_1}=\dfrac{D_1}{D_2}$,

because $v_2D_2=v_1D_1=\text{mass}=\text{a constant}$.

Hence $\dfrac{P_1}{P_2}=\dfrac{D_1}{D_2}$; or, $\dfrac{P_1}{D_1}=\dfrac{P_2}{D_2}=$ a constant.

Therefore, in the formula, $V=\sqrt{\dfrac{1{\cdot}41P}{D}}$, the fraction $\dfrac{P}{D}$ remains unchanged. Hence, *the velocity of sound in a gas is independent of any change of pressure when temperature remains constant.*

(II) *Effect of Temperature.*—With change of temperature, there is a change of density and so the velocity of sound should be different. Let D_0 and D_t be the densities of a gas at 0°C. and t°C. respectively. Now by Charles' law,

$$D_0=D_t(1+\alpha t),$$

where α = coeff. of cubical expansion of the gas $=\dfrac{1}{273}$.

That is, $\dfrac{D_0}{D_t}=1+\dfrac{t}{273}=\dfrac{273+t}{273}$ ... ... (1)

Let V_0 and V_t be the velocities of sound in the gas at 0°C. and t°C. respectively and let the pressure of the gas have the same value P. So we have,

$$V_0=\sqrt{\frac{1{\cdot}41P}{D_0}}, \text{ and } V_t=\sqrt{\frac{1{\cdot}41P}{D_t}}.$$

$$\therefore \frac{V_t}{V_0} = \sqrt{\frac{D_0}{D_t}} = \sqrt{\frac{273+t}{273}} = \sqrt{\frac{T}{T_0}}, \text{ from (1)} \quad \ldots \quad \ldots \quad (2)$$

when T and T_0 are the absolute temperatures corresponding to t°C. and 0°C. respectively

Therefore, the velocity of sound in a gas is directly proportional to the square root of its absolute temperature. So the velocity of sound in a gas increases with the rise of temperature.

We have, from eqn. (2) above,

$$V_t = V_0\left(1+\frac{1}{273}t\right)^{\frac{1}{2}} = V_0\left(1+\tfrac{1}{2}\times\frac{1}{273}\times t\right), \text{ neglecting the terms}$$

containing t^2 and higher powers of t.

In the case of air, $V_0 = 332$ metres per second.

$$\therefore \qquad V_t = 332\left(1+\frac{1}{546}t\right) \text{ metres per second}$$

$$= (332 + 0{\cdot}61t) \text{ metres per second.}$$

Hence, *for each centigrade degree rise in temperature, the velocity of sound in air increases by about 0·61 metre or 61 cms., i.e. about 2 ft. per second.*

(III) *Effect of Humidity*.—The density of water-vapour is less than the density of dry air at ordinary temperatures in the ratio of 0·62 : 1 Therefore, the presence of water-vapour in the air lowers the density of air and so increases the velocity of sound in it Hence, for a given temperature, *the velocity of sound in damp air is greater than that in dry air.*

Correction for the Presence of Moisture in the observed Value of the Velocity of Sound in Air.—

If V_m = velocity in moist air at pressure P mm. and temperature t°C

V_d = velocity in dry air at pressure 760 mm. and temperature t°C.

D_m = density of moist air at pressure P mm and temperature t°C.,

D_d = density of dry air at pressures 760 mm. and temperature t°C.,

then, $\quad V_m = \sqrt{\dfrac{\gamma P}{D_m}}\,; \quad V_d = \sqrt{\dfrac{\gamma \times 760}{D_d}}$

Now, if f = saturation pressure of water-vapour at t°C., we have D_m = weight of 1 c.c. of moist air at pressure P mm. and temperature t°C = wt. of 1 c.c. of dry air at pressure $(P-f)$ mm and temperature t°C + wt. of 1 c.c. of moisture at pressure f mm and temperature t°C. (Dalton's law).

We know that the mass of 1 c.c. of water-vapour = 0·622 × mass of 1 c.c. of dry air.

Now, because the density of a gas at a constant temperature varies directly as its pressure, we have,

$$D_m = \frac{P-f}{760} \times D_d + 0{\cdot}622 \times \frac{f}{760} \times D_d = \frac{D_d}{760}(P - f + 0{\cdot}622 \times f)$$

$$= \frac{D_d}{760}(P - 0{\cdot}378f) \quad \ldots \quad \ldots \quad \ldots (1)$$

$$\therefore \quad \frac{V_d}{V_m} = \sqrt{\frac{760 \times D_m}{P \times D_d}} = \sqrt{\frac{P - 0{\cdot}378f}{P}} = \sqrt{1 - 0{\cdot}378\frac{f}{P}}.$$

$$\therefore \quad V_d = V_m\sqrt{1 - 0{\cdot}378\frac{f}{P}}.$$

26. The Velocity of Sound in Different Gases :—We know that the velocity of sound in air, $V_a = \sqrt{\frac{\gamma P}{D_a}}$, where D_a is the density of air and V_a the velocity of sound in it. Under similar conditions of pressure and temperature, the velocity in another di-atomic gas (for which the value of γ is the same), say hydrogen,

$$V_h = \sqrt{\frac{\gamma P}{D_h}}. \quad \therefore \quad \frac{V_a}{V_h} = \sqrt{\frac{D_h}{D_a}}.$$

So the velocity of sound in a gas is inversely proportional to the square root of its density. Thus if V_o, V_h be the velocities, and D_o, D_h the densities of oxygen and hydrogen respectively under the same conditions of temperature and pressure, we have,

$$\frac{V_o}{V_h} = \sqrt{\frac{D_h}{D_o}} = \sqrt{\frac{1}{16}} = \frac{1}{4}.$$

27. The Velocity of Sound in Water :— The velocity of sound in water was determined by Colladon and Strum in 1825 in the lake of Geneva, where a large bell, hung below the surface of water from the side of a boat, was struck by a hammer. The sound was received through a sort of ear-trumpet fixed in the water to another boat, which was placed at a distance of 2 miles. There was an arrangement in the first boat such that, at the instant the hammer was struck, a charge of gunpowder was ignited giving a flash in the air which could be seen by the observer in the second boat. The interval between the flash and the report was noted and the velocity was calculated in the usual way.

Theoretical Calculation.—

Velocity of sound in water, $V_w = \sqrt{\frac{\text{adiabatic elasticity}}{\text{density}}}$. For water density=1 gm. per c.c. and the adiabatic volume elasticity of water $=2{\cdot}1 \times 10^{10}$ dynes per sq. cm.

$$\therefore \quad V_w = \sqrt{\frac{2{\cdot}1 \times 10^{10}}{1}} \text{ cms. per sec.} = 1449 \text{ metres per sec.}$$

This agrees fairly well with the experimental result. Note that this is nearly 4 times the velocity of sound in air.

In calculating the velocity of sound in any other liquid, the volume elasticity (bulk modulus and the density of the liquid, which will be different from those of water, are to be considered.

28. The Velocity of Sound in Solids:—Sound travels much faster in solids than in air. The velocity of sound in cast-iron was determined by Biot by striking with a hammer one end of a long series of cast-iron pipes of total length 951 metres joined end to end. The sound travels through the walls of the pipes and through the air inside them with unequal speeds. An observer at the other end noted the interval between the sounds transmitted by the metal and that by the air. The interval between the sounds was 2.5 seconds.

Therefore, if V = velocity of sound in cast iron, and V_1 that in air, the time taken by sound to travel 951 metres through cast-iron $= 951/V$,

and that through air $= \frac{951}{V_1}$. $\therefore\ \frac{951}{V_1} - \frac{951}{V} = 2.5$

Assuming the value of the velocity of sound in air at the particular temperature, the velocity in cast-iron was determined, but the result was not quite accurate.

Theoretical Calculation.— When a compression wave is transmitted along a solid, its velocity is given by $V = \sqrt{Y/D}$, where Y = Young's modulus of elasticity for that material. For annealed steel, $Y = 21.4 \times 10^{11}$ dynes per sq. cm. and $D = 7.63$ gms./c.c.

$$\therefore \quad V = \sqrt{\frac{21.4 \times 10^{11}}{7.63}} \text{ cms. per sec.} = 5221 \text{ metres per sec.}$$

The present accepted value of the velocity of sound in iron is 5130 metres per second.

(a) The Velocity of Sound in other Forms of Solids.—The velocity of longitudinal waves in solids, when in the form of a string, can be experimentally determined in the laboratory as explained in Chapter VII. When the solid is in the form of a rod, the velocity is conveniently determined by Kundt's method (*vide* Chapter VIII) which is based on the principle or resonance.

From the table of velocities of sound it will be seen that sound travels faster in solids and liquids than in air. If the ear is applied to one end of a long wooden or metal board while somebody lightly scratches the other end, the sound of the scratching will be clearly heard, but it may not be audible when the ear is removed from contact with the board, *i.e.* when the sound travels through air.

Similarly, any sound made under water may be easily heard at a considerable distance by means of a submerged hydrophone (Art. 29) which is an under-water microphone receiver with a sensitive metal diaphragm for recording sound-waves. *But sounds do not readily*

pass from one medium to another when the media differ greatly in density. For this reason, when your ears are under water you will not be able to hear the shouts of people around you made in air.

The sounds made by a running horse's hoops will be heard from a very long distance if the ear is appl.ed to the ground th.ugh they may be inaudible when the listener is standing up, and similarly the ear in contact with a railway line catches the sounds of an approaching train long before they can be heard by others. This principle is applied by the water company's inspector in detecting leaks in the water mains under the street. This is done by applying a rod to the ground above the pipe and pressing the ear to the rod, that is, by making a continuous solid connection from the pipe to the ear when the sound of water running in the pipe will be readily audible. Similarly, the doctor presses his stethoscope on the chest in order to make a solid connection between the chest and the ear so that the sound in the lungs and of the heart-beatings may be audible.

The principle may be applied for preventing sound from passing from one room to another of a building by making cavity walls, that is, walls with an air space between them.

29. The Hydrophone :- It is a microphone receiver used for the reception of sound under water and for the finding of the direction of a sound. It is largely used in echo-depth-sounding, location of submerged objects and ice-bergs by methods of echo-soundings, location of submarines by the method of sound ranging in sea-water and similar acts of sound-reception under water.

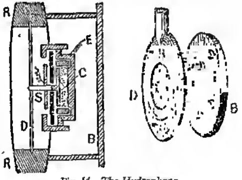


Fig. 14—The Hydrophone.

Ordinarily, it is a carbon-granule type of transmitter adapted for use under water. It consists of a heavy annular metallic ring *R* provided with a central thin diaphragm *D* made also of metal. One end of a stylus *S* is fixed to the centre of the diaphragm and the other end to a carbon-granule box *C*. The diaphragm *D* and the back-end *E* of the box are separately joined to two wires from a cable by which the receiver is dipped into the sea. The ends of the wires at the other end of the cable are connected in series to a headphone and a battery of cells. The back side of the ring *R* is provided with a screen *B*, called the baffle or the deaf side, since it cuts off the reception of sound at that end. The movement of the

diaphragm, due to the incidence of any vibratory disturbance on it causes a fluctuation of resistance in the carbon granule box and so of the current in the headphone circuit. For correct reception of sound the receiver is rotated in all possible directions until the maximum sound is heard in the headphone. The direction of the sound is normal to the plane of the diaphragm at this position.

30. Sound Ranging:—In war engagements the position of an enemy gun can be located by noting the times taken by the report of the gun to reach several sound-detecting stations.

These stations are usually selected on a common base line at a distance of some miles from the enemy front line, separated from each other by intervals of few hundred yards. Each station is provided with **a hot-wire microphone** which is a sensitive electrical apparatus for detecting sounds. These microphones are electrically connected to a central station where the instant of reception of sound by each microphone is automatically recorded.

Suppose there are three different stations A, B and C (Fig. 15). If the report of the gun reached B a second later than it reaches A, then taking 1100 ft. per sec. for the velocity of sound, the gun at G, say, must be 1100 ft. farther from B than from A, or $(GB-GA)=1100$ ft. If, again, the report reaches C three-fifths of a second later than at A, then

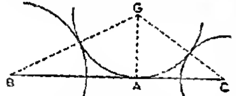


Fig. 15—Location of a Gun

$(GC-GA)=\frac{3}{5}\times 1100=660$ ft. If now circles with radii 1100 ft. and 660 ft. respectively are drawn with centres B and C, the gun G will be at the centre of a circle passing through A and touching each of the other circles. This new circle is usually drawn by trial.

31. Determination of Ship's Position:—In foggy weather when a ship finds it difficult to get its bearing, it sends out simultaneously two signals—a wireless signal and another under-water sound signal—to the stations on the coast, which are suitably equipped for their reception and which in turn inform the ship by wireless the interval between receiving the two signals. Thus, if the interval is 2 secs. at the station A, then the ship is $(2\times 4714)=9428$ ft. from A, while an interval of 4 secs. at B would indicate that the ship is at a distance of $(4\times 4714)=18856$ ft. from B, where the velocity of sound in sea-water is 4714 ft./sec. Therefore, the ship's position will be obtained by intersecting arcs drawn on the chart with centres A and B and radii 9,428 ft. and 18,856 ft. respectively.

Examples. *(1) 10 seconds have elapsed between the flash and the report of a gun. What is its distance, the temperature being 15°C. ? (Velocity of sound in air at 0°C. = 332 metres per second.)*

From formula, $V_t = V_0\left(1+\frac{1}{2}\times\frac{1}{273}t\right)$; so we have $V_{15} = 332\left(1+\frac{1}{546}t\right)$ $= (332+0{\cdot}61\,t) = 332+0{\cdot}61\times 15 = 332+9{\cdot}15 = 341{\cdot}15$ metres.

Hence in 10 seconds the sound would have travelled $341{\cdot}15\times 10 = 3411{\cdot}5$ metres.

$\therefore$ The distance required $= 3411{\cdot}5$ metres.

(2) A piece of stone is dropped into a well and the splash is heard after 1·45 seconds. Calculate the depth of the well, assuming the velocity of sound in air to be 332 metres per second. (*Pat. 1919*)

If t be the time taken by the stone in falling, the depth of the well $x = \frac{1}{2}gt^2$.

Hence the time taken by the report to reach the mouth of the well from water $= (1{\cdot}45 - t)$ sec. So the distance travelled by the sound,

$x =$ velocity $\times$ time $= V(1{\cdot}45 - t)$, $\quad\therefore\ V(1{\cdot}45-t) = \frac{1}{2}gt^2$.

or, $332(1{\cdot}45-t) = \frac{1}{2}\times 9{\cdot}81\times t^2$ ($\because\ g = 981$ cms. $= 9{\cdot}81$ metres) ;

or, $332\times 1{\cdot}45 - 332t = 4{\cdot}9t^2$; or, $4{\cdot}9t^2+332t-481{\cdot}4 = 0$. $\therefore\ t = 1{\cdot}42$ seconds.

Hence the depth of the well, $x = 332(1{\cdot}45-1{\cdot}42) = 332\times 0{\cdot}03 = 9{\cdot}96$ metres.

(3) Calculate the velocity of sound in air at 10°C. when the pressure of the atmosphere is 76 cms.

A sound is emitted by a source at one end of an iron tube 950 metres long and two sounds are heard at the other end at an interval of 2·5 secs. Find the velocity of sound in iron.

We know that, $V = \sqrt{\dfrac{1{\cdot}41P}{D}}$.

$\therefore$ The velocity of sound in air at 0°C. and 76 cms. pressure.

$V_0 = \sqrt{\dfrac{1{\cdot}41\times 76\times 13{\cdot}6\times 981}{\cdot 001293}}$ cms. per sec. $= 332{\cdot}5$ metres per sec.

$\therefore$ The velocity of sound at $10°C. = 332{\cdot}5+0{\cdot}61\times 10 = 338{\cdot}6$ metres per sec.

If V be the velocity of sound in iron expressed in metres per sec., the time taken by the sound to travel 950 metres along the iron tube is $950/V$ secs. The time taken by the sound to travel through the same distance in air at 10°C. is $950/338{\cdot}6$ secs. where the velocity of sound in air at $10°C. = 338{\cdot}6$ metres per sec.

The velocity of sound in solids is greater than that in air ; hence the time taken by the sound to travel through the iron of the tube is smaller than the time taken to travel through the air inside the tube.

$\therefore\ 2{\cdot}5 = \dfrac{950}{338{\cdot}6} - \dfrac{950}{V}$, whence $V = 3107{\cdot}92$ metres per sec.

(4) A man sets his watch by the noon whistle of a factory at a distance of 1 mile. How many seconds is his watch slower than the time-piece of the factory ? (Velocity of sound = 332 metres per sec.) (*Pat. 1941*)

The man when setting his watch by the whistle did not take the time taken by the sound to travel over a distance of 1 mile into consideration. Hence his watch is slower than the factory time-piece by the above time.

Velocity of sound = 332 metres per sec. = 1088 ft. per sec.

1 mile = 5280 ft. Therefore the time taken to travel 1 mile $= \dfrac{5280}{1088} = 4{\cdot}85$ seconds. Hence the watch is 4·85 seconds *slower*.

Questions

1. How will you determine the velocity of sound in air ? Will the result be the same when a strong wind is blowing ? How will you eliminate the effect due to wind ? Will the result be the same in summer and in winter ? Give reasons for your answer. (All. 1931)

[Hints.—As velocity increases with temperature, the value of it will be found to be greater in summer than in winter.]

2. State the law connecting the velocity of sound through a gas with its density. Compare the velocities of sound in hydrogen and oxygen under similar conditions.
[*Ans* 4 : 1] (C. U 1912)

3. Calculate the velocity of sound in air on a day when the barometer stands at 75 cms (the density of air is ·00129 gm./c.c.).
[*Ans* 330 7 metres per second]

4(a). Give Newton's expression for the velocity of sound in a gas. (All. 1946)

(b) Explain clearly the different steps in the reasoning which led to the introduction of Laplace's correction in Newton's original expression for the velocity of sound in air. (Pat 1927 , *cf.* All. 1946 ; *cf.* Del 1940)

(c) Prove that in the case of a perfect gas the ratio between the adiabatic and isothermal elasticities is the same as the ratio between its two specific heats. (R. U. 1950)

5. If the velocity of sound in air be given by $V = \sqrt{\frac{E}{D}}$, show that E is equal to the atmos
as isotherma
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6 Find the barometric pressure on a day when the velocity of sound in air is 340 metres/sec. and density of the air is $1·22 \times 10^{-3}$ gm./c.c., given that the value of $\gamma = 1·41$. (U P. B. 1955)
[*Ans.* 75 cms. of Hg]

7. Give Newton's expression for the velocity of sound in air. Does it tally with the experimental value ? If not, why not ?

How is the velocity of sound affected by change of pressure, temperature and humidity ? (G. U 1949)

8 Discuss the effects of temperature and pressure on the velocity of sound (Del. 1938 ; Utkal, 1952)

9 Describe in general terms the effects of wind, pressure and temperature on the velocity of sound in air. (G. U 1950)

10. How can the velocity of sound in atmospheric air be measured ? Give any two methods. How is the velocity affected by changes of pressure and temperature ?
(C. U. 1917, '37, '41 , All 1945, '46 ; *cf* Pat. 1921, '30, '40, '43 , U. P. B 1944)

11 Indicate how you could find the distance of a storm by noting the temperature of the air and the interval between the flash of lightning and the sound of thunder coming from the storm.

What evidence could you give that the velocity of sound is practically independent of the amplitude and frequency of the air vibrations. (Pat. 1934)

[Hints.—When some one speaks or sings, the sound is not a simple one It is a compound sound, *i.e* it consists of tones of different amplitudes and frequencies But as every tone takes the same time to reach us, evidently the velocity of sound does not depend on the amplitude and frequency of the air vibrations]

12. The interval between the flash of lightning and the sound of thunder is 3 secs. when the temperature is $10°C$. How far away is the storm? (Velocity of sound in air at $0°C$. is 1090 ft. per sec.)

[*Ans.* 1110 yds.]

13. The thunder accompanying a lightning is heard 6 secs. later than the flash. Assuming the temperature of air to be $27°C$., calculate the distance at which the lightning must have occurred. Velocity of sound in air at $0°C. = 331·3$ metres/sec.

[*Ans.* 2086 metres approx.] (M. U. 1920)

14. A cannon is fired from a station *A* at the top of a mountain and observers are placed at two points *B* and *C* equidistant from *A*, *B* is at the top of another mountain, while *C* lies in the valley between the two. Assuming the temperature of air to fall as we descend, explain which of the observers will hear the cannon first. (Pat. 1922)

15. An observer sets his watch by the sound of a gun fired at a fort 1 mile distant. If the temperature of the air at the time is $15°C$., what will be the error? Mention other causes which are likely to lead to errors in the setting. (Velocity of sound in air at $0°C. = 1090$ ft. per sec.)

[*Ans.* 4·7 secs.]

16. An echo from a cliff is heard 5 secs. after the sound is made. If the temperature of the air is $15°C$., how far away is the cliff? The velocity of sound at $0°C. = 1090$ ft./sec. (Pat. 1950)

[*Ans.* 2800 ft. approx.]

17. If the velocity of sound in air at $0°C$. and 76 cms. of mercury pressure is 330 metres per sec., calculate the velocity at $27°C$. and 74 cms. pressure.

[*Ans.* 346·6 metres per sec.] (C. U. 1935)

18. On what factors, and how, does the velocity of sound in a given medium depend?

19. The densities of dry air and moist air are in the ratio $10 : 8$. On a dry day a sound travels a certain distance in 6 secs. How long will the sound travel the same distance on a moist day?

[*Ans.* 5·36 secs.]

20. On one occasion when the temperature of air was $0°C$., a sound made at a given point was heard at a second point after an interval of 10 seconds. What was the temperature of the air on a second occasion, when the time taken to travel between the same two points was 9·652 seconds?

[*Ans.* $19·7°C$]

21. An observer sets his watch by the sound of a signal gun fired at a distant tower. He finds that his watch is slow by two seconds. Find the distance of the tower from the observer. Temperature of air during observation is $15°C$. and the velocity of sound in air at $0°C$. is 332 metres/sec. (Pat. 1939; *cf.* Utkal, 1953)

[Hints.—$V = 332(1+0·61\times15) = 341·15$ metres/sec.

$\therefore$ Distance $= 341·15\times2 = 682·3$ metres.]

22. Calculate the velocity of sound in hydrogen gas, assuming the velocity in air to be 332 metres/sec. and having also given that 1 litre of hydrogen weighs 0·0896 gm. and 1 litre of air 1·293 gms.

[*Ans.* 1262 metres/sec. approx.]

23. An explosive percussion signal on a railway is set off by a locomotive passing over it. A listener 1 km. away with one ear on the rail hears two reports. Explain the phenomenon and calculate the time interval between the two sounds. Given γ for steel $= 2\times10^{12}$ dynes/sq. cm.; ρ for steel $= 7·8$ gm./c.c.; ρ for air $= 0·0013$ gm./c.c.; γ for air $= 1·4$; $P = 10^6$ dynes/cm². (G. U. 1953)

[*Ans.* 2·8 secs.]

24. How would you show that sound travels faster in air than in carbon dioxide and slower in air than in iron? (Pat. 1918; Utkal, 1952)

25. Explain: If an observer places his ear close to one end of a long iron-pipe line, he can hear two distinct sounds when a workman hammers the other end of the pipe line. (C. U. 1950)

26. Explain how sound-waves have been used to determine the position of a ship in a sea in foggy weather.

CHAPTER IV

REFLECTION AND REFRACTION OF SOUND

32. Sound and Light Compared:—When a disturbance occurs in open air, sound-waves proceed radially outwards in all directions from the source as the centre, just as light radiates out from a centre in all directions around it. But there is a fundamental difference between the methods of their propagation. Sound is propagated in the form of longitudinal waves, whereas light is propagated in the form of transverse waves. The term *rays of light* is used to express the directions in which light-waves proceed from a source. Similarly, any line, along which a sound-wave is propagated, may be called a *sound ray*. These terms are, however, only a convenient way of speaking and have no reference to the actual modes of propagation. Light-waves are reflected from plane and spherical surfaces obeying certain laws; sound waves are also reflected according to the same laws, *viz.* that the angles of incidence and reflection are equal and that the incident and reflected rays and the normal at the point of incidence are in the same plane; but conditions under which reflections of these two waves take place are widely different on account of the lengths of light-waves and the lengths of sound-waves being greatly different. It must also be marked that light can travel through vacuum whereas sound-waves require a material medium for their transmission.

Under favourable conditions sound-waves can also be *reflected* like light-waves, and there may be also *interference* due to two waves of sound as due to two appropriate waves in the case of light.

Light from a luminous source is usually complex being composed of simple colours mixed up in some proportion. Sounds emitted by common sources are also complex. The quality of a sound (*vide* Chapter VI) depends upon the number of simple tones present in the sound, their order, and also on their relative intensities. The colour of a light, say, red or blue, depends upon the frequency of the waves produced; similarly, the pitch of a sound (*vide* Chapter VI) depends on the frequency of vibration produced.

Sound-waves are detected by the *auditory* nerves of the ear while light-waves are detected by the optic nerves.

33. Reflection of Sound:—In order that appreciable reflection of a wave may take place from any surface, the area of the surface

should be fairly large in comparison with the wavelength of the wave incident on it. Sound-waves are much larger than light-waves. The lowest audible note has got a wavelength of about half-an-inch, and the highest audible note has got a wavelength of about 32 ft.—for example, the wavelength corresponding to the note C is nearly 4 ft. whereas the wavelengths of visible light are included between 16 and 30 millionths of an inch. Consequently, it is evident that larger surfaces are required for complete reflection of sound-waves than are required for light-waves. On the other hand, the sound-waves being larger do not require the reflecting surface to be so smooth as may be required for light-waves. For this reason, a brick wall, a wooden board, row of trees or a hill-side, all serve as reflectors of sound-waves. The following experiments will illustrate how the reflection of sound-waves takes place like light-waves.

(1) Reflection at a Plane Surface.—Fix a large plane wooden board AB vertically and place a long hollow tube T_1 with its axis pointing to some point C on the board making a definite angle with the plane of the board (Fig. 16). Now place another similar tube T_2 with its axis pointing towards C. Hold a small watch just in front of the tube T_1 and put your ear at the end of the receiving tube T_2 which is turned with the point C as centre in all possible positions till the sound of the watch appears maximum, a board S being placed between the tubes to cut off the direct sound. It will be found that sound obeys the same laws of reflection as light, *viz.*—

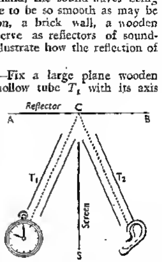


Fig. 16—Reflection of Sound.

(*i*) The angle of reflection is equal to the angle of incidence; that is, the axes of T_1 and T_2 make equal angles with the normal to AB at C.

(*ii*) The reflected sound ray, the incident ray, that is, the axes of T_1 and T_2, and the normal at the point C of incidence on the board lie in one plane.

(2) Reflection by Concave Surfaces.—Two large concave spherical mirrors M and M' are placed coaxially on a table facing each other. A watch is placed at the focus of one of them, M. The sound-waves proceeding from the watch being reflected from the first mirror will fall on the second mirror, and will be converged at the focus of M' where the sound-waves can be received by

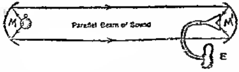


Fig. 17

the ear E by means of a funnel tube. The ticking of the watch will be distinctly heard at the focus, and it will be inaudible at other points, or at the same point, by displacing the mirror a little

33(a). Practical Examples :— The principle of reflection of sound is applied in *speaking tubes, air-trumpets, doctors' stethoscopes*, etc. In these cases the sound-waves are reflected repeatedly from side to side of the tubes (Fig. 18) Here the sound-waves can not spread, so the energy of the waves, instead of being distributed through a rapidly increasing space, remains more or less confined within the limits of the tubes, and so an ear, placed at the distant end, can hear the sound distinctly.

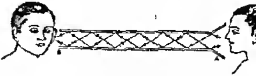


Fig 18 —Reflections in a Tube.

Reflection in an Auditorium.—Sometimes the rooms and halls of buildings with arched ceiling serve as reflectors of sound-waves The walls of large halls also often reflect the sound-waves which interfere with the words of a speaker, and the effect is confusing This may be avoided by the hanging up of screens and curtains which are bad reflectors for sound-waves The interference is also avoided to a certain extent when the hall is filled with an audience whose bodies serve to dampen the sound, and for this reason it is often easier to speak before a large audience than in an empty hall On the other hand, it has been found that in the open air, where there is no echo, it is rather difficult to make oneself heard to a large crowd, and this is not so in a big hall as a certain amount of reflection helps in increasing the volume of sound. It has been practically seen that the effect is better when the echo is heard nearly about 2 seconds after the original sound.

In churches there is often a concave reflecting board above the pulpit which reflects the sound made by the preacher down to the congregation

If a source of sound is placed at the focus of a parabolic reflector, the sound-rays are rendered parallel whereby they can reach great distances.

It is known to everyone that the hollow of the hand held at the back of the ear in a curved way serves to concentrate the sound-waves and thus helps one to hear a distant sound.

34. Echo :— *When sound returns back after reflection from an obstacle, it is called an echo.* A speaker's own words at a place are often repeated by reflection from a distant extended surface, such as a distant cliff, a row of buildings, a row of close trees, etc. The

phenomenon is known as an *echo* and is a very familiar example of the reflection of sound waves. A sound made near a wall, or hill-side will be reflected and heard as two distinct sounds, provided the distance between the observer and the reflecting surface is large enough to allow the reflected sound to reach him without interfering with the direct sound. The impression of a sound persists for about $\frac{1}{10}$th of a second after the exciting cause ceases to exist. This period may, therefore, be regarded as *the period of persistence of the sensation of sound.* Taking the velocity of sound in air to be 1100 ft. per second approximately, a sound-wave can travel 110 ft. in $\frac{1}{10}$th of a second. So, in order that the echo of a sound may be distinctly heard, the reflecting surface should be at a distance not less than 55 ft. from an observer, in order that the reflected sound-wave may reach the ear of the observer not earlier than $\frac{1}{10}$th of a second after the first sound is heard.

Velocity of Sound by the Method of Echo.—*By means of echoes it is possible to obtain a rough estimate of the velocity of sound.* Suppose you stand some hundreds of yards from a hill and try to find out the time between a shout and its echo. If you are 500 yds. from the hill and the echo comes back in 3 seconds, the sound has travelled twice 500 yds. or 3000 ft. in 3 seconds and therefore has travelled 1000 ft. in a second. So the velocity of sound is 1000 ft. per second.

Series of Echoes.—Suppose a person at A is placed between two reflectors B and C situated at a distance of 330 ft. from each other, so that the distance AB is 110 ft. and AC 220 ft. Now if a pistol is fired at A, the wave travels to B, is reflected and comes back to A reaching in $\frac{2\times110}{1100} = \frac{2}{10}$ second. The wave then travels to C and comes back to A in $\frac{2\times220}{1100} = \frac{4}{10}$ second after the first echo, *i.e.* $\frac{6}{10}$ second from the beginning. The wave again travels to B and is reflected. This goes on.

But in the beginning the sound-wave also directly travels to C, and comes back to A, after reflection in $\frac{4}{10}$ second. It then goes to B and comes to A, $\frac{2}{10}$ second later and so on. So we get a series of echoes resulting from B, in $\frac{2}{10}, \frac{6}{10}, \frac{8}{10}, \frac{12}{10}, \frac{14}{10}$, etc. second, and another series of sound resulting from C in $\frac{4}{10}, \frac{6}{10}, \frac{10}{10}, \frac{12}{10}, \frac{16}{10}$, etc. second.

Articulate Sounds.—In the case of *articulate* sounds, however, the distance of the obstacle should be at least twice, that is, 110 ft. instead of 55 ft., as observed above. It is so, because a person cannot pronounce more than 5 syllables distinctly in one second, and the ear also cannot recognise them if more than 5 syllables are pronounced in one second. If a person pronounces a, he takes $\frac{1}{5}$th of a second for it by which time the sound can travel through 220 ft., taking the velocity of sound to be 1100 ft. per second. So, an echo will be heard only if the reflecting surface be at least at a distance of 110 ft. from the observer. If the person pronounces any 5 syllables, say a, b, c, d and e,

and if the reflecting surface be at a distance of 110 ft, then he will hear the echo of the first syllable just as he is about to pronounce the second syllable *b*. Similarly, the echoes of *b*, *c*, *d*, would come to him by $\frac{1}{5}$th of a second, just as he is about to pronounce the next one. So only the echo of the last syllable will be distinctly heard. This echo which enables us to hear only one syllable distinctly is called a **mono-syllabic echo**. If the reflecting surface be at a distance of two or three times, the echo will be **di-syllabic** or **tri-syllabic** and so on. Evidently, if the distance be *n* times 110 ft., then the echo of the last *n* syllables can be heard. Echoes which enable us to hear two or more syllables are sometimes called **poly-syllabic echoes**.

35. Echo Depth-sounding:— The phenomenon of reflection of sound has been applied in measuring the *depth of the sea*. For this purpose a *hydrophone* is placed under water and a small under-water charge of some explosive is placed near it. Two sounds are heard when the charge is fired, the direct sound of explosion coming to the hydrophone and the echo of it coming a little after by reflection from the sea-bed. The instants of reception of the two sounds by the hydrophone are automatically recorded by a suitable device and the interval between them found out. If this is t sec, then taking the velocity of sound in water to be 4714 ft per sec., the distance of the surface to the sea-bed and back must be $4714 \times t$ ft, *i.e.* the depth of the sea is $2357t$ feet.

An instrument constructed on the above principle known as a **fathometer** is used for depth-sounding in oceans. Echo of radio-waves is used to explore the upper atmosphere.

Examples. (*1*) *A man stationed between two parallel cliffs fires a gun. He hears the first echo after two seconds and the next after 5 secs. What is his position between the cliffs and when he hears the third echo?* (*All, 1910; Utkal, 1951*)

Let V be the velocity of sound in air, x the distance of one of the cliffs from the man, and y the distance of the other cliff. Then, if the first echo be heard after two seconds, $2 = \frac{2 \times x}{V}$; or, $V = x$

The sound-wave will also be reflected by the other cliff and come back after 5 seconds. $\therefore$ $5 = \frac{2 \times y}{V}$; or, $V = \frac{2}{5}y$ $\therefore$ $x = \frac{2}{5}y$; or, $\frac{x}{y} = \frac{2}{5}$

That is, the position of the observer divides the distance between the cliffs in the ratio of 2:5. The third echo will be heard 7 seconds after the firing of the gun, for the sound wave reflected from either of the cliffs will be reflected from the other cliff and take 7 seconds to come to the man.

(*2*) *An engine is approaching a tunnel surmounted by a cliff, and emits a short whistle when half a mile away. The echo reaches the engine after $4\frac{1}{2}$ seconds. Calculate the speed of the engine assuming the velocity of sound to be 1100 ft. per second.*

Let A be the first position, B the second position, when the echo of the whistle is heard and C the position of the cliff.

Then $AC = \frac{1}{2}$ mile $= 2640$ ft. In $4\frac{1}{2}$ second the distance to be travelled by sound $= 1100 \times \frac{9}{2} = 4950$ ft. $\therefore$ The distance $(AC + BC) = 4950$ ft.

So $BC = 4950 - 2640 = 2310$ ft. and $AB = (2640 - 2310) = 330$ ft.

This distance is travelled by the train in $4\frac{1}{2}$ secs.

$\therefore$ Speed of engine $= \dfrac{330 \times 2}{9}$ ft. per sec. $= 50$ miles per hour roughly.

(3) *An echo repeats 5 syllables, each of which requires $\frac{1}{5}$ of a second to pronounce and $\frac{1}{2}$ a second elapses between the time the last syllable is heard and the first syllable is echoed. Calculate the distance of the reflecting surface, the velocity of sound being 332 metres per second.*

The five syllables will take $(5 \times \frac{1}{5}) = 1$ second to pronounce. Now $\frac{1}{2}$ a second elapses after the last syllable is pronounced in order that the echo of the first syllable is heard; so the time taken by the sound of the first syllable to travel to the reflecting surface and back to the observer is $1 + \frac{1}{2} = \frac{3}{2}$ seconds.

In $\frac{3}{2}$ secs. the sound will travel $332 \times \frac{3}{2} = 498$ metres. This distance is twice that between the observer and the reflecting surface; therefore the required distance $= \dfrac{498}{2} = 249$ metres.

(4) *A man standing before a cliff repeats syllables at the rate of 5 per second. When he stops, he hears distinctly the last 3 syllables echoed. How far is he from the cliff? (The velocity of sound in air is 1100 ft. per sec.).*

It has been explained already that in the case of a mono-syllabic echo the distance of the reflecting surface must be 110 ft. Now because the last 3 syllables are heard distinctly, the man must be at a distance of about $3 \times 110 = 330$ ft. from the cliff.

36. Nature of the Reflected Longitudinal Wave :— Whenever a longitudinal wave passing through one medium meets another medium of different density, it will be partly reflected, *but the type of the reflected wave will depend upon the density of the second medium.* This can be understood from the following illustrations:

Reflection at a rigid surface.— Let a number of light and heavy steel balls be arranged successively in one line, the light balls representing the particles of a lighter medium and the heavy balls those of a denser medium. If a forward push be given to one of the lighter balls, it will strike the next ball, which in turn will strike its neighbour, and in this way, energy will be handed on from one to the other until the last light ball strikes a heavy ball. After the impact, the light ball will rebound and strike a ball just behind it, and thus set up a *reflected pulse* backwards. It should be noticed that at the time of proceeding forward, one ball was pressing against another and it appeared as if a compression wave was moving onwards.

After the impact, also, the same process is repeated backwards. Therefore the nature of the pulse is not changed. Similar thing happens in the case of longitudinal sound-waves. *When such a wave meets a fixed end, or the surface of a denser medium, a wave of compression is reflected back as a wave of compression, and a wave of rarefaction is reflected as a wave of rarefaction, that is, in reflection from a fixed and rigid surface, there is no change of the type of the wave.*

Reflection at a yielding surface.—

Now, in the above experiment, if a forward impulse be given to *one of the heavy balls* the direction of motion of the heavy ball after impact with a light ball will remain the same, *i.e.* forward. But the second ball, being lighter than the striking heavy ball, after impact, will move with greater speed, so it will create *rarefaction* behind it. Consequently, *in the case of a longitudinal wave meeting a less dense medium, the reflected wave suffers a reversal of type; a compressed wave is reflected back as a rarefied wave, and vice versa.*

If both ends of a spiral are free, a pulse of condensation travelling to the other end is reflected along the same path as a pulse of rarefaction. So also a pulse of rarefaction returns as a pulse of condensation.

37. Refraction of Sound :— When sound-waves cross the boundary separating two media in which the velocities of transmission are different, they are refracted obeying the same laws of refraction as for light. The refraction of sound may be demonstrated by taking a lens-shaped India-rubber bag filled with any gas, say, carbon dioxide, whose density is different from that of air. Refraction of sound, however, has got very little important application in our daily life.

(a) Effect of Temperature.— As the density of air changes due the change of temperature and so the velocity, it follows that change of temperature of air causes refraction of sound-waves. During the day time the lower layers of air are at a higher temperature than those higher up. So the sound-waves, as they travel, will be refracted upwards, *i.e.* their line of advance will be bent away from the ground, and hence the intensity at a distance will be diminished due to this effect. On the other hand, at night time when the lower layers are colder than those above, as with layers of air over the surface of water, the bending of the *line of advance will be towards* the ground and the intensity will be increased. So, in this case, sound from a longer distance will be heard much more clearly than in day time.

(b) Effect of Wind.— A sound-wave travels a longer distance near the surface of the earth in the direction of the wind than against it.

This is due to refraction of sound. Each vertical column of air on the earth's surface moves, during a strong wind through a greater distance at the top than at the bottom. When the sound moves in the direction of the wind, the velocity of sound is *augmented* more in the upper layers than in the lower layers of such a column. The direction of propagation of the sound being normal to the column, the sound bends downwards, *i.e.* there is a concentration of sound near the surface of the earth.

When the wind blows against the sound, the velocity of the sound is *diminished* more in the upper layers of the air than in the lower layers of each column of air. So the sound is refracted upwards.

Questions

1. Describe an experiment to demonstrate the reflection of sound. (C. U. 1946)

Name a few appliances based on the reflection of sound-waves. (Pat. 1944 ; *cf.* C. U. 1946)

2. What is an echo ? Give an instance where echoes are a disturbance and mention briefly the measures that would be adopted as a remedy. (Utkal, 1952)

3. What is an echo ? (C. U. 1946 ; Pat. 1947)

Why is a succession of echoes sometimes observed ?

A man fires a gun on the sea-shore in front of a line of cliffs, and an observer, equidistant from the cliffs and 300 ft. away from the firer notices that the echo takes twice as long to reach him as does the report. Find by calculation or graphically the distance of the man from the cliffs. (Pat. 1922)

[Hints.—A is the position of the firer and C that of the observer ; B is the place on the cliffs where reflection takes place. (*See* Fig. 18, Part IV.)

From the question, $AN = NC = 150$ ft. if A, N and C are in the same st. line ; and $AB = BC$. Hence calculate NB, which is the distance of the man from the cliffs.]

[*Ans.* 259·8 ft.]

4. Explain how *echoes* are produced. How may the phenomenon be used to measure the velocity of sound in air ?

5. A boy standing in a disused quarry claps his hands sharply once every second and hears an echo from the face of the opposite cutting. He moves until the echo is heard midway between the claps. How far is he then from the reflecting surface, if the velocity of sound at that time is 1120 ft. per sec. ?

[*Ans.* 280 ft.]

6. Explain—"A brick wall reflects waves of sound but not waves of light, whereas a small mirror will reflect waves of light but not of sound." (C. U. 1952)

7. How would you show that sound waves get reflected and obey the law that the angle of incidence is equal to the angle of reflection ? Explain how the formation of an echo and the action of a physician's stethoscope are due to the reflection of sound waves. (Del. H. S. 1954)

8. How are echoes produced ? Give a practical application of the use of echoes. (Utkal, 1947 ; *cf.* Pat. 1949)

9. At what distance from the source of sound must a reflecting surface be placed so that an echo may be heard 4 secs. after the original sound ? (The velocity of sound in air is 1100 ft. per second.)

[*Ans.* 2200 ft.]

10. A man standing between two parallel cliffs fires a gun. He hears one echo after 3 secs. and another after 5 secs. ; what is the distance between the cliffs ?

[*Ans.* 4400 ft.]

11. Six syllables are echoed by a reflecting surface placed at a distance of 650 ft. What is the temperature ? ($V_0 = 1090$ ft. per sec.)

[*Ans.* $-3{\cdot}34°$C.]

12. A cannon is placed 550 yards from a long perpendicular line of smooth cliffs. An observer at the same distance from the cliffs hears the cannon shot 4 seconds after he sees the flash. If the velocity of the sound is 1100 ft. per second, when will he hear the echo from the cliffs ?

[*Ans.* 1 second after hearing the direct report.]

13. Explain the production of echoes. An echo repeated 6 syllables. The velocity of sound is 1120 ft. per sec. What was the distance of the reflecting surface ?

[*Ans.* 672 ft.] (C. U. 1940)

14. An echo repeats four syllables Find the distance of the reflecting surface, if it takes one-fifth of a second to pronounce or hear one syllable distinctly. (Vel. of sound = 1120 ft. per sec) (Pat. 1914)

[*Ans.* 448 ft]

15. A man standing between two parallel cliffs fires a rifle. He hears the first echo after $1\frac{1}{2}$ secs, then a second $2\frac{1}{2}$ secs after the shot, then a third echo Explain how these three echoes are produced Calculate how many seconds elapsed between the shot and the third echo, and calculate the distance apart of the two cliffs.

[*Ans.* $t = 4$ secs.; distance $= 2 \times$ vel of sound] (C. U. 1944)

16. How is echo employed to measure depths of oceans ? (C. U. 1946)

17. Describe experiments to demonstrate reflection and refraction of sound. A stone dropped into a well reaches the water with a velocity of 80 ft /sec and the sound of its striking the water surface is heard $2\frac{7}{12}$ secs. after it is let fall Find the depth of the well and the velocity of sound in air. (Bihar, 1955)
($g = 32$ ft./sec^2).
[*Ans* 100 ft. ; 1200 ft./sec]

CHAPTER V

RESONANCE : INTERFERENCE : STATIONARY WAVES

38. Free and Forced Vibrations :—All bodies, no matter what their size, shape, or structure, vibrate in their own natural periods, when slightly disturbed from their positions of rest and left to themselves Such vibrations are called *free vibrations.* The bob of a simple pendulum, when slightly moved to one side and then released, vibrates with its own period depending on its length; so also large structures like bridges, tall chimneys, and large ships on oceans have got their own natural periods of vibration

If a periodic force be applied to a body capable of vibration, and if the period of the force is not the same as the free period of the body, the body at first tends to vibrate in its own way but will ultimately vibrate with a period equal to that of the applied force. Such vibrations of the body are called *forced vibrations.*

Examples.—If a vibrating tuning-fork is held by the stem in the hand, the sound will be most inaudible even from a small distance, but, if the st[illegible] is much intensified The [illegible] communicated to the table [illegible] rate Due to the vibrations of the table a large volume of the air in contact is made to vibrate, and the waves thus set up are added to those originating from the fork, and, consequently, the sound becomes louder.

The diaphragm of a gramophone sound-box is a common example of forced vibration, where the diaphragm vibrates with frequencies corresponding to the tones conveyed from the record. The vibrations from the sounding boards of musical instruments like violin, piano,

etc. are also forced vibrations. The sounding board of a violin is first set into forced vibration by the vibration of the strings, and then the large mass of air inside the board also vibrates and intensifies the sound.

39. Resonance :—When a body is forced to vibrate, due to an applied external force, it vibrates with a very small amplitude, if the period of the applied force is different from that of the free period of the body; but when these two periods are the *same*, the body vibrates with a much greater amplitude. The latter phenomenon is known as resonance. Thus *resonance is a particular case of forced vibration and is produced when one body forces vibrations on a second body whose natural frequency of vibration is equal to that of the first.* The principles of forced vibration and resonance may be illustrated by the following experiment:—

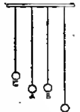


Fig. 19

Expt.—Four simple pendulums A, B, C and D are suspended from a flexible support. The lengths of A and B are equal, and so they have got the same period of vibration; C is slightly shorter, and D slightly longer than A or B (Fig. 19). When A is set in vibration the flexible support is also set in forced vibration of the same period, but of smaller amplitude. As a result of the vibration of the support, a periodic force of the same period is applied to each of the pendulums B, C and D which are made to vibrate. It will be found that B, whose length is equal to that of A, readily vibrates with an equal amplitude. *This is the case of resonance.* The pendulums C and D at first swing slowly and irregularly and then come to rest, but, ultimately vibrate steadily with the same period as that of A, but with smaller amplitude. They show forced vibration.

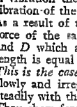


Fig. 20—Resonant Air-column.

40. Resonance of Air-column :—The air-column within a tube may also be made to vibrate by resonance, when a vibrating tuning-fork is held close to the upper end of the tube.

Take a vibrating tuning-fork A and hold it horizontally, as shown in the figure, over a tall glass jar B (Fig. 20). Now gradually pour water into the jar and note that for a certain length ED of the air-column inside the jar a maximum sound is heard. Pour more water in, and the sound disappears. This strengthening of the sound is called *resonance*, which in this case, takes place when the period of vibration of the tuning-fork is equal to the natural period of vibration of the enclosed column of air.

It will be found that, for forks having different

frequencies of vibration, the length of the air-column giving maximum resonance will be different. It will be greater or less as the frequency of vibration of the fork is lower or higher (for explanation, *vide* Chapter VIII).

41. Sounding (or Resonance) Boxes:— Tuning-forks are often mounted on hollow wooden boxes, called sounding or *resonance boxes*. The sizes of these boxes are so arranged that the enclosed mass of air has a free vibration whose natural period is the same as that of the fork. When the fork is struck, it sets the wood into forced vibration of the same period, and this agrees with the natural period of vibration of the enclosed mass of air; so the sound becomes louder due to resonance

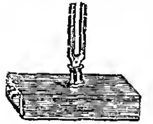


Fig. 21—Resonance Box.

Here the energy of the vibrating fork is quickly used up in setting the wood with the enclosed air into vibration, whereby loudness is gained at the cost of duration of sound. So the action does not violate the principle of conservation of energy Instruments like the *sonometer, violin, sitar, esraj*, etc. are always provided with a large hollow wooden board known as a **sounding board** whose principle of action is similar to what is explained above. When the handle of a tuning-fork vibrating feebly is held on a table, the sound is intensified Here the intensification is due to the vibration of a large volume of air which is made to vibrate by the forced vibration of the table.

42. Resonators:—The great German scientist Helmholtz (1821-1894) constructed globes of brass, each having a large aperture B for receiving sound-waves and a small one A at the other side against which the ear is placed (Fig. 22). He utilised the principle of resonance in his investigations on the quality (*vide* Art 54) of notes emitted by various sources. These globes of various sizes are called **Helmholtz resonators.** In a given set of these resonators the size of each resonator is such that it can respond to a tone of given fixed frequency and the tuning is so perfect that the particular tone, if present in a complex note, can be picked up with distinctness, by placing the ear at the small aperture A.

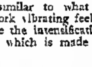


Fig 22—
A Resonator.

43. Sympathetic Vibration:—If two stringed instruments are tuned to the same frequency and if one of them is sounded, the second one also is automatically excited when placed close by. The induced vibration of the second is known as sympathetic vibration

Let two tuning-forks of the same vibration frequency fitted to two resonance boxes be placed near each other. One of them is bowed strongly and then the vibration is stopped by touching it, when the other will be found to emit the same note, although it has not been bowed at all. This is a case of resonance. The vibration of the second fork is called *sympathetic vibration*. The phenomenon will not happen, if the frequencies of the forks are not exactly the same.

If a sequence of small repeated impulses be applied to a vibrating pendulum, and if each push be given exactly at the end of one complete swing, or, in other words, if the period of the impulse be exactly equal to the period of vibration of the swing itself, the pendulum will vibrate so that each succeeding swing will be greater than the previous one. It is for this reason that soldiers are ordered to break step when crossing a suspension bridge, as otherwise the regularity of the impulse due to the steady marching may agree with the natural period of vibration of the bridge, which will set up dangerous oscillation. Similarly, a ship at sea may be thrown into dangerous oscillations when the frequency at which the ship is struck by the waves is equal to the natural frequency of vibration of the ship.

44. Interference of Sound :—When two systems of waves travel through the same medium simultaneously, the actual disturbance at any point of the medium at any instant is the resultant of the component disturbances produced by the waves separately i.e. *the actual displacement of a particle at any point of the medium is the algebraic sum of the displacements which the waves would separately produce*. This is known as the **principle of superposition.** If the crests of the two waves arrive simultaneously at the same point, *i.e.* if they are in the *same phase*, then they will combine to produce large crests; and similarly two troughs arriving at the same point at the same instant will produce deeper troughs. But, if the two waves are exactly *similar*, and if conditions are such that the *troughs* of one wave fall upon the crests of the other, *i.e.* if they are in *opposite phases*, then they completely annul one another and the result will be the absence of any disturbance in the medium at that place at that instant and the *two sound-waves*, in such a case, *will produce silence*. This is the **principle of interference of sound.**

By dropping two stones into a pond simultaneously at two neighbouring points, two sets of ripples are produced and when these ripples meet one another, a definite *interference pattern* is observed. Some lines can be seen along which the water particles are undisturbed and there are other intermediate lines along which a maximum disturbance occurs. Similarly, for sound-waves, the compressions of one set may serve to neutralise the rarefactions of another set at some points of a medium and to reinforce the compressions of the other set at other points of the medium.

45. **Beats**:—When two sounds preferably of the same type and intensity but with slightly different frequencies are produced together, a fluctuation of loudness (waxing and waning of sound) occurs at any place in the neighbourhood of the sources of sound due to the mutual interference of the two notes. In the resulting sound-wave the component waves periodically reinforce each other at some instant of time and destroy each other at some other instant of time and so the sound heard possesses a characteristic throbbing or beating effect. *This phenomenon is known as* **beats.** The phenomenon may be represented graphically as follows:—

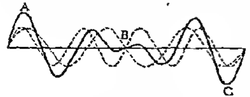


Fig. 23—Formation of Beats

In Fig. 23, the dotted curves represent two wave systems (arranged on the same axis) produced by two vibrating tuning-forks of slightly different frequencies. At the beginning of a given second, the two forks are swinging together so that they simultaneously send out condensations, and the result of the two condensations will produce a double effect upon the ear (as at *A*, Fig. 23). But as the frequencies of the forks differ, the subsequent effects upon the ear is represented by the continuous curve, which is the result of combining these two wave systems, and is obtained by finding the algebraic sum of the separate displacements, as time passes.

It is evident from the nature of the continuous curve that its amplitude varies in a periodic manner, being maximum at *A* and *C* and minimum at *B*, due to which there is a periodic change in the intensity of the sound heard. At *A*, when the vibrations are in the *same phase*, the resultant displacement is the sum of the displacements of the two components waves, and at *B* these are in *opposite phases*, and the resultant displacement is given by their difference. As the loudness depends upon the amplitude of vibration, the sound heard, for small intervals, corresponding to instants, *A* and *C*, is the loudest when the amplitudes are maximum, and it is minimum at *B* when the amplitude is minimum. Such fluctuations of loudness of the sound are known as *beats*.

Suppose two tuning-forks having frequencies, 256 and 257 per second respectively, are sounded together. If, at the beginning of a given second they vibrate in the *same phase* so that the compressions (or rarefactions) of the corresponding waves reach the ear together, the sound will be strengthened. Half a second later when one makes 128 and other $128\frac{1}{2}$ vibrations, they will be in the *opposite phase*, i.e. a compression of one wave will unite with a rarefaction of the other and will tend to produce silence. At the end of one second they

will again be in the *same phase* and the sound will be augmented, and by this time, one fork will gain one vibration over the other. Thus, in the resultant sound the observer will hear the maximum of loudness at every interval of one second. Similarly, a minimum of loudness will be heard at an interval of one second. As we may consider a single beat to occupy the interval between two consecutive maxima or minima, *the beat produced in the above case is one in each second.* It is evident, therefore, that when two sounds of nearly the same vibration frequencies are heard together, the number of beats per second is equal to the *difference* of the frequencies of the two vibrating sources. Thus, if n_1 and n_2 $(n_1 > n_2)$ be the frequencies of the two sources, then the number of beats per second is equal to $(n_1 - n_2)$. Thus, *the number of beats heard per second is numerically equal to the difference in frequencies of the two sounds.*

45(a). Number of Beats heard per Second is equal to the Difference between the two Frequencies:—

Let the smaller of the two frequencies be n_1 and the other greater than it by n. Assuming that they start with the same phase, displacements produced by the two wave-systems at a point at some instant of time t will be given by,

$$y_1 = a \sin 2\pi n_1 t, \text{ and } y_2 = b \sin 2\pi(n_1 + n)t.$$

By the principle of superposition, the resultant displacement will be given by,

$$\begin{aligned} y = y_1 + y_2 &= a \sin 2\pi n_1 t + b \sin 2\pi(n_1 + n)t \\ &= \sin 2\pi n_1 t(b \cos 2\pi nt + a) + b \cos 2\pi n_1 t . \sin 2\pi nt. \end{aligned}$$

This equation represents a wave-equation which may be condensed into the form $y = F \sin (2\pi n_1 t + \alpha)$, where F is its amplitude and α, the epoch. The values of F and α can be found by comparing the two equations and equating the coefficients of $\sin 2\pi n_1 t$ and $\cos 2\pi n_1 t$. That is,

$$F \cos \alpha = b \cos 2\pi nt + a, \text{ and } F \sin \alpha = b \sin 2\pi nt.$$

By squaring both sides and adding,

$$\begin{aligned} F^2 &= b^2 \sin^2 2\pi nt + b^2 \cos^2 2\pi nt + 2ab \cos 2\pi nt + a^2 \\ &= a^2 + b^2 + 2ab \cos 2\pi nt \qquad \ldots \qquad \ldots \qquad \ldots \quad (1) \end{aligned}$$

$$\text{Also,} \quad \tan \alpha = \frac{b \sin 2\pi nt}{b \cos 2\pi nt + a} \qquad \ldots \qquad \ldots \qquad \ldots \quad (2)$$

It is evident from (1) that the amplitude of the resultant wave varies with time. It assumes maximum and minimum values as follows:—

when $t = 0$, $\cos 2\pi nt = 1$, $F = a + b$ (maximum);

when $t = \frac{1}{2n}$, $\cos 2\pi nt = -1$, $F = a - b$ (minimum);

when $t = \frac{1}{n}$, $\cos 2\pi nt = 1$, $F = a + b$ (maximum).

Thus, in an interval of $\frac{1}{n}$ second, two maxima and an intermediate minimum take place. Similarly, it can be shown that between two minimum sounds, a maximum occurs in a period of $\frac{1}{n}$ sec. So the number of beats (two successive maxima or two successive minima produce one beating effect) per sec. is $= n =$ the difference of the frequencies.

46. Tuning Instruments :— It should be *remembered* that beats can be heard only when the frequencies of the notes are nearly equal to each other; if their difference is greater than 15 or 16, separate beats cannot be heard and a discordant unpleasant noise is the result. It is for the above reason that musical instruments are tuned by means of beats. If beats are heard between the first overtone (*vide* Ch. VII) of a lower note and its octave, say in the case of a piano or organ, it is a sure test that the instrument needs tuning. Beats are not heard when the frequencies of the two sounds are exactly equal.

47. Determination of the Frequency of a Fork by the Method of Beats :— Two forks having nearly the same frequency are mounted on sounding boxes and sounded together. The number of beats in any time is counted by means of a stop watch, and, from this, the number of beats per second is determined, which is equal to the difference of the frequencies of the forks. By knowing the vibration frequency of one of them, that of the other can be determined. *To know whether the frequency of the given fork will be higher or lower than that of other, one of the prongs of the given fork is loaded with a little wax, and the number of beats per second is again determined.* The frequency of the fork is diminished by loading its prong. Hence, if the number of beats per second obtained after loading the fork is *greater* than the number obtained before, the frequency of the given fork must be *less* than that of the known fork; if the number be *less*, then the frequency of the unknown fork is *greater* than that of the known fork.

N.B.— The frequency of a fork is increased by filing it.

The uses of beats are in (*a*) *finding frequency*; (*b*) *tuning instruments*.

Examples. (*1*) *Two tuning forks A and B, the frequency of B being 512, are sounded together and it is found that 5 beats per second are heard. A is then filed and it is found that 5 beats occur at shorter intervals. Find the frequency of A.* (*All. 1916; C. U. 1936*)

Since A is filed, its period is diminished, and its frequency is increased; but because beats occur at shorter intervals, *i.e.* the number of beats increases by increasing the frequency of A, it is clear that the frequency of A is greater than that of B.

If n_1 and n_2 be the frequencies of A and B respectively, we have

$$n_1 - n_2 = 5\text{, or } n_1 - 512 = 5; \quad \therefore \quad n_1 = 512 + 5 = 517.$$

(*2*) *The interval between two tones is $\frac{16}{15}$ and the higher tone makes 64 vibrations per second. Calculate the number of beats occurring per second between the tones.*

The interval is the ratio of the two frequencies (*vide* Art. 56). Let the frequency of the first be n, then we have,

$$\frac{15}{16} = \frac{n}{64}. \quad \therefore \quad n = 60. \quad \therefore \text{ The number of beats} = (64-60) = 4 \text{ per sec.}$$

(3) *A fork of unknown frequency when sounded with one of frequency 288 gives 4 beats per sec., and when loaded with a piece of wire again gives 4 beats per sec. How do you account for this and what was the unknown frequency?* (*Pat. 1945*)

The experiment shows that the unknown frequency n in the begining was higher by 4, and after loading the fork with a piece of wire the frequency n' was lower by 4, *i.e.*, it became $(288-4) = 284$. So the unknown frequency $n = 288+4 = 292$.

48. The Conditions for Interference of Two Sounds:—

(1) The component waves must have the same frequency and amplitude.

(2) The type of the two waves should be preferably similar.

(3) The displacements caused by them must be in the same line.

49. Experimental Demonstration of Acoustical Interference:— Two separate sources producing waves satisfying the conditions for interference cannot be realised in practice. That is why, in practice, the waves from a single source are divided at a point and made to reunite again at some other region after travelling paths of different lengths. **Quinke** based his arrangement on this principle, and his apparatus consisted of a mouth piece A connected to the two limbs B and C which combine again into one tube EF against which the ear is placed (Fig. 24). D is a sliding tube, by drawing which in or out the length of the path $ACDE$ can be suitably altered. A vibrating tuning-fork T is held at A and the resulting sound at F is heard. When the sliding tube is at D, the paths ABE and $ACDE$ are equal so that the two waves passing through them meet in the same phase at F and produce a maximum sound. The path $ACDE$ is then increased by drawing out the sliding tube D until a position D_1 is obtained when a minimum sound is produced. The difference in path between $ACDE$ and ACD_1E is half the wavelength. By further drawing out the tube D from D_1 to D_2, again a maximum sound is obtained, the shift so made being equal to half the wavelength again. Thus the full wavelength of the sound used is obtained.

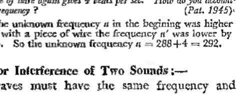


Fig. 24

50. Progressive and Stationary Waves:

Progressive Wave.—In a *progressive wave* a particular state of motion is continuously transferred forward from one part of the medium to the next by similar movements performed one after

another by the consecutive particles, and so the particles pass through the same cycle of movements when the wave advances forward.

Thus, though the motions of the particles are otherwise similar (as the distance from the source of disturbance increases, the amplitude of motion, will however, decrease, the phases of the particles change continuously from one to the next along the direction of propagation of the wave

An ordinary sound-wave in air is an example of longitudinal progressive wave and an ordinary water-wave is a transverse progressive wave.

Stationary Waves.—When two sets of progressive waves, having the same amplitude and period, but travelling in opposite directions with the same velocity, meet each other in a confined space, the result of their superposition is a set of waves, which only expand and shrink but do not proceed in either directions. These waves are called *stationary waves* They are so called because they remain confined in the region in which they are produced and are non-progressive in character Moreover, the nature of vibration at each position along such a wave is fixed.

Stationary vibrations may be longitudinal as well as transverse in character In the case of an organ pipe the longitudinal waves travelling from one end of it get reflected from the other end and travel back. These direct and reflected waves, identical in character but opposite in direction of travel, have also the same velocity and so they produce longitudinal sationary waves within the organ pipe. When a string stretched on a sonometer between two bridges is plucked, the transverse vibrations travel along the string and being reflected from the bridges travel in the opposite direction with the same velocity (*vide* Art 61) The direct and the reflected disturbances are identical in character, but travelling in opposite directions, produce stationary vibrations transverse in character which remain confined within the string

Unlike to a progressive, wave, here the particles in the confined space lying along a line do not successively pass through similar movement, but each particle vibrates in a simple harmonic manner with an amplitude which is fixed for it. The amplitude is minimum at equidistant fixed positions along the confined space, *i e.* the particles at such positions are permanently almost at rest. Such positions are called **nodes.** From one node to the next, the amplitude of vibration of the sucessive particles gradually increases to a maximum (double of the maximum for each constituent wave) midway between the two nodes and then decreases to a minimum at the next node, but the particles between the two connective nodes are always vibrated in the same phase The positions of maximum amplitude are called the **antinodes.** If the displacement is positive in the region between two consecutive nodes, the displacement is negative in the region between

the next two nodes, *i.e.* the particles between two consecutive nodes differ in phase by 180° from the phase of the particles between the next two consecutive nodes. The positions of the nodes and antinodes are fixed and the features are invariable. The distance between two consecutive nodes (or between two consecutive antinodes) is equal *to half the wavelength* of either of the two superposing waves.

Graphical Representation of Stationary Waves:—

In Fig. 25 is shown graphically the addition of two identical transverse simple harmonic progressive waves travelling in opposite directions. The full curve represents the resultant wave obtained by adding the ordinates, *i.e.* the displacements of the two dotted curves. The second diagram in the figures shows the two waves and their resultant, at a time $\frac{1}{8}$ T later than the first; that is, each wave has advanced one-eighth of a wavelength λ, one $\frac{1}{8}\lambda$ to the right and the other $\frac{1}{8}\lambda$ to the left. The third diagram shows the waves $\frac{1}{8}T$ later than the second, *i.e.* $\frac{2}{8}T$ later than the first and one of the dotted curves has moved $\frac{1}{8}\lambda$ to the right further than the preceding one and the other $\frac{1}{8}\lambda$ to the left farther than the preceding one. The dotted curves exactly neutralize one another and the resulting disturbance is represented by a straight line. Similarly, the fourth and the fifth diagrams represent the waves and their resultants respectively after times $\frac{3}{8}T$ and $\frac{1}{2}T$. By taking times $\frac{5}{8}T$, $\frac{6}{8}T$, etc it will be seen that the same changes are produced in the reverse order.

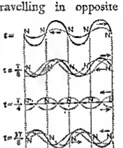


Fig. 25—The Formation of Stationary Waves.

Note that the points of the full curves marked N through which dotted vertical lines pass are always at rest. These points are the *nodes*. The points midway between the nodes are the *antinodes* or loops. These are points of maximum disturbance. The resultant disturbance simply shows a change of form from instant to instant as given by the full curve, but there is no forward motion of the wave as a whole. Such waves in which the positions of the nodes and antinodes are fixed are stationary waves.

51. Progressive and Stationary Waves Compared:—

Progressive Waves	*Stationary Waves*
(*i*) All particles of the medium execute periodic motions about their mean positions, and have identical motions (with the dis-	(*i*) All particles of the medium (except at some equidistant points) execute periodic motions having amplitudes which are

Progressive Waves	*Stationary Waves*
tance from the source increasing, the amplitude will, however, decrease gradually from one particle to the next).	fixed for them. From a definite particle along the line of propagation, the amplitude increases gradually from a minimum to a maximum at some other definite particle and then decreases in the [illegible] minimum are called *nodes* and the points (midway between the nodes), where the amplitude is maximum, are called *antinodes*. The period of motion for the particles is the same as that of the component waves.
(*ii*) The wave travels onward with a definite velocity.	(*ii*) The wave is not bodily transferred from one part of the medium to another; and the compressions and rarefactions or the crests and troughs, in the case of longitudinal waves or transverse waves as the case may be, merely appear and disappear without progressing in either direction.
(*iii*) The movement of one particle begins just a little later than its predecessor, or, in other words, the phases of the particles change continuously from one particle to the next.	(*iii*) At any instant all the particles in any one segment, *i.e.* between two consecutive nodes or antinodes, are in the same phase, but the particles in two consecutive segments are in opposite phases.
(*iv*) Each particle of the medium in turn goes through a similar movement, *i.e.* similar changes of pressure, density, etc. as a complete wave passes through it and is restored to its initial condition *after each periodic time*.	(*iv*) The particles at nodes undergo maximum change of pressure and density while those at antinodes undergo minimum change of pressure and density throughout a periodic motion.
(*v*) In a complete vibration there is no instant when all the particles are stationary.	(*v*) Twice in each complete vibration all the particles are at rest at the same moment (*vide* line 3, Fig. 25).

52. Hermann Helmholtz (1821—1894):—A German physicist and Physiologist of very outstanding calibre. He made extensive researches on light, sound and electricity. He was son of a school teacher in Potsdam and began his life as an army doctor after studying Medicine in Friedrich Wilhelm Institute in Berlin. In 1848 he joined the University of Königsberg as Professor of Physiology. He here invented the Ophthalmoscope for examination of the retina of the eye. After successively serving at Bonn (1855) and Heidelberg (1858) as Professor of Anatomy and Physiology he was then called to Berlin as the first Professor of Physics. In 1888 he became president of the newly-founded Physikalisch Technische Reichsanstalt (corresponding to the English National Physical Laboratory).

He had a wide ranging capability for different domains of knowledge and a rare aptitude for Mathematics. Two of his earlier works, **Physiological Optics** and **Theory of Sound**, made him a popular scientist in his time. Some Laboratory instruments such as Helmholtz Coil Galvanometer, Helmholtz Resonators, etc. still bear his name. His most outstanding contribution to Physics, however, lies in his explanation of the quality of musical sounds. He has shown that the quality of musical sounds depends wholly on the number, order of succession and the relative intensities of the overtones present and is independent of their phase relationships. He also investigated on the physiological effects of overtones and found that a note which possesses the few overtones only, not exceeding the sixth, besides the fundamental, has a pleasing effect on the ear, while notes containing more overtones are generally discordant.

Questions

1. (*a*) Explain clearly the difference between forced vibration and resonance. Give mechanical and acoustical illustrations.
(*cf.* C. U. 1909 ; Bomb. 1952, '55 ; Pat. 1951)

(*b*) Write notes on 'Forced vibrations'. (Utkal, 1953 ; Pat. 1947, '52)

2. Describe experiments to illustrate the principle of forced and free vibrations and give illustrations in case of sound.
(Pat. 1951 ; R. U. 1955 ; Poo. U. 1952 ; East Punjab, 1953)

3. Explain the principle of resonance.
(Del. H. S. 1948, '50, '52 ; Guj. U. 1953 ; Rajputana, 1952 ; Utkal, 1953 ; All. 1925, '29, '45 ; Pat. 1929, '30 ; C. U. 1929)

4. Explain why, when the handle of a vibrating tuning-fork is pressed against a wooden board, the intensity of sound is greatly increased.
(C. U. 1915 ; *cf.* 1920, '31, '47)

5. Explain what you mean by 'resonance' and 'resonator'.
(Pat. 1929 ; *cf.* '31, '33 ; All. 1918 ; G. U. 1949)

6. Explain how resonators are used for the analysis of sound.

7. What are 'beats' ? (Pat. 1947)

How are they produced ? If two tuning-forks sounded together produce beats, how would you determine which was of the higher pitch ?
(All. 1925, '32, '44 ; Dac. 1930 ; *cf.* Pat. 1932, '40, '41, '45 ; *cf.* C. U. 1933, '39)

8. What are 'beats'? How are they produced? Illustrate your answer by suitable diagrams and mention some uses of beats. (Dac. 1951; Utkal, 1951)

9. How has the phenomenon of beats been used to determine the unknown frequency of a tuning-fork? Explain. (A. B. 1952)

10. A standard fork A has a frequency of 256 vibrations and when a fork B is sounded with A there are four beats per second. What further observation is required for determining the frequency of B? (C. U. 1939)

[*Ans.* The frequency is either 260 or 252. To know exactly the frequency of B, we should know whether the frequency of A is greater or less than that of B.]

11. A tuning-fork originally in unison with another tuning-fork of frequency 256 produces 4 beats per sec. when a little wax is attached to it. What is its frequency now? (Pat. 1952)

[*Ans.* 252]

12. Calculate the velocity of sound in a gas in which two waves of length 1 and 1·01 metres produce 10 beats in 3 seconds. (U. P. B. 1954; Rajputana, 1949)

[*Ans.* 336·67 metres/sec.]

13. A set of 24 tuning-forks is arranged in a series of [illegible]

14. You are provided with two tuning-forks of nearly equal frequencies. Explain how you would proceed to find out which of the two has the greater frequency. (Pat. 1941; R. U. 1952)

15. Explain the phenomenon of 'beats' in sound. How will you prove that the number of beats produced by two sounding bodies is equal to the difference of their frequencies? (R. U. 1952)

16. Distinguish between a progressive and a stationary wave, giving an example of each and illustrating your answer by diagrams.

(G. U. 1953, U. P. B. 1950; C. U. 1933, '55; *cf* Pat. 1931, '35, '52, All. 1931, '39, '46)

17. What are beats and stationary vibrations? Explain by composition of vibrations the production of beat and stationary vibrations. (Pat. 1937)

18. What are stationary waves? (C. U. 1947)

19. Explain the terms, 'nodes' and 'antinodes'; 'forced vibration' and 'resonance'. (G. U. 1949; C. U. 1950, Pat. 1949)

20. Distinguish clearly between 'node' and 'antinode'. (Utkal, 1952, Del. H. S. 1949, Pat. 1943, '50)

21. Write a note on stationary undulations. (Guj. U. 1952, '55, Bomb. 1950; Dac. 1942, Benares, 1953)

22. What are nodes and antinodes? How will you demonstrate their existence? What will be the effect on the distance between successive antinodes in a column of a gas by increasing its temp. and pressure? (Del. U. 1939; Pat. 1929; C. U. 1950)

CHAPTER VI

MUSICAL SOUND: MUSICAL SCALE: DOPPLER EFFECT

53. Musical Sound and Noise:—Sound may be divided into two classes (*i*) *Musical Sound* and (*ii*) *Noise.*

A *musical sound* is a continuous pleasing sound which is produced by regular and periodic vibrations; sounds produced by a tuning-fork, a violin or a piano are all musical sounds.

Noise is a general term including all sounds other than musical sounds. It is discordant and unpleasant to the ear.

The essential *difference* between a musical sound and a noise, generally speaking, lies in the fact that in the former case the vibrations are *regular* and *periodic;* while in the case of a noise, the vibrations are *irregular* and *non-periodic* in character. It is, however, difficult to draw up a clear line of demarcation between a musical sound and a noise; for, in practice, musical sounds too are seldom free from irregularities of vibration; while, on the other hand, in noises sometimes there is also regular periodicity of the motion. Sometimes noise is accompanied by musical vibrations as in the clang of a bell. Moreover, the difference is only subjective. The same sounds may appear to be musical or noisy to different persons and under different conditions. Therefore, the difference is more artifical than real.

54. Characteristics of Musical Sound:—Musical sounds may be said to differ from one another in the following three particulars:—

(1) *Intensity* or *Loudness*; (2) *Pitch*; (3) *Quality* or *Timbre.*

(1) Intensity.—It is the measure of loudness or volume of a note. It is an objective consideration and depends on the energy contained per unit volume of the medium through which sound waves pass. It may also be measured by the energy which passes per unit area placed normal to the direction of propagation of the sound. It is a characteristic of all sounds whether musical or not.

(*i*) Loudness depends upon *the square of the amplitude* or the *extent of vibration* of the sounding body. When the body vibrates with greater amplitude, it sends forth a greater amount of energy to the surrounding medium, and, hence, energy received by the drum of the ear is also greater. So the sound becomes louder.

The energy e of a body of mass m vibrating with velocity v and amplitude a is given by,

$$e=\tfrac{1}{2}mv^2=\tfrac{1}{2}m\left(\frac{2\pi a}{T}\right)^2=\frac{2\pi^2ma^2}{T^2} \quad (\textit{vide}\ \text{Art. 12}); \quad \therefore \quad e\propto a^2.$$

Therefore, *the loudness of a note*, which depends upon the energy of the vibration, *is proportional to the square of the amplitude of the vibration.*

(*ii*) The loudness of a sound is *inversely proportional to the square of the distance* of the observer from the source of the sound **(Inverse Square Law).**

Thus, the energy received by the observer at a distance of 2 metres from the source is only one-fourth of the energy which the observer would receive *when at a distance of 1 metre from the source*

[Suppose it is required to compare the intensities of the sound at two points A and B, distant r_1 and r_2 from a source of sound from which the total sound energy emanating per second uniformly all around is E Draw two spheres with the source as centre with radii r_1 and r_2 respectively The amount of energy flowing per second per unit area at A normal to the surface of the sphere$-I_A-$intensity at $A=E/4\pi r_1^2$.

Similarly, the intensity at $B-I_B=\frac{E}{4\pi r_2^2}$ $\frac{I_A}{I_B}=\frac{r_2^2}{r_1^2}$ That is, the *intensity* at a point is inversely proportional to the square of the distance]

(*iii*) The loudness of a sound depends upon *the density of the medium* in which the sound is produced It is seen that the greater the density of the medium, the greater is the loudness of the sound heard

It is seen that some effort is to be made to make oneself heard by another in aeroplanes or balloons when flying high up from the surface of the earth as the density of air therein is small For the same reason the sound is more intense in carbon dioxide than in air.

(*iv*) The loudness of a sound depends upon the *size of the vibrating body.*

If the size be larger, than a larger volume of the medium is put into vibration, and greater amount of energy will pass per unit area. So the sound heard will be louder.

(*v*) The loudness of a sound is increased by the *presence of resonant bodies.*

The sound of a tuning-fork, or a vibrating string in air, is much intensified when placed on a sounding-box which undergoes forced vibration

(2) Pitch.—The pitch of a note is that physical cause which enables us to distinguish a shrill (acute or sharp) sound from a dull (flat or grave) sound of the same intensity sounded on the same musical instrument It depends on the frequency of vibration of the emitted sound The higher the frequency the more shrill is the sound and we say that the sound rises in pitch As pitch is directly pro-

portional to frequency, it is customary to express the pitch of a note by its frequency.

The pitch is a fundamental property of a musical sound and a noise has no definite pitch.

(3) Quality or Timber.—The *quality* or *timbre* is that characteristic of a musical note which enables us to distinguish a note sounded on one musical instrument from a note of the same pitch and loudness sounded on another instrument.

A musical note consists of a mixture of several simple tones; of these the one having the lowest frequency, called the fundamental, is relatively the most intense. Its frequency determines the pitch of the note. *Notes of the same pitch and loudness sounded on two different musical instruments differ in quality from each other owing to the difference in the number of other tones (or overtones) besides the fundamental, their order of succession, and their relative intensities.* Any difference in respect of these factors introduces a difference in the wave-form of a sound. So simply it may be said that the quality of two sounds will differ if their wave-forms differ (*vide* Fig. 26). Now even if two sounds are similar in respect of these factors, a change in their wave-form occurs, if the phase-relations between the overtones present in the two sounds are different. Helmholtz found experimentally that when any change in wave-form is due to difference in the phase-relationship of the overtones, the quality of the two sounds do not differ. That is, quality will differ when the wave-form differs only on account of difference in respect of the number of overtones, order of their succession, and their relative intensities. Helmholtz investigated on the physiological effects of overtones also. He found that a note possessing the fundamental and the first few overtones not exceeding the sixth is very pleasing to the ear, while a note in which the fundamental has mixed up in it more overtones than the sixth and which are relatively more intense, produces a metallic and harsh effect.

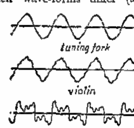


Fig. 26

Since the quality of a note depends on the number of overtones, their order and their relative intensities, two notes, similar in pitch and loudness, but differing in quality, will have different wave-forms, though the wavelength and amplitude of their fundamentals may be the same, and so their pitch and loudness are also the same. So the nature of the displacement curve of a note represents its quality.

55. Determination of Pitch :—The pitch of a musical note is determined by the frequency of vibration of the source of the note. Determination of frequency may be made by the following methods:—

(1) Savart's Toothed Wheel.—This consists of four toothed wheels of equal diameter mounted concentrically on a spindle fitted to a whirling table (Fig. 27). The number of teeth on each wheel conforms to a certain ratio, *e.g.* 20, 30, 36, 48.

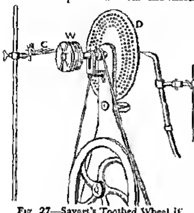


Fig 27—Savart's Toothed Wheel *W* and Seebeck's Siren *D*

A thin metal plate or a card-board *C* is clamped in front of the wheel so that it lightly presses against the teeth of one of the wheels *W* when it is in motion, and a sound formed by a series of taps is heard. On increasing the speed of rotation, a musical sound is produced, the pitch of which depends on (*a*) *the number of taps made in a given time*, and (*b*) *the speed of rotation*.

To determine the pitch of a note, the speed of rotation of the wheel is gradually altered while the card is lightly pressed against a particular wheel until the note emitted by the wheel is in unison (*vide* Art 5) with the given note. Now, if m be the number of teeth in the wheel used, and n the number of revolutions per second, the frequency N of the note is given by, N=number of taps made per sec $=m \times n$.

(2) Seebeck's Siren.—Seebeck's Siren or Puff Siren consists of a circular metal disc *D* (Fig. 27) through which a number of equidistant small holes have been drilled along concentric circles of varying diameter. The disc is mounted on a whirling table. A stream of air blown through a narrow tube ending in a nozzle, by means of foot-bellows, is directed to pass through the holes in one of the rings. As the disc rotates, the stream of air through the tube is alternately stopped and allowed to pass through the holes, producing a series of puffs at regular intervals. To determine the pitch of a note, the rotation of the siren is adjusted until the note produced by the siren is exactly in unison with the given note. Now, if m be the number of holes in the ring used, and n the number of revolutions per sec. made by the whirling table, the frequency N of the given note is given by, N=number of puffs made per sec.$=m \times n$

Note.—The highest frequency up to which a note is audible varies from 20,000 to 30,000 per second and the lowest is about 20 per second

Example. *The disc of a siren is making 10 revolutions per second. How many holes must it possess in order that it may produce four beats per second with a tuning-fork of frequency 484? Which has the greater frequency, the siren or the fork?*

The number of beats per second is numerically equal to the difference of frequencies of the two notes. Hence the note emitted by the siren must have a frequencies of $(484+4)=488$; or $(484-4)=480$.

The frequency of the note emitted by the siren, N=no. of holes in the siren × no. of revolutions per second. $\therefore$ N = no. of holes × 10.

As the number of holes must be a whole number, N must be multiple of 10. So the value 488, which is not a multiple of 10, cannot be accepted. Hence, $N=480$.

$\therefore$ 480 = no. of holes × 10 ; $\therefore$ The number of holes $=\frac{480}{10}=48$. Evidently the fork has the greater frequency.

(3) Cagnaird de la Tour's Siren.—This is a much improved form of siren by which the pitch of a note can be fairly and accurately determined. In this siren (Fig. 28) a current of air is blown through a pipe into a wind-chest A, from which it issues through a ring of equidistant holes cut in the circular top of the wind-chest. Another disc having holes exactly corresponding with the holes in the top of the wind-chest, and very close to it, in such a way that it can rotate freely about a vertical axis. The two sets of holes are drilled so as to slant in opposite directions, as shown in Fig. 28, so that the pressure of the air at the time of escaping through the holes causes the upper disc to rotate, the number of rotations being counted by a speed-counter S geared to the axle of the disc.

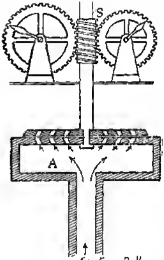


Fig. 28—Cagnaird de la Tour's Siren.

Every time the holes in the two rows coincide at the time of rotation of the upper disc, a jet of air escapes from each hole in the upper disc and, if there are m holes in each of the discs, there will be m puffs for one revolution of the disc. Of course, each puff will consist of m separate jets, but, as they take place simultaneously, they are regarded as a single puff. Now, if n be the number of revolutions of the disc per second, the frequency N of the note emitted is equal to $m \times n$.

(4) Resonance of Air Column.—In the relation $V=4nl$ in Art. 75, if V and l are known, the frequency n can be determined.

(5) Method of Beats.—The frequency can also be determined by the method of beats, as explained in Art. 47.

(6) Sonometer.—By tuning a vibrating string of a sonometer to unison with a given note, the frequency of the note can be determined by the formula, given below, if the tension T, and m, the mass per unit length of the vibrating string, are known—

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} \quad \text{(vide Art. 68).}$$

(7) Direct or Graphical Method; (Duhamel's Vibroscope).—The frequency of a vibrating fork can be determined by the graphical method. A sheet of smoked paper is wrapped round a cylindrical drum which can be rotated uniformly by means of a handle attached to it (Fig. 29). A thin metal style is attached to one prong of the tuning-fork which is so arranged that it can vibrate parallel to the axis of the drum and the style just touches the smoked paper. As the drum is rotated, the style will trace a wave line on the paper. If at the time of the vibration of the fork, two points can be marked on the wave line on the smoked paper at an interval of half-a-second, or one second, the frequency of the fork can be determined by actually counting the number of complete vibrations between the points.

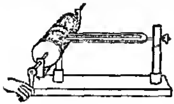


Fig. 29—The Duhamel's Vibroscope

In order that the amplitude of the waves traced out by the style may not decrease owing to effects of friction, in actual practice the fork is excited and its vibrations are maintained electromagnetically. The tracing time is recorded by means of an electric pendulum which is so arranged as to produce a spark at the expiry of a rated interval. Corresponding to successive sparks, spots are made by the end of the style on the wavy line traced on the smoked paper. The number of vibrations made by the fork in the rated interval (and hence the frequency of the fork) is given by the number of complete waves found between any two consecutive spots.

(8) Falling Plate Method.—In this expt. an arrangement is so made that a plate may fall freely under gravity. A glass plate P blackened preferably by camphor-smoke is suspended vertically by means of a thread from two hooks fixed on a vertical piece of wood B, as shown in Fig. 30(a). A tuning-fork F to one prong of which a very light style S is fixed is clamped in front of the plate in such a way that the style just touches the smoked plate during the fall. The fork is set into vibration by striking it with a violin bow and then the plate is released by burning the thread between the hooks. As the plate falls under gravity, the style draws a wave-trace [Fig. 30(b)] of steadily increasing wavelength upon the smoked glass.

Ignoring waves at the beginning which are very crowded (and not suitable for counting), choose two lengths AB and BC *having the same number of complete waves.*

Let the velocity of the plate at the point $A=u$, and the time required by the plate to fall through the distance AB or $BC=t$. Calling $AB=l_1$ and $BC=l_2$, we have $l_1=ut+\frac{1}{2}gt^2$. The velocity at B being $(u+gt)$, we have $l_2=(u+gt)t+\frac{1}{2}gt^2$;

$$\therefore \quad (l_2-l_1)=gt^2\ ;\ \text{or,}\ t=\sqrt{\frac{l_2-l_1}{g}} \quad \ldots \quad (1)$$

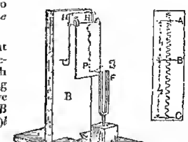


Fig. 30—The Falling Plate Method.

If n be the frequency of the fork F and m the number of complete waves between AB or BC, we have $nt=m$.

$$\therefore \quad n=\frac{m}{t}=m\div\sqrt{\frac{l_2-l_1}{g}} \quad \ldots \text{ from (1)}$$

N.B.—This method has the disadvantage that by attaching the style to the prong, the frequency of the fork is altered. Also there may be some friction between the plate and the style by which the free rate of fall of the plate is affected.

Example. *A small pointer, attached to one of the prongs of a tuning-fork, presses against a vertical smoked glass plate. The fork is set vibrating and the glass plate is allowed to fall. If 30 waves be counted in the first 10 cms. find the frequency of vibration of the fork. ($g=980$ cms./sec^2.).* (Pat. 1943)

We have, the distance fallen through, $S=ut+\frac{1}{2}gt^2$, but here $u=0$, $S=10$;

$$\therefore \quad 10=\tfrac{1}{2}gt^2\ ;\ \text{or,}\ t^2=\frac{20}{g}=\frac{20}{980}=\frac{1}{49};\quad \text{or}\ t=\frac{1}{7}\ \text{sec.}$$

As 30 waves are counted in $\frac{1}{7}$ second, we have frequency, $n=30\div\frac{1}{7}=210$.

56. Musical Scale:—We express the pitch of a note by the number of vibrations per second, but pitch can also be expressed by what is known as the musical method. In this method certain sounds constitute what we call a *musical scale.* This musical scale used for many centuries by most of the European countries is called the **major diatonic scale,** which affords the simplest and the most pleasing succession of notes in an ascending order of frequency. This scale

consists of eight notes, the lowest one, *i.e.* the fundamental note, is named *do*, and others *re*, *me*, etc. and they are generally designated by the letters *C, D, E, F, G, A, B, C'*. The note from which the scale starts is called the **tonic or key note.**

Interval.—The ratio of the frequencies of two notes expresses the *interval* between them. Thus the interval of two notes having frequencies 256 and 192 is $\frac{256}{192} = \frac{4}{3}$; 512 and 256 is $\frac{512}{256} = \frac{2}{1}$, and so on. It is the interval which is detected by the ear. In changing from one frequency to another, the change over is not recognised by the ear if the ratio between them is constant, whatever might be the actual frequencies concerned. Certain intervals have names; thus $\frac{2}{1}$ is called the **octave**; $\frac{3}{2}$ the **fifth**; $\frac{4}{3}$ the **fourth**; $\frac{5}{4}$ the **major third**; $\frac{6}{5}$ the **minor third.**

Any two intervals are added together by taking the product of their frequency ratios. For examples, major third and minor third $= \frac{5}{4} \times \frac{6}{5} = \frac{3}{2}$ = *fifth*. Again, *fifth* and *fourth* $= \frac{3}{2} \times \frac{4}{3} = \frac{2}{1}$ = *octave*.

57. Some Acoustical Terms:—When the two notes have the same frequency, *i.e.* their interval is 1, they are said to be in *unison*. Two notes, when sounded together, are said to be **concord** or **consonance** when they give a pleasing sensation to the ear. This happens when the interval between them is a simple ratio such as, 2 to 1, 3 to 2, etc. But, if the ratio is complex such as, 9 to 8, 15 to 8, etc. they produce an unpleasant or harsh effect and they are said to be in **discord** or **dissonance.**

According to Helmholtz the cause of dissonance is the production of beats by the interference of the notes. The beats produce a jerking effect on the ear-drum and are discordant, just as flicking of light is disagreeable to the eye.

The pleasing effect produced by sounding two notes, which are in concord, one after another, is called **melody**; and when they are produced simultaneously, the pleasing effect is called **harmony.** When three notes of frequencies in ratios 4.5.6 are sounded together, they form a concordant combination which is called a musical **triad** (e.g. C·E.G), and, if a triad is sounded with an additional note which is the octave of the lowest note of the triad, the combination is known as a **chord.** When one musical instrument alone, such as a violin or a flute, is played upon, the performance is called a **solo.**

Octave.—One note is an *octave* (GK. *okto*, eight) higher than ...her, when their intervals is 2:1. These notes when played ...ther produce the most pleasing combination in the musical scale.

The names and the relations between the notes of an Octave are given as follows—

Name (Western system)	Do	re	me	fa	sol	la	te	do
„ (Indian „)	Sa	re	ga	ma	pa	dha	ni	sa
Symbol	C	D	E	F	G	A	B	C′
Actual Frequency ...	256	288	320	341·3	384	426·7	480	512
Relative Frequency	24	27	30	32	36	40	45	48
Interval between C and each note	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2
Interval between each note and its predecessor		$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{16}{15}$

Intervals and their special names

1 : 1 Unison	4 : 3 Fourth
16 : 15 Semitone (or limma)	3 : 2 Fifth
10 : 9 Minor tone	5 : 3 Major sixth
9 : 8 Major tone	8 : 5 Minor sixth
6 : 5 Minor third	15 : 8 Seventh
5 : 4 Major third	2 : 1 Octave

It will be noticed that there are five black keys inserted between all the consecutive notes except the 3rd and 4th, 7th and 8th. The first of these is named *C sharp*, second *D sharp*, third *F flat*, fourth *G flat*, and fifth *A flat*.

N.B.—The intervals in the **Major diatonic** scale are a major tone, minor tone or semi-tone. As there are three major tones ($D:C$, $G:F$, and $B:A$), the major diatonic scale is so called.

Example. *Taking the frequency of vibration of C to be 256, find the note which makes 320 vibrations per sec.*

Let x be the vibration ratio or the interval between the two notes, then

$$256 \times x = 320\ ; \quad \therefore\ x = \frac{320}{256} = \frac{5}{4}.$$

$\therefore$ $\frac{5}{4}$ is the interval between the notes E and C; hence the required note is E.

58. Tempered Scale:—Modern music requires frequent change of the *tonic*. For such changes to be possible in the Major Diatonic Scale, a very large number of keys has to be employed and this will make the instrument unmanageable. The problem with the diatonic scale was, therefore, how to maintain flexibility of the scale without undue complication being necessary. The solution was **the tempered scale** in which, besides the usual eight keys, five additional intermediate keys have been introduced, and the intermediate keys have been slightly altered in frequencies to make all successive intervals equal. Thus, to a very large extent, change of tonic **(modulation of scale)** has been made possible and, at the same time, the simplicity of the instrument has been retained.

59. Doppler Effect:—It will be noticed that the pitch of the whistle of a train appears to rise when the train approaches the hearer and it falls as the engine recedes from him. Similar effect is noticed when a motor car passes at a high speed. Such apparent change in the pitch of a note as perceived by an observer due to the relative motion of the source, the observer or the medium is called the **Doppler effect** after the name of the Austrian physicist Christian Doppler (1803-53).

Apparent Frequency.—The apparent change in pitch perceived by an observer due to the motion of the source, observer and the medium calculated as follows:—

$$\overrightarrow{S\;S_1\;\;M\;M_1} \qquad\qquad \overrightarrow{O\;O_1\;\;W\;W_1}$$

Let S and O represent the positions of the source and the observer respectively and the distance SM or OW be equal to the velocity of sound V in still air. Suppose that the source, the observer, and the medium are all moving in the same direction from left to right. Let the velocity of the source (V_s) be equal to SS_1, i.e. the distance passed over by the source in one second, and similarly the velocity of the observer OO_1 $(=V_o)$ and the velocity of wind, $MM_1=WW_1=w$.

At some instant of time, when the observer is at O, let a wave reach him for the first time. After one second, that wave will be at W_1, for the wave travels a distance OW in still air and the air medium moves through a distance WW_1 in that second in that direction. All the waves received by the observer in that second are confined between O_1W_1, since the position of the first wave of that second is at W_1 while the last wave is received by the observer when at O_1. The length occupied by the waves, $O_1W_1=V+w-V_o$.

Now turning to the source end, the first wave was sent out by the source while at S and the last wave while at S_1 in the particular second under consideration. All the waves emitted in that second are confined between S_1 and M_1, because the first wave reaches M_1 in that second having travelled over SM in still air and the air-medium having moved through MM_1. Thus all the waves sent out by

the source in that second are contained within the length $S_1M_1 = V+w-V_s$.

If n be the real frequency of the source, it emits n waves in one second which occupy a length $V+w-V_s$. If the apparent frequency as perceived by the observer under the circumstances as stated above be n_1, then n_1 waves are contained within the length $V+w-V_o$.

So, we have, $n_1 : n = \frac{V+w-V_o}{V+w-V_s}$; or, $n_1 = n \times \frac{V+w-V_o}{V+w-V_s}$.

N.B.—The velocity of the source, or of the observer, or of the medium, will be zero when at rest. Proper signs positive or negative, shall have to be assigned to them depending on the directions in which they move. *Remember* the observer moving away from the source is positive, the source moving towards the observer is positive and the wind moving towards the observer is positive in the above calculations and so opposite directions will be negative.

Examples. *(1) What is the apparent frequency of the sound of a whistle of frequency 600 from an engine which is approaching an observer at rest at 10 metres per sec.? (Velocity of sound = 332 metres per sec.).*

Here $V_o = 0$, $w = 0$ and $V_s = 10$ metres per sec.

$\therefore$ Apparent frequency, $n_1 = n \times \frac{V+w-V_o}{V+w-V_s}$

$= 600 \times \frac{332+0-0}{332+0-10} = 600 \times \frac{332}{322} = 619$ (approximately) per sec.

(2) Calculate the apparent frequency of the note of a whistle of frequency 1000 per sec. heard from a train which is approaching the station at 45 ft./sec. where the whistle is blown (velocity of sound = 1100 ft./sec.).

Here $V_s = 0$, $w = 0$ and $V_o = -44$ ft./sec.

$\therefore$ Apparent frequency, $n_1 = n \times \frac{V+w-V_o}{V+w-V_s}$

$= 1000 \times \frac{1100+0-(-44)}{1100+0-0} = 1000 \times \frac{1144}{1100} = 1040$ per sec.

Questions

1. What are the factors determining the loudness of a musical note? (East Punjab, 1952, '53)

2. Distinguish clearly between 'loudness and pitch' of musical note. On what physical conditions of the sounding body do they respectively depend? (C. U. 1909, '12, '14, '19, '21; Pat. 1924, '28; All. 1924; Dac. 1929, '51)

3. On what do loudness, pitch and quality of musical sound depend? (C. U. 1931; All. 1926; Dac. 1928, '31; Pat. 1928, '39)

4. What is the essential feature of a musical note which distinguishes it from noise? (Pat. 1948, '49; East Punjab, 1953; C. U. 1931)

5. How would you distinguish between (a) musical sound and noise, and (b) one note from another?

6. Distinguish clearly between 'musical sound' and 'noise'. (Ana. U. 1950; Utkal, 1952)

7. How will you explain the difference between pitch and loudness of sound by comparing the roar of a lion and the buzzing of a mosquito? (All. 1927)

[Hints.—The buzzing of a mosquito is due to the motion of its wings which vibrate several hundred times a second ; so the frequency and consequently the [illegible]

7(a) What is the difference between a noise and a musical note ? How do you know that musical notes of different pitches travel with the same velocity ?

An under-water microphone is attached to the prow (fore part) and another [illegible]

[illegible] (Bihar, 1954)

8. How do you explain why audible notes from different sources can generally be distinguished one from another, even when they have the same intensity or pitch ? Describe experiments in order to demonstrate the correctness of your answer.

9. What is meant by 'musical scale' ? (All. 1946)

10. Trace the sounds coming from a violin, a flute, a harmonium and a piano to their ultimate source. How do these sounds differ from one another and why ? (Pat. 1932)

11. Write notes on 'Timbre' (Pat 1947)

12. What do you understand by the pitch of a note ?

Explain a method of experimentally determining the pitch of the note emitted by a given tuning-fork.

(Del. H S 1947, '52 ; C. U. 1917, '32 ; Pat. 1926, '47, '48, '49 ; All 1919, '21, '24 , Del. 1942, '47)

13 Give a brief account of the various methods of determining the frequency of a fork and discuss their merits (All 1928 , Pat. 1936)

14. Describe a siren, giving a diagram and explain how you would use it to determine the frequency of a given tuning-fork.

(C U. 1921, '28, '30, '40, '53 ; Pat. 1920, '21, '37, '40 ; Dac 1927 ; G U. 1919)

15 The disc of a given siren has 32 holes. A tuning-fork makes 512 vibrations per second. What must be the speed of rotation per minute of the siren disc so that the note emitted by the siren may be in unison with that emitted by the tuning-fork ? (C. U. 1910 ; G. U. 1949)

[*Ans.* 960 per min.]

16. The disc of a siren is making 10 revolutions per second. How many holes must it possess in order that it may be in unison with a tuning-fork of frequency 480 ?

[*Ans.* 48] (Dac. 1932)

17. A cog-wheel containing 64 cogs revolves 240 times per minute. What [illegible] is held against the revolving [illegible]

18. How would you determine experimentally the absolute value of the frequency of a tuning-fork ? Illustrate your answer with a neat sketch of the arrangement described. (Pat. 1941)

19 Give a brief account of the various methods employed in measuring the frequency of a tuning-fork and describe one method in detail.

(All. U. 1921 ; Pat 1951 ; C. U. 1955)

20. Define musical interval, harmony, melody and chord. Show that the [illegible] *sa* and *ga* obtained by multiplying the intervals *sa* and *re* and *ga* but not by adding them. (Patna, 1928)

21. Explain what is meant by the pitch of a note. A note of frequency 384 is said to be a 'fifth' higher in pitch than one of 256. What is the frequency of the note a 'fifth' higher than the 384 note, and what is the difference in pitch between it and the 256 note?
[*Ans.* 576 ; 320]

22. A siren having a ring of 200 holes is making 132 revolutions per minute. It is found to emit a note which is an octave lower than that of a given tuning-fork. Find the frequency of the latter. (C. U. 1944 ; G. U. 1955)
[*Ans.* 880]

CHAPTER VII

VIBRATION OF STRINGS

60. Vibration of Strings:—In sound a *string* is usually understood to mean a wire or a cord of any material, which is flexible and uniform in cross-section. These conditions are found to be satisfactorily fulfilled by thin metallic wires or catgut. Strings may vibrate in two ways: *tranversely* and *longitudinally*. A string can be vibrated longitudinally by rubbing it along the length with a piece of chamois leather covered with resin, or by a piece of wet flannel. It can be vibrated transversely by plucking it to a side, by bowing it with a violin bow, etc. When a stretched string is plucked to one side, it tends to return to its original (straight) position of rest. But owing to inertia that it possesses, it overshoots the mark like the motion of a pendulum and goes over to the other side and goes on swinging to-and-fro with gradually decreasing amplitudes and after sometime it stops. The vibration in this case is mainly due to the tension in the string, which, when the string is deflected, tends to bring it back to its initial straight position. *In stringed musical instruments, only the transverse vibrations of strings* are employed.

61. Reflection of Waves in Transverse Vibration:—(*a*) *Reflection of waves in a string.*—Let a wave travelling along a wire, say from left to right, meet a fixed support and let the wave meet the support in the form of a crest. The end of the wire will exert a force on the support tending to move it in the direction of the force. Then according to Newton's Third Law of Motion, the support will react and exert an equal and opposite force on the wire which causes a rebound, so that the pulse is thrown over the other side of the string and starts a reversed pulse travelling back along the string from right to left. Thus, **in this case reflection takes place at the fixed ends with change of type;** *a crest is reflected back as a trough and a trough is reflected back as crest.* It should be noted, however, that in the case of water-waves, which are *transverse waves*, a crest meeting a rigid wall is reflected back as a crest and a trough is reflected back as a trough like longitudinal sound-waves (*vide* Art. 36), and the important difference between the reflections of sound waves at the close and open ends of a pipe should also be noted (*vide* Ch. VIII).

(*b*) *Reflection of Water-wave.*—When a water-wave travels along, it has both potential and kinetic energy. Part of the energy is ***potential,*** because a force must have been applied to and work done upon the water to raise it above its normal level, and a part is ***kinetic,*** because the molecules are in motion When the waves strike a rigid wall or a denser medium, the motion of the molecules towards the wall is arrested, their kinetic energy is reduced, which is then converted into potential energy, thus increasing the amount of potential energy So the average elevation of the water in the crest is increased and the water is piled up against the obstruction, which then runs down and away from the wall producing a crest like the original wave and travelling in the opposite direction Thus, in the case of a water-wave meeting a rigid wall, a *crest is reflected as a crest and similarly a trough is reflected as a trough*

62. Stationary Waves in a String:—When a stretched string is plucked aside, a wave will travel along its length with a definite velocity The transverse wave will be propagated to both ends and will be reflected at these points If a complete wave consisting of a crest and a trough is sent along a string crest first, it will return as trough first after reflection at the fixed end These reflected waves will return to the centre of the string when they pass each other and go on to the ends to be once more reflected, and so on These incident and reflected waves, travelling to-and-fro along the string in opposite directions with equal velocities combine to form *transverse stationary waves whose positions of nodes and antinodes are fixed* (*vide* Art 50)

63. The velocity of Transverse Waves along a String:—When a string stretched under tension, is displaced laterally, transverse waves are set up in it The waves travel along the string with a velocity dependent on the tension and the linear density of the string

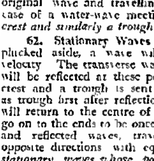


Fig. 31

Suppose the string *cc* stretched under tension T [Fig 31] is displaced perpendicularly to its length so as to make transverse vibrations (through plucking, bowing or striking), due to which, suppose, the summit, aEb of the displaced position, is bent into the arc of a circle The transverse wave travelling along the string from left to right with *velocity V may be imagined to be due to the hump* also travelling with the same velocity. For the circular motion of an element near the summit E of the hump, the necessary centripetal force is supplied by the tension at a and b

Suppose $aE=Eb$ and O, the centre of the curvature aEb Let the -le aOE be θ. Join Oa and Ob. Draw tangents T at a and b,

representing the tension of the string which produced backwards meet at P on OE. Suppose the length aEb is S, mass per unit length of the string is m, and the radius of the curvature is R.

The components of the tension T at a and b in the direction PO (each equal to $T \sin \theta$) constitute the centripetal force $\frac{m.\ S.\ V^2}{R}$ on the hump, while the components of T at rt. angles to PO cancel each other. Therefore,

$$\frac{m.\ S.\ V^2}{R} = 2\ T \sin \theta = 2\,T\,\theta \text{ (approximately)}$$

$$(\because\ \theta \text{ is very small}) = 2T \times \frac{S/2}{R} = \frac{TS}{R}.$$

$$\therefore\quad V = \sqrt{\frac{T}{m}}.$$

64. Frequency of Transverse Vibration of Strings.— The velocity of a transverse wave along a stretched string is given by,

$$V = \sqrt{\frac{T}{m}} = \sqrt{\frac{Mg}{m}} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

where T=tension of the string expressed in dynes; m=mass in grams per unit length of the string; M=mass of load on the string.

When the string gives out its fundamental, *i.e.* the note of the lowest pitch, the length of the string, l cm.=distance between two consecutive nodes=$\lambda/2$ (*vide* Fig. 32).

$\therefore$ From Art. 8, $V = n\lambda = 2nl$.

Substituting the value of V in (1), we get,

$$2nl = \sqrt{\frac{T}{m}};\ \text{or,}\ n = \frac{1}{2l}\sqrt{\frac{T}{m}} \quad \ldots \quad \ldots \quad (2)$$

Again, if ρ be the density of the material of the wire and r be its radius, then $m = \pi r^2 \rho$, and so we have from (2),

$$n = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2 \rho}} = \frac{1}{2rl}\sqrt{\frac{T}{\pi \rho}} = \frac{1}{dl}\sqrt{\frac{T}{\pi \rho}} \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

where d is the diameter of the wire.

65. Laws of Transverse Vibration of Strings:—From formula (2), we get the following laws for the transverse vibration of strings:—

(1) *Law of length.*—The frequency of a note emitted by a string varies inversely as the length, the tension remaining constant; that is, $n \propto 1/l$, when T and m constant.

(2) *Law of Tension.*—The frequency of a note emitted by a string varies directly as the square root of the tension, the length being kept constant; that is, $n \propto \sqrt{T}$, when l and m are constant.

(3) *Law of Mass.*—The frequency of a note varies inversely as the square root of the mass per unit length of the string, the length and tension remaining constant; that is, $n \propto 1/\sqrt{m}$, when l and T are constant.

Again, from formula (3), the law of mass may be put into two additional laws for strings of round section as given below:

3(*a*). *Law of Diameter.*—The frequency of the note produced by a string varies inversely as the diameter of the string, length and density of the material of the string and tension remaining constant; that is, $n \propto 1/d$, when l, ρ and T are constant.

3(*b*) *Law of Density.*—The frequency of the note emitted by a string varies inversely as the square root of the density of the material of the string, length and diameter of the string and tension remaining constant, that is, $n \propto 1/\sqrt{\rho}$, when l, d and T are constant.

66. Experimental Verification of the Laws of Transverse Vibration of Strings (*by Sonometer*):—The laws of transverse vibration of strings can be verified by means of an instrument, called the sonometer. It consists of a hollow wooden box AA, on which one or more wires can be stretched (Fig 32). Each wire is attached to a peg at one end and passes over two wedge-shaped hard wood B, B_1, called the *bridges* and a pulley at the other end. The string is kept taut by weights E attached at this end. A third bridge C can be placed in any position between the other two in order to set any desired length of the string into vibration

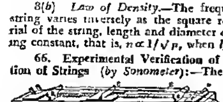


Fig 32—The Sonometer

Law 1. *To verify* $n \propto 1/l$.—To verify the law of length, two tuning-forks of known frequencies n_1 and n_2 are taken One of the forks is made to vibrate, and, altering the position of the movable bridge C, the length BC of the sonometer wire (under a given tension) is so adjusted that the note emitted by that length of the wire, when plucked in the middle, is in unison with the note yielded by the fork (*vide* Art. 67) Then the frequency n_1 of the fork is equal to the frequency of the wire of length l_1. Repeating the experiment with the other tuning-fork, another length of the wire is similarly determined. Let n_2 be the frequency of this fork, and l_2, the corresponding length of the wire, it will be found by experiment that $\frac{n_1}{n_2}=\frac{l_2}{l_1}$; or, $n_1l_1=n_2l_2$ Repeating the experiment with other forks, it will be found that $n_1l_1=n_2l_2=n_3l_3$, etc.; i.e. nl= a constant which verifies the law.

Note that the same wire is used and the tension is kept the same while adjusting the length of the wire for unison with the different forks

Law 2. *To verify* $n \propto \sqrt{T}$.—Stretch another wire called the comparison wire, by the side of the first wire Let T_1 be the tension on the first wire. A length of the comparison wire is then adjusted which is in unison with the note yielded by the first wire.

Let the length be l_1. Now increase the tension on the first wire

to T_2 ; so the frequency of the note emitted increases. Again another length l_2 of the comparison wire is found which is in unison with the note of the first wire. Let n_1 and n_2 be the frequencies of the notes of the comparison wire of lengths l_1 and l_2, and so of the first wire corresponding to tensions T_1 and T_2 respectively. We have, by the law of length, $\frac{n_1}{n_2}=\frac{l_2}{l_1}$. Again, it will be found by the experiment that $\frac{l_2}{l_1}=\sqrt{\frac{T_1}{T_2}}$. So, $\frac{n_1}{n_2}=\sqrt{\frac{T_1}{T_2}}$.

By applying different tensions T_1, T_2, T_3, etc. to the first wire and determining corresponding the attuned lengths l_1, l_2, l_3, etc. for which the respective frequencies are n_1, n_2, n_3, etc. it may be shown that $n/\sqrt{T}$ is constant. This verifies the law of tension.

Law 3. *To verify* $n \propto 1/\sqrt{m}$.—To verify the law of mass, two wires of different mass per unit length are taken. The wires may be of the same material or of different materials. One of them is stretched by the side of the comparison wire by a suitable load. Taking any length of the wire whose mass per unit length is m_1, a length l_1 of the comparison wire is determined, which is in unison with the note of the first wire. Replacing the first wire by the second wire of mass m_2 per unit length, and keeping the *tension* the *same*, the above experiment is repeated, taking the *length* of the second wire *same* as that of the first. A length l_2 of the comparison wire is found which is in unison with the note of the second wire. Then a measured length of each of the two wires is taken, and each of them is weighed. From these weights, mass per unit length (m_1 and m_2) for the two wires is found.

Let n_1 and n_2 be the frequencies of the lengths l_1 and l_2 of the comparison wire. We have, by the law of length, $\frac{n_1}{n_2}=\frac{l_2}{l_1}$, and it is found by the experiment that $\frac{l_2}{l_1}=\sqrt{\frac{m_2}{m_1}}$.

Hence, $\frac{n_1}{n_2}=\sqrt{\frac{m_2}{m_1}}$; or, $n_1\sqrt{m_1}=n_2\sqrt{m_2}$. Repeating the experiment with other wires of different mass per unit length, it will be found that $n\sqrt{m}=$ a constant. This proves the law of mass.

Law 3(a). *To verify* $n\propto 1/d$.—Take two wires of different diameters but of the same material and proceed just as in the above experiment (Law 3). Let l_1 and l_2 be the lengths of the comparison wire which are found to be in unison with notes produced by equal lengths of the two wires having diameters d_1 and d_2 respectively. Now, measure d_1 and d_2 with a screw-gauge. From Law 1, we have $n_1/n_2=l_2/l_1$, and it will be found by experiment that $l_2/l_1=d_2/d_1$.

Hence, $\frac{n_1}{n_2}=\frac{d_2}{d_1}$, which verifies the law.

Law 3(b). *To verify* $n\propto 1/\sqrt{\rho}$.—Take two wires of different

materials but of the same diameter, and repeat the experiment exactly as in the verification of Law 3(a). It will be found that $\frac{n_1}{n_2} = \sqrt{\frac{\rho_2}{\rho_1}}$. This verifies the law.

N.B.—It should be noted that this experiment gives a method of determining acoustically whether two wires are made of the same material or not.

67. Notes on Tuning :— In tuning two strings, or a tuning-fork and a string or any two notes, the following two methods may generally be adopted:

(i) "By Resonance".—Tune as nearly as possible by ear. Then place an inverted *V*-shaped paper rider, or a thin wire rider, on the middle of the string, and place the stem of the vibrating tuning-fork on the sonometer box. It will set the string into vibrating by resonance and the rider will be thrown off, if the tuning be accurate. If, however, this does not occur, adjust the length of the string by moving the movable bridge until the rider is thrown off.

(ii) "By Beats".—By adjusting the length of the string by the movable bridge until the two notes (of the string and of the fork) are very nearly of the same frequency, beats will be heard, *i.e.* the resultant sound will appear to give alternate maxima and minima of loudness. On adjusting the length still further, beats will become slower, and will cease entirely when tuning is exact, *i.e.* when the frequencies of two notes are exacty equal.

68. Determination of Pitch of Sonometer :—**(a)** The frequency of a note can be determined either by keeping the length of the sonometer wire constant and adjusting the tension, or by adjusting the length of the wire keeping the tension constant, until the string is in unison with the note, the pitch of which is to be determined. The latter method is, however, convenient. If the frequency of a tuning-fork is to be determined, its stem is lightly pressed against the sonometer box after it is made to vibrate. The resonant length of the wire is then measured and the mass of the string per unit length is determined. The stretching weight is noted; the tension is calculated by multiplying it by the acceleration due to gravity. The frequency n is then calculated by the formula, $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$.

N.B.—By knowing n, the *density* of the material of the wire can be determined from formula (3), Art. 64

(b) The pitch of a tuning-fork can also be determined by taking another standard fork of known frequency and then determining as above a length of the same wire stretched by the same weight until this fork and the wire are in unison again. If n be the frequency of the standard fork, n' the unknown frequency, and l and l' be the corresponding lengths of the wire, then, we have, $\frac{n}{n'} = \frac{l'}{l}$ whence n' can be determined.

69. Certain Terms :—

Note, Tone.—A *note* is a general term denoting any type of musical sound. The musical sound is a complex sound made up of two or more simple component sounds of different pitches. Each of the simple component sounds is called a *tone*. A tone cannot further be divided into simpler components and, therefore, has a single frequency. In other words, a tone is a sound of single frequency, while a note consists of some pure tones.

Fundamental, Overtone, Harmonic and Octave.—When a body vibrates, generally there are present in the note several tones of frequencies which are multiples of the frequency of a *fundamental*, which is the tone of the lowest pitch. The other tones, except the fundamental, are called *overtones*. When the frequencies of the overtones are exact multiples of the frequency of the fundamental, they are, in particular, called *harmonics*.

The tone whose frequency is twice that of the fundamental is said to be an *octave* higher, or called the first harmonic, of the fundamental. All tones of frequencies between any number n and $2n$ are said to be in the *same octave*.

70. The Harmonics of a Stretched String :—(*i*) A string can be made to vibrate in different modes. When it vibrates as a whole it is the simplest mode of its vibration. Such vibration is produced when the string is plucked at its centre. It has been pointed out in Art. 62 that when a string vibrates the waves generated are reflected from the fixed ends, and the incident and the reflected waves give rise to transverse stationary waves having definite nodes and antinodes. In the present case there will be produced *two nodes N, N at the two fixed ends, and one antinode A* in the middle as shown in Fig. 33 (I); in this case the length of the string, $l = \lambda/2$.

$\therefore \quad n = \dfrac{V}{\lambda} = \dfrac{V}{2l}$. But the string may vibrate in other ways also.

(*ii*) If the string be plucked at a point *one-fourth* the length of the wire from one end, and at the same time the middle point of the wire is lightly touched, it will vibrate in two segments. In this manner of vibration there are *three nodes* and *two antinodes* as shown in Fig. 33(II). In this case $l = \lambda$ $\therefore \quad n_1 = V/l$; or, $n_1 = 2n$.

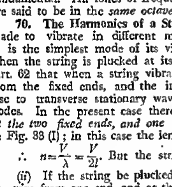


Fig. 33

This tone is an *octave higher* and is called the *first harmonic of the fundamental tone.*

(*iii*) In the next mode of vibration, if the string is held at one-third of its length and if the middle of the shorter segment is bowed the wire will vibrate in three segments, and in that case there

Questions

1. The sonometer is stretched with a force of 200 gms.-weight.

(a) The force is increased to 800 gms.-wt. ; (b) the length of the string is halved. How is the pitch of the note emitted by the string affected in each case ? (C. U. 1912)

[*Ans.* (a) $n_2 = 2n_1$; (b) $n_2' = 2n_1$, i e the pitch is doubled in each case.]

2. The string of a monochord vibrates 100 times a second. Its length is doubled and its tension altered until it makes 150 vibrations a second. What is the relation of the new tension to the original ? (C. U. 1924)

[*Ans.* $T_2 : T_1 :: 9 : 1$]

3 What will be the frequency of the note emitted by a wire 50 cms. in length when stretched by a weight of 25 kilograms, if 2 metres of the wire are found to weigh 4 79 grams ? (C. U. 1934)

[*Ans* 320 per sec.]

4 Find the frequency of the note emitted by a string 50 cms. long stretched by a load of 10 kgms , if 1 metre length of the string weighs 2·45 gms. ($g = 900$ cm /sec.^{2}).

[*Ans.* 200 per sec] (East Punjab, 1953)

5 [illegible] weight produces when plucked [illegible] tension in gm.-wt. in the wire ; (Pat. 1952)

6. Two tuning forks *A* and *B* produce 4 beats per second when sounded together. *A* resounds to 32 4 cms. of stretched wire and while *B* is in unison with 32 cms. of the same wire. Find the frequencies of the forks (Mysore, 1952)

[*Ans* 320 , 324]

7. [illegible]

Whe[illegible]
On sho[illegible]
is the fr[illegible]

$$\left[n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2\times 73}\sqrt{\frac{T}{m}} ;\ (n+3) = \frac{1}{2(72\ 5)}\sqrt{\frac{T}{m}} ;\ \therefore\ \frac{n}{n+3} = \frac{72\ 5}{73} ;\ n = 435\right]$$

8 A wire 50 cms. long vibrates 100 times a second. If the length is shortened to 30 cms and the stretching force quadrupled what will be the frequency ?

[*Ans* 333 3] (All 1927)

[illegible]

10. A stretched string 1 metre long is divided by two bridges into three parts so as to give notes of the common chord whose frequencies are in the ratio of 4 5 6 Find the distance between the bridges.

[*Ans.* 32 432 cms]

11 A string 24 inches long weighs half an ounce and is stretched on a sonometer with a weight of 81 lbs. Find the frequency of the note emitted when struck.

[*Ans* 101 8] (Dac 1934)

12. What is the fundamental frequency of transverse vibration of a steel wire 1 mm in diameter and 1 metre long, hanging vertically from a rigid support with a mass of 20 kilograms attached to its lower end. Density of steel = 7 9 gms /c c.

[*Ans.* 85 5] (Utkal, 1947)

13. State and explain the laws of vibration of a stretched string. Why are strings of musical instruments mounted on hollow wooden boxes ? (G. U. 1950)

14. A brass wire (density 8·4) 100 cms. long and 1·8 mm. in diameter is stretched by a weight of 20 *kilograms*. Calculate the number of vibrations which it makes per second when sounding its fundamental tone ($g = 980$ cms. per sec.^{2}).
[*Ans.* 47·88 nearly] (C. U. 1930)

15. State the laws of transverse vibration of a stretched string and describe experiments to verify them.

(Bihar, 1955 ; C. U. 1925, '34, '36, '41 ; All. 1927, '29, '45 ; Pat. 1940, '42, '49)

A sonometer is in tune with a fork. On shortening the wire by 1% the tension remaining constant, 4 beats per second were hea . What is the frequency of the fork ? (Bihar, 1955)

[*Ans.* 396 per sec.]

16. A stretched wire under tension of 1 kgm.-weight is in unison with a fork of frequency 320. What alteration in tension would make the wire vibrate in unison with a fork of frequency 256 ?

$$\left[\frac{n_1}{n_2}=\sqrt{\frac{T_1}{T_2}};\quad \therefore\ \frac{320}{256}=\sqrt{\frac{1}{T_2}}\quad \text{or,}\quad T_2=\frac{16}{25}.\right.$$

So the tension should be reduced by $\left(1-\frac{16}{25}\right)$ or $\frac{9}{25}$ kgm.-wt.]

17. A sitar wire is 80 cms. long and it emits a note of 288 vibrations per second. How far from the top it may be pressed so that it may emit a note of 312 vibrations per second ? (Pat. 1951)

[*Ans.* 6·2 cms.]

18. How would you verify with a sonometer the law connecting the frequency of a stretched string with its tension ? If an additional weight of 75 lbs. raises the pitch by an octave, what was the original tension ?

[*Ans.* 25 lbs.-wt.]

19. Given two tuning-forks, how would you determine the pitch of the note emitted by one of them if that of the other is known ? (C. U. 1919 ; Pat. 1930)

20. How would you verify the relation between the pitch of the note emitted by a stretched string and its tension. (Pat. 1943)

21. Describe experiments showing how the note given by a stretched string depends on (*i*) the tension, and (*ii*) its mass per unit length. (Utkal, 1952)

22. Explain how you would find acoustically whether two wires are made of the same material or not.

23. Wires of brass and iron are stretched on a sonometer and are adjusted to emit the same fundamental tone. The two wires are of equal length, but the tension of the brass wire is 5 kgms.-weight and that of the iron 3 kgms.-weight. Assuming that the iron wire has a diameter of 0·8 mm. find that of the brass. (C. U. 1946)

$$\left[\quad \textit{Ans.}\quad 0{\cdot}8\sqrt{\frac{5\times(\text{density of iron})}{3\times(\text{density of brass})}}\ \text{mm.}\quad\right]$$

24. Two exactly similar strings *A* and *B* of a sonometer are stretched by means of weights. Describe two distinct arrangements by which the note given by *A* would have twice the frequency of the note given by *B*. Account for your arrangement. (C. U. 1950)

25. Show how the frequency of a tuning-fork is determined with the help of a stretched string. (Pat. 1937 ; All. 1945 ; C. U. 1945)

26. What are harmonics ? How will you demonstarte their formation in a sonometer wire ? What important part is played by them in musical notes ?
(U. P. B. 1950 ; *cf.* G. U. 1952)

CHAPTER VIII

VIBRATION OF AIR-COLUMNS: LONGITUDINAL VIBRATIONS OF RODS (DUST-TUBE EXPERIMENT)

71. Stationary Vibration of Air-Column within Organ Pipes:— The column of air enclosed in a pipe can be set into momentary vibration when any sudden disturbance is communicated to it, or the pressure at the mouth of the pipe is suddenly altered. For example, a sound is produced by suddenly withdrawing a cork from a tightly-corked cylindrical bottle, because the sudden withdrawal of the cork disturbs the air-pressure at the mouth of the bottle which is the cause of the vibrations of air in the bottle. The whistling sound produced by blowing across the open end of the barrel of a key is also another example of vibration of air-column. In various musical instruments such as the flute, clarionet, etc. the musical sound is produced and maintained by vibrating the air-column enclosed within the pipe. *Air-column in a pipe, closed or open, vibrates longitudinally when disturbed at the mouth.*

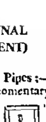


Fig. 84—A Closed Organ Pipe.

An organ pipe is the simplest form of a wind instrument. Fig. 84 shows a longitudinal section of an organ pipe. It consists of a hollow tube *BD* in which air can be blown through a pipe *A*. The air issues through a narrow slit *B*, and strikes against the sharp edge *C*, called the *lip*, of the mouthpiece. This sets up vibration in the air-column enclosed in the pipe. When the blast is directed into the pipe, it produces compression, and, when directed outwards it can, by suction, produce rarefaction at the lower end of the air-column. **An organ pipe is called closed or open according as it is closed at one end or open at both ends.**

(a) Closed Organ Pipe.— As air is blown through the pipe (Fig. 84), it strikes the edge, and a slight upward deviation of the air blast produces a compressed wave which travels to the closed end (which is a rigid wall), and so the air near the end is compressed to a pressure greater than the atmospheric pressure. This compressed air forces back the air behind it in order to return to atmospheric pressure, and in so doing it starts a compressed wave which returns along the pipe. Thus a compressed wave is reflected from the closed end as a compressed wave, and returns to the mouth. But the mouth being open, and the air free to expand, the pressure of the compressed wave . the sheet of air outside and so the layers of air relieve them-

selves from a strained state and as a result there is reversal of the type of the wave and so a wave of rarefaction starts inside. The wave of rarefaction again comes back to the mouth as a rarefied wave after being reflected at the closed end. This is again reflected as a compressed wave at the mouth, which is a free end, and is also intensified by the compressed wave directed inwards by the blast of the air outside. In this way, of the vibrations of various frequencies set up by the impact of the air blast with the lip *C* of the pipe, the air-column inside the pipe takes up only those with which it can resound, and pulses pass up and down the length of the pipe, the result being the propagation of a musical note and the pipe is found *to speak*.

The result of the reflected pulse meeting with the direct one *is a stationary longitudinal wave set up inside the pipe, and nodes and antinodes occur at definite places. The air at the open end is free to move inwards or outwards with the maximum freedom and, therefore, is a seat of* **antinode**. *The closed end being a rigid wall, the air in contact with it has the least freedom of movement and so the closed end is always a* **node**.

(b) Open Organ Pipe.—In an open pipe, when a compressed wave reaches the far end, the air at that point is for an instant at a pressure greater than ordinary atmospheric pressure, and the mouth of the tube being open, the air there can vibrate with the utmost freedom and so suddenly expands into the surrounding air. Thus the pressure diminishes so quickly that it falls somewhat below the pressure of the surrounding air, which causes a sudden rarefaction at the end of the pipe. This sets up a rarefied wave which passes back along the pipe. This rarefied wave is reflected back as a wave of compression at the other free end. Within the tube, the reflected pulses meet with the direct ones blasted into the mouth from outside and the result is the formation of a stationary longitudinal wave having nodes and antinodes at definite intervals. Both the open ends of the tube are seats of antinodes, the air there being most free to move either inwards or outwards. For the fundamental tone emitted by the tube, there is one node between those two antinodes.

72. Fundamentals of a Closed and of an Open Organ Pipe of the Same Length :—

Closed Pipe.—In the simplest mode of vibration in the case of a *closed organ pipe*, there is a node at the closed end and an antinode at the open end [Fig. 35(*b*)]. *In a stationary wave the distance between two consecutive nodes, or two consecutive antinodes, is equal to one-half the wavelength:* so in this case the length of the tube is one-fourth of the wavelength, *i.e.* the wavelength is *four times* the length of the tube. This is the **fundamental tone**.

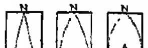
Let n_1 and λ_1 represent frequency and wavelength of the fundamental tone given by a closed organ pipe of length l. Hence $\lambda_1 = 4l$; and $V = n_1\lambda_1$, where V is the velocity of sound.

$$\therefore \quad n_1 = \frac{V}{\lambda_1} = \frac{V}{4l}.$$

Open Pipe.— In the case of the fundamental of an *open pipe*, i.e. a pipe open at both ends, there is an antinode at each end of the pipe with a node in the middle [Fig. 36(*a*)]. So the length of the pipe is half the wavelength. If n' and λ' be the frequency and wavelength of the fundamental tone for the open pipe, we have $\lambda' = 2l$. Again, $V = n'\lambda'$.

$$\therefore \quad n' = \frac{V}{\lambda'} = \frac{V}{2l} = 2n_1.$$

Hence, *the pitch of the fundamental of an open organ pipe is twice, i.e. one octave higher than that of a closed organ pipe of the same length.*



Frequency
n_1 $3n_1$ $5n_1$
(*a*) (*b*) (*c*)
Fig. 35—Closed Pipe.

N.B.— If an open pipe, while giving out a note, is suddenly closed, *the pitch of the note at once decreases* and the sound emitted becomes less sharp. If an organ pipe is closed at one end by a movable shutter, the pitch of the note emitted by the pipe is found to rise on slowly opening the shutter and to fall as the shutter is gradually closed.

(a) Overtones (or Harmonics) of Organ Pipes.— Production of harmonics depends to some extent on the nature of excitation of the tube. If the air is blown more and more powerfully, the nature of the stationary waves remains the same no doubt but the number of nodes and antinodes is increased, *i.e.* higher and higher harmonics are also produced.

(i) Closed Pipe.— In the case of a closed pipe, the closed end is always a node and the open end always an antinode [Fig. 35(*a*)]. The next possible mode of vibration, after the fundamental, is to have *one* intermediate note and *one* antinode [Fig. 35(*b*)], i.e. the length of the pipe l is three-fourth of the wavelength λ_2; so in this case, $\lambda_2 = \frac{4}{3}l$.

If n_2 be the frequency of the note, $n_2 = 3V/4l$. Hence, $n_2 = 3n_1$, where n_1 is the frequency of the fundamental.

For the *next* higher overtone, there will be *two* intermediate nodes, and *two* intermediate antinodes alternately placed [Fig. 35(*c*)].

In this case $\lambda_3 = \frac{4}{5}l$; and the corresponding frequency, $n_3 = 5V/4l$. Hence, $n_3 = 5n_1$, and so on. In the case of a closed pipe, therefore, *only harmonics proportional to the odd natural numbers are present and this makes the quality of the note given out by a closed pipe lacking in fullness.*

Harmonics of Closed Pipe

No.	Wavelength in air	Frequency of the note	Relation with the fundamental
1	$4l$	$n_1 = \frac{V}{4l}$	Fundamental
2	$\frac{4}{3}l$	$n_2 = \frac{3V}{4l}$	$n_2 = 3n_1$
3	$\frac{4}{5}l$	$n_3 = \frac{5V}{4l}$	$n_3 = 5n_1$
&c.	&c.	&c.	&c.

Therfore in a closed pipe the possible frequencies of vibration are in the ratio 1 : 3 : 5, etc.

(ii) Open Pipe.—We have already seen that in the case of the fundamental of an open pipe, there is an antinode at each end and a node in the middle [Fig. 36(*a*)]. If n' be the frequency of the fundamental,

$$n' = V/2l.$$

For the *next overtone*, there will be *two* intermediate nodes and *one* intermediate antinode between them [Fig. 36(*b*)]. In this case $\lambda'' = 2l/2 =$ the length of the pipe, and the frequency, $n'' = V/l = 2n'$, i.e. it stands an octave higher than the fundamental.

In the *next overtone*, there will be *three* intermediate nodes and *two* intermediate antinodes [Fig. 36(*c*)]. In this case $\lambda''' = 2l/3$, and the frequency, $n''' = 3V/2l = 3n'$; and so on.

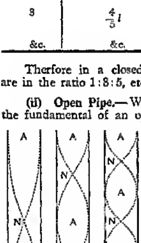


Fig. 36—Open Pipe.

Hence, in the case of an open pipe, both *odd* and *even* harmonics are present.

escapes through the other pipe terminating in a pin-hole jet where the gas is burned. Any vibration of the air inside the pipe, which forms the other side of the chamber *C*, throws the membrane *M* in contact with it into a similar state of vibration and which again causes corresponding vibration in the pressure of the gas in the chamber *C*, and thus a corresponding change takes place in the length of the flame. If the change in pressure be periodic, the length of the flame also varies periodically. But the change in pressure being very rapid, the alterations in the length of the flame cannot be followed by the eye due to persistence of vision. To render them distinct, the light is received on a cubical box having plane mirrors on its four sides [Fig. 39, (*a*)] which may be rotated rapidly about a vertical axis in front of the flame, and the successive steps of the flame are seen by looking at the reflection of the flame in the rotating mirror. When the flame [Fig. 38, (*b*)] burns steadily, a continuous band of light will appear on the rotating mirror. So when the manometric flame is at an *antinode*, where there is no variation of pressure of the vibrating air-column (*vide* Art. 51), the membrane will not be agitated and so the flame is quite steady, and a long band of light will appear on the mirror. When, however, the flame is at a *node*, where there is the maximum change of pressure, the flame jumps up and down with a frequency equal to that of the membrane and the reflection in the rotating mirror presents a broken-up-toothed-appearance.

(*a*)

(*b*)

Fig. 39

Fig. 39 represents appearance of the flame in the revolving mirror produced by different tones. Fig. 39, (*a*) represents that due to an organ pipe blown gently, and Fig. 39, (*b*) that due to the pipe blown hard having double the frequency.

Comparison.—The manometric flame method is also applied in *comparing the frequency of two organ pipes.* When a capsule is applied at a node in each pipe and the corresponding flames are examined side by side, it will be found that n teeth in one image will occupy the same length as n' teeth in the other. So the frequencies of the two pipes are evidently in the ratio $n : n'$.

Examples. (1) *If the length of an open organ pipe sounding its fundamental note be one metre what shall be the length of such a pipe in order that it may sound the fifth of the previous note?* (Pat 1926)

If l_1 be the length of the pipe giving out its fundamental and l_2 the length of the pipe when the fifth of this note is sounded (*vide* Art. 56), then, in the first case,

$V = 2nl_1$ where n is the frequency of the fundamental note.

Now because a fifth corresponds to ratio of $\frac{3}{2}$, the frequency in the second case is $\frac{3n}{2}$; hence, $V = 2 \times \frac{3n}{2} \times l_2 = 3nl_2$; $\quad \therefore 3nl_2 = 2nl_1$ ($\because V$ is constant) $= 2n \times 1$ ($\because l = 1$ metre). $\therefore l_2 = \frac{2}{3}$ metre.

Thus the length of the pipe sounding the fifth of the fundamentals is $\frac{2}{3}$ metre or about 66·6 cms.

(2) *The top of an organ pipe is suddenly closed. If it emits next above the fundamentals in both the cases and the difference in pitch be 256, what was the ptich of the note emitted ordinarily by the open pipe?* (*Pat. 1938*)

Let V be the velocity of sound in air and n_1 the frequency of vibration of the open pipe next above the fundamental, then we have $n_1 = V/l$, where l is the length of the pipe. When it is closed, it becomes a closed pipe having a frequency of vibration n_2, say. As the pipe now emits also the frequency next above the fundamental, we have, $n_2 = 3V/4l$; but $n_2 = n_1 - 256$ (n_1 being greater than n_2).

$$\therefore \quad n_1 - 256 = \frac{3V}{4l} = \tfrac{3}{4}n_1 \text{ ; whence } n_1 = 1024.$$

(3) *Two open pipes are sounded together, each note consisting of the fundamental together with two upper harmonics. One fundamental note has 256 vibrations per second and the other 170. Would there be any beats produced? If so, how many per second?* (*C. U. 1931*)

The vibration frequencies of the first pipe are 256, (256×2) or 512, and (256×3) or 768; and those of the other 170, 340 and 510. Of these notes two have got very nearly equal frequencies, *viz.* 512 and 510. So there will be beats, and the number of beats per second = 512−510 = 2.

(4) *Two organ pipes give 6 beats when sounded together in air at a temperature of 10°C. How many beats would be given when the temperature is 24°C? (Velocity of sound in air at 0°C. is 1088 ft. per second.)* (*All. 1932*)

In the case of an open organ pipe the velocity V of sound in it at 10°C. will be given by, $V = 2nl$, where l is the length of the pipe and n is the frequency of the note given out. For another pipe whose length is l', $V = 2n'l'$, where n' is the frequency of the note. Number of beats $= n - n' = \frac{V}{2}\left(\frac{1}{l} - \frac{1}{l'}\right) = 6$ ... (1)

Now if V' be the velocity of sound at 24°C., no. of beats, $N = \frac{V'}{2}\left(\frac{1}{l} - \frac{1}{l'}\right)$ ... (2)

From (1) and (2), $N/6 = V'/V$. But $V = (V_0 + 2 \times t)$ ft. per sec., where V_0 is the velocity of sound at 0°C. = 1088+20 = 1108 ft. per sec., and

$V' = 1088 + 2 \times 24 = 1136$ ft. per sec.,

$$\frac{N}{6} = \frac{1136}{1108}; \quad \text{or, } N = 6{\cdot}15. \quad \therefore \text{ Number of beats} = 6.$$

(5) *Two organ pipes one closed at one end and the other open at both ends, are respectively 2·5 ft. and 5·2 ft. long. When sounded together the number of beats heard was found to be 4 per second. Calculate the velocity of sound.* (*Pat. 1941*)

Let n_1 and n_2 be the frequencies of the closed and open pipes respectively.

Then $n_1 = \frac{V}{4 \times 2{\cdot}5} = \frac{V}{10}$; and $n_2 = \frac{V}{2 \times 5{\cdot}2} = \frac{V}{10{\cdot}4}$; No. of beats $= 4 = n_1 - n_2$

$= \frac{V}{10} - \frac{V}{10{\cdot}4}$; whence $V = 1040$ ft. per sec.

75. Determination of the Velocity of Sound by the Resonance of an Air-column :— A vibrating tuning-fork F is held close to the top of a glass tube which is vertically placed in a long cylinder almost full of water (Fig. 40). On gradually raising or lowering the tube a particular length of air-column in the tube will be found when the sound will be strongly reinforced. Thus it is an arrangement for a closed pipe of adjustable length. Adjust the position of the tube when the intensity of the sound becomes maximum. In that position

the frequency of vibration of the air-column agrees with that of the fork, and the fork and the air-column in the tube are then said to be in resonance. It should be noted that the pitch of the sound heard is independent of the diameter of the tube and of its material, glass or metal. The action may be explained as follows.—

Each movement of a prong of the fork towards the mouth of the tube compresses the air in front of it, and thus sends a compressed wave down the tube. The compressed wave, on reaching the surface of water, which is a denser medium, is reflected back as a *compressed wave* (vide Art. 71). The reflected compressed wave on reaching the open end of the tube is relieved from the strained condition by moving sideways and it is again reflected, but, this time, as a *rarefied wave* which starts down the tube (*vide* Art 71). Now, if the prong reaches the extreme downward position at the same instant and begins to move upwards, a wave of rarefaction will proceed downwards into the tube. The reflected rarefied wave will thus *coincide* with the rarefied wave started down the tube due to the backward motion of the fork and so will be *reinforced*. Again, the reinforced waves will be reflected back from the closed end (water surface) as rarefied waves, which will reach the open end just when the prong begins to move down. So the wave of compression formed by the reflection of the rarefied wave at the open end is helped by the fresh compressed wave sent by the prong. Thus shows that the fork and the air-column of the tube agree in motion (*i.e.* their time-periods are the same), and so *resonance* is produced. Thus resonance causes **the intensification of sound due to the union of the direct and reflected waves.**

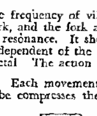


Fig 40

From the above it is evident that when resonance is produced, the wave travels over *twice* the length of the air-column in the time taken by the prong to make half a vibration. Therefore, in a complete vibration of the prong, the wave travels over *four times* the length l_1 of the air-column AN (Fig. 41). We have, therefore, $l_1 = \lambda/4$, or, $\lambda = 4l_1$, where λ is the wavelength, and l_1, the length of the air-column. But, if V be the velocity of sound, and n the frequency of vibration of the fork, we have, $V = n\lambda$; $\therefore$ $V = 4l_1 n$.

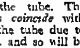


Fig 41

In fact, the antinode
a little outside the tube,
tube. Lord Rayleigh ha

the effective length of the vibrating air-column is $l_1+0{\cdot}6r$, where r is the radius of the tube, and $0{\cdot}6r$ is called the *end correction.*

Hence, $\quad V=4n(l_1+0{\cdot}6r).$

Thus, from the resonant air-column, the velocity of sound can be determined by knowing the frequency of the fork.

If the temperature of air in the tube is t, the velocity of sound at $0°C.$ can be found from the relation,

$V_t=V_0\sqrt{1+\dfrac{t}{273}}$; or, $V_t=V_0\sqrt{\dfrac{T}{273}}$, where T is the temperature on the absolute scale corresponding to $t°C.$

The End-correction can be avoided in the following way— In the first position of resonance, l_1 (Fig. 41) $=\lambda/4$, but if the tube be sufficiently long, then by raising the tube further out of water a second position of resonance, of weaker intensity, may be obtained where the length of the resonant air-column l_2 (Fig. 41) $=3\lambda/4$ [*vide* Art. 72(*a*)].

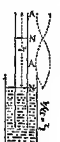


Fig. 42

Since, in the first case, $\dfrac{\lambda}{4}=l_1+0{\cdot}6r$, and

in the second case, $\dfrac{3\lambda}{4}=l_2+0{\cdot}6r$,

we have, $\dfrac{3\lambda}{4}-\dfrac{\lambda}{4}=\dfrac{\lambda}{2}=l_2-l_1. \quad \therefore \quad V=n\lambda=2n(l_2-l_1).$

By this means the wavelength can be determined, eliminating the end-correction.

N.B.—In order to obtain the velocity of sound in dry air the result corrected for temperature should also be corrected for moisture contained in the air by the formula of Art. 25.

Examples. (*1*) *You are provided with a vessel containing water, a glass tube about 40 cms. long open at both ends and a tuning-fork whose frequency is 256. What experimental result do you expect? (The velocity of sound in air is 33280 cms. per second nearly.)* (*C. U. 1914*)

Let l be the length of the air-column which emits the fundamental note. Then, wavelength $=4l$. Velocity of sound $=$ frequency $\times$ wavelength ;

or, $\quad 33280=256\times 4l$; whence $l=32{\cdot}5$ cms.

that is $(40-32{\cdot}5)$ or, $7{\cdot}5$ cms. of the glass-tube should be dipped in water when resonance will be produced.

(*2*) *A tuning-fork is held above the mouth of a closed glass cylinder whose capacity is 150 cubic inches and height 14 inches, and water is poured slowly until the most perfect resonance is obtained. The volume of the water introduced was 20 cu. in. What was the vibration number of the tuning-fork? (Velocity of sound in air = 1120 ft. per sec.)*

Volume of air in the tube for resonance $=150-20=130$ cu. in.

Area of cross-section of cylinder $=\frac{150}{14}$ sq. in. $\therefore$ Length of air-column for perfect resonance, $l=130\div\frac{150}{14}=12{\cdot}133$ in. again ; $\frac{150}{14}=\pi r^2$, where $r=$ radius of cylinder ; thus $r=1{\cdot}85$.

Hence, end-correction $= 0{\cdot}6r = 0{\cdot}6 \times 1{\cdot}85 = 1{\cdot}11$ in. We have $V = 4n(l + 0{\cdot}6r)$ whence n is the required frequency.

$\therefore\ (1120 \times 12) = 4n \times 13{\cdot}21$ ($\because\ V = 1120 \times 12$ in.); whence $n = 253{\cdot}8$.

(3) *A certain tuning-fork first produced resonance in a glass tube with an air-column of 33 cms. and it could again produce resonance with a column 100·5 cms. in the same tube. Calculate the end-correction.* *(All. 1921)*

In the first case, if l_1 be the length of air-column for resonance, the effective length of air-column $= l_1 + x$, where x is the end-correction.

$\therefore\ l_1 + x = \lambda/4$, where λ is the wavelength. In the second case, if l_2 be the length of air-column for resonance the effective length $= l_2 + x$.

$$\therefore\ l_2 + x = \frac{3\lambda}{4}, \text{ or, } \frac{\lambda}{4} = \frac{l_2 + x}{3}; \quad \therefore\ l_1 + x = \frac{l_2 + x}{3}; \text{ or, } 3(l_1 + x) = l_2 + x.$$

Since $l_1 = 33$, and $l_2 = 100{\cdot}5$, we have, $x = 0{\cdot}75$.

(4) *A closed pipe is filled with a gas whose density is 0·00126 gm. per c.c. If the length of the pipe is 50 cms., find the frequency of the note emitted. (The velocity of sound in air at 0°C. is 332 metres per second.)*

As the density of air is 0·001293 gm. per c.c. and as the velocity of sound in any gas is inversely proportional to the square root of its density, the velocity of sound in the gas of the pipe, $V = 33200\sqrt{\dfrac{0{\cdot}001293}{0{\cdot}001260}}$ cms. per sec.

But $V = 4nl$; whence $n = \dfrac{V}{4l} = 168$.

§ 76. Longitudinal Vibration of Rods:— When a rod of wood or glass firmly clamped at its middle point is rubbed lengthwise with a piece of resined cloth, or wet linen it is set in longitudinal vibration, that is, in planes parallel to its axis, and it gives out a shrill note. The rod is alternately elongated and compressed in its course of movement and the vibration takes place exactly in the same manner as the stationary vibration of an open pipe sounding its fundamental.

The free end of the rod being the parts of maximum vibration are *antinodes, whilst,* for the simplest mode of vibration there will be a *node* in the middle where it is clamped. Evidently the length of the rod is half the wavelength (distance between two consecutive nodes and antinodes).

The velocity of sound in the rod is given by, $V = \sqrt{\dfrac{E}{D}}$, where E is the Young's modulus of elasticity and D, the density of the material of the rod. Again we have, $V = n\lambda$, where λ, the wavelength, is in this case, equal to twice the length l of the rod.

$$\therefore\ V = 2nl; \text{ or, } n = \frac{V}{2l}; \text{ or, } n = \frac{1}{2l}\sqrt{\frac{E}{D}}.$$

Thus, knowing the velocity of sound in the rod, the frequency, or the pitch of the sound emitted can be calculated. Again, if the pitch of the sound is determined by comparison with a sonometer wire, the velocity of sound is known from the relation, $V = 2nl$. Thus this also provides a method of determining *the velocity of sound in a solid rod.*

77. Kundt's Dust-tube Experiment :—The velocities of sound in different gases were determined by Kundt by using longitudinal vibration of rods. The velocity of sound in a rare gas is usually determined in the laboratory by this method.

Experiment.—The apparatus consists of a metal or glass rod which is clamped exactly at its middle point C and has a card-board disc D firmly fixed at its end within a long glass tube AB in which it can move without touching its walls. The other end of the tube AB is closed by an adjustable stopper B (Fig. 43).

Fig. 43—Kundt's Dust-tube Experiment

Before fixing the tube in position it is thoroughly dried by blowing hot air through it, and then some dry lycopodium powder is evenly spread along its sides. The rod is now stroked (rubbed lengthwise) with resined cloth, if it be metal, or with a cloth moistened with methylated spirit, if it is of glass, causing it to vibrate longitudinally. Waves are emitted by the disc D which is moving backwards and forwards with the frequency of the note emitted by the rod and thus setting up vibrations in the air within the tube. These waves started from the disc D are reflected back by the surface of the piston B, and thus stationary waves having fixed nodes and antinodes are set up in the tube. The position of the adjustable piston B is carefully adjusted until a *resonance* is produced, when the fundamental note emitted by the rod coincides with a harmonic of the enclosed air-column within the tube.

When resonance is reached, the fine lycopodium powder is seen to be thrown into a state of violent agitation when the powder will be seen to fly away from the *loops* (antinodes), the places of maximum displacement of air-particles, and will collect in heaps at the *nodes*, the places of minimum displacement of air-particles. In general, several nodes and loops will be formed within the tube as shown in the diagram. If l be the mean distance between two consecutive nodes, the wavelength λ of the longitudinal vibration of air is $2l$, and if n be the frequency of the note emitted by the rod it is also the frequency of vibration of the air in the tube, as the rod and the tube are in resonant vibration, and the velocity of the sound in air, $V=n\lambda=n\times 2l$. Now for the simplest mode of vibration of the sounding rod, a node is formed at the middle where it is clamped and two *loops* are formed at the two ends. So, if l' be the length of the rod, the wavelength λ' of the longitudinal vibration within the rod is $2l'$, and if V' be the velocity of sound in the rod, $V'=n\lambda'=n\times 2l'$; so we have,

$$\frac{V}{V'}=\frac{n\times 2l}{n\times 2l'}=\frac{l}{l'}=\frac{\text{length between two consecutive loops or nodes}}{\text{length of the rod}}.$$

The above relation provides a method of calculating V or V' when one of them is known; and if frequency n be found by means of a sonometer and a standard fork, then *velocity of sound in air, and also in the rod, can both be determined.*

Velocity in different Gases.—To compare velocities of sound in two gases, first fill the tube with one of the gases and find out the average distance l_1 between two nodes formed at resonance. Repeat the experiment with the other gas and let the distance in this case be l_2; then, if V_1 and V_2 are the respective velocities in the two gases, we have $\frac{V_1}{V_2}=\frac{n\times 2l_1}{n\times 2l_2}=\frac{l_1}{l_2}$.

77 (a). Determination of the Frequency of a Fork by Stroboscopic Wheel :—

A stroboscopic wheel is simply a metallic disc, having a number of a equidistant rectangular radial slots arranged along the rim, mounted vertically on a horizontal axle which is mechanically driven at a known speed. The fork under test is placed on one side of the wheel, the plane of vibration of the prong being parallel to the plane of the wheel and the longer side parallel to the longer axis of the slot when the latter is vertical. The fork is run electrically and a strong light is focussed on a prong facing the slots. The wheel is set to motion and observation is made horizontally from the other side of the wheel.

The speed of the wheel is gradually increased till the interval, in which one slot is replaced by the next, becomes roughly equal to the *period* of the fork, when the prong would appear to oscillate slowly. Next, when the said interval is adjusted exactly equal to the *period* of the fork, by suitably altering the speed of the wheel, the prong would appear to remain *stationary*. This is what is called the stroboscopic principle.

Knowing the number of slots on the wheel, and the rate of motion of the wheel, the period of the fork and thus the frequency of the fork can be determined.

Questions

1. Describe in detail with a diagram an open organ pipe, and explain its mode of excitation. What effect is produced on the pitch and character of the note, if the open end is suddenly closed? (C. U. 1926; Pat. 1928)

2. (a) Give an account of nodes and antinodes in open and closed organ pipes. (All. 1918, '22; C. U. 1931, '32; *cf.* G. U. 1949)

(b) How are stationary waves produced in (i) an open organ pipe, (ii) closed organ pipe? (G. U. 1947; *cf.* All. 1945)

3. What do you understand by pitch of musical note? The organ pipes of the same length are given, one open and the other closed. What should be the relation between the pitch of the fundamental notes emitted by them? (C. U. 1921, '26; Pat. 1921, '39)

4. What is the frequency of the fundamental note of an open organ pipe 4 ft. long ? (Velocity of sound in air = 1100 ft. per sec.) (C. U. 1950)

What would be the effects of (*a*) covering its open end, (*b*) increasing the temperature, (Pat. 1930 ; C. U. 1950)

(*c*) varying the nature of the gas enclosed in the tube (Pat. 1930) and (*d*) lengthening the pipe ? (C. U. 1950)

[**Hints.** $1100 = n \times (2 \times 4)$; or, $n = 137 \cdot 5$; (*a*) When one end is closed, the pitch will be halved, *i.e.* will be lowered an octave ; (*b*) velocity will be increased (*see* also Art. 53) with the increase of temperature ; hence pitch will be increased ; (*c*) pitch increases or decreases with the increase or decrease of velocity which again varies inversely as the square root of density of the gas ; and (*d*) pitch decreases with increase of length.]

5. What will be the effect on the pitch of the note of an organ pipe, if the air in the pipe is replaced by carbon dioxide ? (G. U. 1949)

6. What is meant by resonance ? Calculate approximately the length of the resonence box closed at one end on which a tuning-fork is to be mounted, the pitch of which is 256, the velocity of sound in air being 1120 ft. per sec. Would the same resonance box answer for a fork of another pitch ? If so, of what pitch ? (All. 1926)

[**Hints.**—The resonance box acts as a closed organ pipe ; so $V = 4nl$; or, $1120 = 4 \times 256 \times l$; or $l = \frac{35}{32}$. The box will also speak for a fork whose frequency is 3 or 5 times the fundamental frequency.]

7. The velocity of sound in hydrogen is 1296·5 metres per second. What will be the length of a closed organ pipe, filled with hydrogen, which gives a note having a vibration frequency of 512 per second ? (C. U. 1915 ; Dac. 1933)
[*Ans.* 63·3 cms. (approx.)]

8. What is the frequency of the note emitted by a siren having 32 holes and making 1575 revolutions per minute ? A closed organ pipe sounding its fundamental is in unison with the above note. What is the length of the pipe ? (Velocity of sound in air = 1120 ft. per sec.)

[*Ans.* 840 ; $\frac{1}{3}$ ft.]

9. Calculate the shortest length of a pipe 4 cms. in diameter which will be set in resonant vibration by a tuning-fork making 256 vibrations per second. (Velocity of sound in air = 340 metres per sec.)

[*Ans.* 32 cms.]

10. Two organ pipes, open at both ends, are sounded together and four beats per second are heard. The length of the short pipe is 30 inches. Find the length of the other. (Velocity of sound = 1120 ft. per sec.) (C. U. 1935)

[*Ans.* $30\frac{6}{11}$ inches.]

11. What are the fundamental and harmonic notes of organ pipes, open and closed ? (C. U. 1947, '50)

12. What effect is produced on the frequency and quality of a note given by an organ pipe if the top is suddenly closed ? If the frequencies of the first overtones of the two notes so obtained differ by 440, what was the original frequency ? (All. 1924)

[*Ans.* 880]

13. The pitch of the fundamental note of an open pipe 100 cms. long is the same as that of a sonometer wire 200 cms. long which has a mass of one gram per centimetre. Find the tension of the wire. (Pat. 1937)

[*Ans.* $4 \cdot 356 \times 10^8$ dynes, taking $V = 330$ metres per sec.]

14. Calculate the change of pitch of an open organ pipe 3 ft. long when the temperature changes from 10°C. to 15°C.
[*Ans.* $n_2 : n_1 = 1 \cdot 009$].

15. The frequency of the fundamental of an open and closed organ pipe is 128 c.p.s. What are the frequencies of their first three overtones ? (G. U. 1955)

[*Ans.* 256, 384, 512 and 384, 640, 896 c p s.]

16. The frequency of a note given by an organ pipe is 312 at 15°*C*. At what temperature will the frequency be 320 supposing the pipe to remain unchanged in length ?

[**Hints.**—$V_{15} = 312\lambda$, and $V_t = 320\lambda$ $\therefore$ $\dfrac{V_t}{V_{15}} = \dfrac{320}{312} = \dfrac{40}{39}$.

Again, $V_t = V_0\left(1 + \frac{1}{2}\cdot\frac{t}{273}\right)$; and $V_{15} = V_0\left(1 + \frac{1}{2}\cdot\frac{15}{273}\right)$

$\therefore$ $\dfrac{V_t}{V_{15}} = \dfrac{546+t}{561}$. So, $\dfrac{546+t}{561} = \dfrac{40}{39}$; whence $t = 29\cdot4°C$.]

17 If an organ pipe gives a note of 256 when the temperature of air is 40°*C*., what will be the frequency of the note when the temperature falls to 20°*C*. ?

[*Ans.* 247 3]

18. Distinguish between forced vibration and resonance and mention two practical applications of each.

What should be the length of an open organ pipe which sounded together with another similar pipe of length 30 inches would produce 4 beats per second ?

(Velocity of sound in air = 1,120 ft. per sec) (Bihar, 1956)

[*Ans.* $29\frac{7}{9}$ inches, or $30\frac{2}{7}$ inches]

19. How can the existence of nodes and antinodes in a sounding organ pipe be demonstrated ? (C. U. 1937, '50)

20 Describe experiments demonstrating the existence of nodes and antinodes in an open organ pipe (G. U. 1949)

21. Suggest any experiment by which you can determine the wavelength of any note in air. (Pat. 1926)

Show how the phenomenon of resonance can be used for directly determining the wavelength of a given note of sound in air. (R. U. 1952)

22. How would you demonstrate that the best resonant length is one-fourth the wavelength in the case of a closed pipe and one-half the wavelength in the case of an open pipe ? (Pat 1929)

[**Hints.**—Describe the resonant column experiment (*vide* Art. 75). The tube [illegible]

for the overtones.

In the second case, hold the same tuning-fork in front of an open pipe (both ends open), the length (say, about 10 inches) of which is made adjustable by slipping up and down over it a tightly fitting roll of ordinary writing paper Adjusting the length and proceeding as above, it will be observed that sound is maximum for $l = \lambda/2$, and gets fainter and fainter for the overtones, *i e.* for $l = 2\lambda/2$ and $3\lambda/2$, etc.]

23 Explain the mode of vibration of an air-column closed at one end thrown into resonance by a tuning-fork. (Utkal, 1952)

24 A vibrating tuning-fork is placed at the mouth of an open jar, and water is poured into the jar gradually. Explain what will happen. (*cf* G U. 1949)

25. What is meant by the end-correction of the length of a resonant air-column ? (R. U. 1955)

26. Explain how you would determine the velocity of sound in air by an experiment of this kind. (C. U. '31, '47 ; Pat. 1941, '49, '51, '53 ; Dac. 1933, '34, '52 ; And. U. 1951 ; Anna. U. 1950 ; Utkal, 1948, '49, '53)

27. Describe an experiment to find out the velocity of sound in carbon dioxide. (Pat. 1939 ; All. 1922 ; *cf.* Dac. 1931)

28. What is meant by resonance ? Show how the phenomenon of resonance may be used to measure the velocity of sound in a gas. (C. U. 1945)

29. A cylindrical tube 100 cms. long, closed at one end, and of one cm. internal radius, is placed upright and filled with the water, and a tuning-fork of frequency 510 is sounded continuously over its open end. Assuming the velocity of sound in air to be 340 metres per sec., describe exactly what you would expect to observe if the tube were gradually emptied. (Pat. 1936)

[*Ans.* The tube will speak when the length of the air-column is 16, 49·4, 82·7 cms.]

30. A tuning-fork, whose frequency is 410, produces resonance in a glass tube of diameter 2 cms. when lowered vertically in water ; on lowering the tube further down another point of resonance is found. Find the lengths of the air-column producing resonance. ($V = 340$ metres per sec.)

[*Ans.* $l_2 = 61·59$ cms. ; $l_1 = 20·13$ cms.]

31. When a fork of frequency 512 is sounded, the difference in level of water in a tube between two successive positions of resonance is found to be 33 cms. What is velocity of sound in air ? (G. U. 1949)

[*Ans.* 33,792 cms./sec.]

32. Write a note on organ pipes. (Vis. U. 1955)

33. Describe a stroboscopic wheel. How can the frequency of a tuning-fork be determined with it ? (R. U. 1953)

CHAPTER IX

MUSICAL INSTRUMENTS : PHYSIOLOGICAL ACOUSTICS

78. Musical Instruments :—The musical instruments can be divided mainly into *three* classes—(a) *Wind instruments*; (b) *Stringed instruments* ; (c) *Percussion instruments.*

(a) Wind Instruments.—The working of these instruments depends upon the vibration of an air-column. These again can be divided into *two* classes: (*i*) Instruments without reeds such as the flute, piccolo, etc. ; (*ii*) Instruments with reeds such as the clarionet.

harmonium, etc. The most familiar example of the wind instruments is the organ pipe, which may be of the above two types: (*a*) one without reeds, known as the flue pipe, and (*b*) the other with reeds, known as the reed pipe.

It is already stated that only a column of air of right length may be made to respond to a particular note. But in the case of a column of air contained in a pipe resonance can be produced by making a flutter in the air at one end of the pipe. The pipe selects from the flutter (which is merely a combination of pulses of various wave-lengths) that particular pulse with which it can resound in order to produce a musical note. This is the *principle of various musical instruments* in nearly all of which the sounding part is a column of air.

The Flue Pipe.—The simplest form of this type is an ordinary organ pipe the principle of which has been described in Art. 71. The note emitted by this pipe depends primarily upon the length of the pipe. The fundamental note is given out at a certain minimum blowing pressure by increasing which higher harmonics are given out.

In the **Organ**, there is a set of pipes of fixed pitch and the instrument is provided with a keyboard as in harmoniums.

The Reed Pipe.—In this instrument the air blast impinges on a flexible metal strip (Fig. 44) called the *reed*, which controls the amount of air passing to the pipe by wholly or nearly covering the aperture through which the air passes. The reed which completely closes the aperture of the pipe is called a *beating reed*, which behaves as a stopped end of the pipe, and the other by which the aperture is nearly but not fully closed, is called a *free reed*. Free reeds are used in harmoniums and American organs, where the wind is forced into a rectangular air-chamber at one side of which the reed is attached. The air presses against the reed and causes it to vibrate. A single beating reed made of cane is used at the mouthpiece of a *clarionet*.

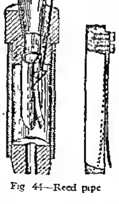


Fig 44—Reed pipe

(b) Stringed Instruments.—In this class the note is produced by the vibration of strings kept under tension, such as the harp, *piano, violin, esraj, setar, etc.*

(c) Percussion Instruments.—These are tuned to a fixed pitch, ch as the kettle-drum, tambourine, etc. in which the vibration of air is produced by striking with a hammer a stretched membrane or a metal plate.

79. The Phonograph:—Long before the invention of the phonograph, Thomas Young, an English scientist, succeeded in recording sound vibrations on a rotating drum. It was Thomas Alva Edison, an American, who in 1807 invented the phonograph by which it was possible to record as well as reproduce sound vibrations.

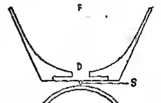


Fig. 45—The Phonograph.

The *phonograph* consists of a funnel *F*, which is closed at the lower end by a thin glass or mica diaphragm *D* (Fig. 45). When sound vibrations are directed into the funnel, they set the diaphragm into vibration, and with it, a pointed steel or a chisel-shaped sapphire crystal *S*, attached at the centre, also vibrates. The chisel is in contact with a cylinder *C* of paraffin wax, and, at the time of vibrations, cuts a groove of *varying depth* on the cylinder which is rotated, and at the same time moved lengthwise by clock-work. The depth of the groove is not uniform but corresponds to the strength and complexity of the vibrations communicated to *D*. *The cylinder is thus a faithful record of the sound vibrations directed at F.*

To *reproduce* the sound, a smooth sapphire point, attached to a similar diaphragm fitted in a frame, called the *sound-box*, is placed at the beginning of the groove of the cylinder which is rotated and shifted sideways at the same speed as before. The sapphire point rises and falls in accordance with the height and depth of the groove, and thus the diaphragm of the sound-box reproduces exactly the movements of the diaphragm *D* of the recorder. These movements communicated to the air produce the same sound which was originally directed into the funnel *F*.

The materials with which the phonograph records are prepared being very soft, the records do not last long and so the reproduction is not very faithful.

80. The Gramophone:—It is a machine for the recording and reproduction of sounds, vocal or instrumental, such as, music, speech, etc. It is a more improved apparatus than the phonograph. The sound records are made in the form of flat discs in which spiral grooves representing sound-tracks run from the rim to the centre. The grooves are of *varying width* and *not of varying depth*, as a result of which the resistance to the movement of the needle along the furrow is much less than in the phonograph and so the reproduction of sound is much more faithful. Moreover, the discs are made of a matrix (composed of shellac, tripoli powder and other ingredients) which is

much harder than the wax used in the phonograph and so they do not deteriorate with use so easily.

Recording of Sound.—The modern method of recording is electrical. The source of sound is placed in front of a microphone by whose mechanism the current passing through it is fluctuated. This fluctuated current is amplified to the required extent by the use of thermionic valves. The amplified fluctuating current is used to actuate a cutting chisel upon a disc of wax by the principle of electromagnetic action. This record of wax is called the negative. An electro-plate of it is made on a copper disc by electrolysis. This electro-plate is called the 'mother shell' or the 'parent record', or the positive. Two 'working matrices' of two different musics are made from two such mother shells and are fixed to the top and bottom plates of a hydraulic press with their recorded surfaces facing each other. The recording material (the disc), previously warmed a little, is placed in between the two working matrices and the two records are stamped on the two faces of the recording material by pressure.

Reproduction of Sound.—This is done through the mechanism of a *sound-box* which has a needle, with a pointed end, rigidly screwed to the shorter arm of lever system (Fig 46). The needle slides on the spiral grooves of the record, the record being made to rotate at a uniform speed with the help of an adjustable *governor*, by the action of the energy of wound spring. The end of the longer arm of the lever is fixed to the centre of a circular mica diaphragm. The diaphragm is mounted between rubber rings called *gaskets*, and form the front of a cylindrical metal box called the *sound-box*. The vibration of the needle running on the furrows sets the diaphragm to motion, reproducing the recorded sound. The sound-box is connected to a metallic conical pipe called the *tone-arm*, which is capable of moving freely about a vertical axis. The tone-arm with the sound-box gradually moves to the centre of the record as the needle slides on it. The sound from the tone-arm is finally magnified through a horn which is usually housed within the cabinet. The lever system is balanced on a knife-edge forming the fulcrum. The vibration of the lever is controlled by two springs as shown in the figure.

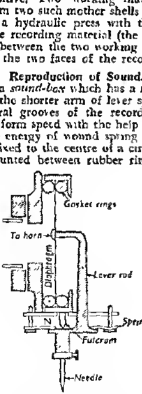


Fig 46—The Sound-box

In the **Radio Gramophone**, the mechanical sound-box is replaced

by an electric 'pick-up' by the mechanism of which a periodically modulated feeble current is obtained as the needle slides on the grooves of the record. This feebble modulated current, after suitable amplification through a combination of thermionic valves, is led through a **loud speaker** by which a voluminous sound commanding a large assembly of audience is reproduced.

Physiological Acoustics

81. The Ear:— The human ear (Fig. 47) consists of *three* parts —(*a*) the *external ear* (or **pinna**) by which the sound-wave is collected; (*b*) the *middle ear* (or **drum**) in which the vibrations are transmitted from the external ear to (*c*) the *internal ear* (or **labyrinth**).

(a) External Ear.—Starting from the outside, there is, in the first place the *external ear E* (the part external to the head) from which extends the ear passage, called the **external auditory meatus** *M*, down which the air-vibrations travel. This is closed at its end by a stretched membrane called the **membrana tympani** *T*, beyond which lies the cavity, called the *ear drum* or *tympana* or the middle ear.

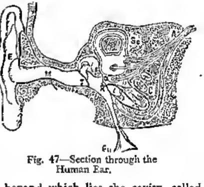


Fig. 47—Section through the Human Ear.

(b) Middle Ear.—This cavity is bounded upon its outside by the tympanic membrane and its inner side by bony walls except at two places, the **fenestra ovalis** *O* and the **fenestra rotunda** *R* where membranes are stretched. A combination of three little bones or **ossicles**, the first of which is the **malleus** *m* or the hammer bone, extends from the inside of the tympanum. This bone communicates with the internal ear through two other bones, the *anvil i* (or **incus**) and the *stirrup s* (or **stapes**), the base of which is joined to the *fenestra ovalis*, which separates the middle ear from one parr of the inner ear. The middle ear is connected to the throat by an **eustachian tube** *Eu*. This tube is usually closed, but the action of swallowing opens a valve in this tube and serves to keep the air-pressure inside the middle ear equal to that of the atmosphere. Ear-ache is often caused when the valve does not work and due to which the outside pressure becomes greater than that inside so that the bones are pressed hard causing painful results.

(c) Labyrinth.— It is a complicated structure having a set of cavities. The cavities have bony walls, called the osseous **labyrinth**, and internal membranes, known as the **membranous labyrinth**.

The osseous labyrinth consists of the following—(1) *Vestibule* '*v*' in the outer wall on which lies the *fenestra ovalis*. Through the inner wall of the vestibule the divisions of the auditory nerve *A* enter into the internal ear. (2) **Cochlea** (*t C*) at the entrance to which lies the *fenestra rotunda*. It is a spiral canal like the form of a snail shell. It contains a fluid which receives and transmits vibrations to the auditory nerve. In this canal there is a membraneous partition, called the **basilar membrane** which plays an important part in the act of hearing. The semi-circular canals *Sc* serve to maintain equilibrium and do not take part in the hearing.

The *membraneous labyrinth* contains a fluid, *endolymph*, and between it and the *osseous labyrinth* is another fluid *perilymph*.

82. How we hear:—The waves produced in the air by the vibrations of the sounding body are collected by the *pinna* and these waves passing through the *auditory meatus* strike the *tympanic membrane* which is forced to execute corresponding vibrations. These vibrations are transmitted through the three little bones in succession, the *malleus*, the *incus*, and the *stapes*, to the membrane of the *fenestra ovalis* of the inner ear. The vibrations of the fenestra ovalis start waves which reach the *cochlea* where the vibrations are handed on by the fluid to the basilar membrane. The vibrations, so generated, actuate the auditory nerve and the brain, and give rise to the sensation of sound.

83. The Human Voice:—The vocal organ can be compared to a double reed organ pipe. The voice is produced by forcing air from the lungs through the space between two stretched membranes *V*, *V* called the **vocal chords**, which are stretched across the top of a wind pipe, called the **trachea**, with a narrow slit, called the **vocal slit**, between them, the two edges of the slit acting as reeds (Fig. 48). The two membranes are attached to muscles by which their tension and vibration frequency can be altered. The trachea, or the wind pipe of the throat, leads to the lungs at one end, and at the front part of the throat it forms the vibrating part, called the larynx or the *voice-box*.


Fig. 48—Vocal Organ.

The edges of the membranes are set into vibrations like reeds by the air from the lungs and thus sound is produced, the pitch of which can be altered by altering the tension of the vocal chords, and the quality of which depends upon the air-cavities of the nose, throat, and mouth, which act as resonators, the shape and the size of which the speaker can vary at will.

The vocal chords are much longer in men then those in women and children, and so the wavelength of sound emitted by a man is

much longer than that emitted by a woman or child. Thus a *female voice* is *of higher pitch* than that of a male voice.

Questions

1. Describe a phonograph and explain its action.
(And. U. 1951 ; Dac. 1912 ; C. U. 1932, '47)

2. Describe the gramophone. What is the function of the horn ?
(All. 1923, '32)

3. Summarise *your knowledge about a gramophone sound-box.*
(U. P. B. 1938)

4. Describe a gramophone. How is sound recorded and reproduced ?
(East Punjab, 1942 ; Nag. U. 1950 ; U. P. B. 1949 ; Pat. 1948, 1949, '52 ; *cf.* Benares, 1953)

5. Give a brief account of the different parts of a gramophone and describe the various stages in the propagation of the sound from the origin to the ears of the hearer.
(Pat. 1931 ; *cf.* C. U. 1946 ; G. U. 1950)

6. Give a brief description of the human ear with a neat diagram and mention the functions of the different parts.
(C. U. 1933, '38)

APPENDIX (A)

AERONAUTICS

CHAPTER I

THE ATMOSPHERE

1. Aerodynamics and Aeronautics:—*Aerodynamics* is a general name for that part of Physics which deals with the properties of *any* gaseous medium in motion. *Aeronautics* is a specialised branch of it which, in particular, deals with the behaviour of atmospheric air when an *aircraft moves* through it

2. Facts about Atmosphere:—Before proceeding further to study the principles on which the flight of an aircraft depends, the following facts about the atmosphere should be well remembered.—

(A) Extent of the Atmosphere and the Variations of Pressure and Temperature with Altitude.—The composition of the atmosphere has been dealt with in Part I. and it will be noted from there that nitrogen, oxygen, argon, and small traces of some other gases are the constituents of air and the percentage of composition slightly varies from one place to another As the atmosphere extends upwards, the density of the air diminishes. Opinions, however, vary as to how high the atmosphere reaches Some estimate the height to be as great as 200 miles even (*vide* Art. 304, Part I) In Art. 303, Part I, it has been described how the temperature of the atmosphere falls as the height increases Roughly speaking, in the lower belt of the atmosphere which is known as the *troposphere,* the temperature steadily falls at the rate of about 1°*F.* for every 300 ft. increase in height, and in the upper belt which is known as the *stratosphere,* the temperature is more or less steady near about—60°*F.* and does not alter with the increase of height

The average pressure of the atmosphere at sea-level is about 14.7 lbs.-wt. per sq. inch, which changes from place to place and from day to day with changes of weather and temperature. The pressure ...ases with increase of altitude. It has been estimated that about ...e-half of the total weight of the atmosphere is concentrated in the first 18,000 ft In Art. 303, Part I, greater details about the variation of pressure with altitude is given It should be remembered, however, that the pressure exerted by *air in motion* may be greater or less *than the pressure exerted by air when stationary,* according to the ...ature of its motion, and from these pressures the forces of *lift* and *drag* (discussed later) on an aircraft are obtained.

(B) Air Resistance.—Due to the fact that air has weight and that it is always subject to convection currents, air offers resistance to

any body which moves through it, and this resistance, for a body of given shape, and given relative motion, depends upon the properties of (*i*) viscosity, (*ii*) elasticity, and (*iii*) inertia, of the air.

(*i*) **Viscosity.**—It is an inherent property of all fluids and has been dealt with in Arts. 223 to 229, Part I. Due to its existence, when any relative motion occurs between parts of a fluid, internal forces of frictional character are set up within the fluid which tend to retard the relative motion. This phenomenon clearly shows that the molecules of a fluid are mutually interlocked, the strengths of the bonds of interlocking vary, however, from one fluid to another depending on the viscosity. So when a body moves through air (which is a kind of fluid) and the layers of air in contact with it are moved, they also cause layers next to them to move to some extent. The types of movements that are caused in the neighbouring layers depend on the shape of the moving body and the magnitude of its motion relative to the air. When this relative motion is high, *eddies* or *vortices* are formed in the air around the body. It will be seen later that these eddies cause many phenomena connected with *flight*.

(*ii*) **Elasticity.**—The tendency of the air-particles to re-occupy former space from which they are disturbed is due to that property of air which is known as its *volume elasticity* (*vide* Art. 217, Part I). With increase of altitude when the pressure falls, the tendency of air to expand and thus to reduce in density arises out of this property.

(*iii*) **Inertia.**—It is a property common to all matter (arising out of mass or density) due to which air tends to be at rest or in steady motion, and resists any attempt to change such rest or motion.

(C) Density.—The density of the air depends on the atmospheric pressure. It is greatest at the sea-level and decreases with altitude. At sea-level the density of air is about 0·08 lb. per cu. ft., and at 20,000 ft, it is only 0·042 lb. per cu. ft., which is about one-half of the first value. It is the density of air which makes all flight possible, as an aircraft is supported in the air by forces entirely dependent on the density; the less the density the less the weight lifted and more difficult does flight become, and in vacuum any flight is impossible.

An idea about how the density of air decreases with increase of altitude will be obtained from the following table:—

Altitude	Density (lb./cu. ft.)	Altitude	Density (lb./cu. ft.)
Sea-level	0·0800	15,000 ft.	0·0503
1,000 ft.	0·0778	20,000 „	0·0426
5,000 „	0·0689	30,000 „	0·0298
10,000 „	0·0590	40,000 „	0·0197

(D) Humidity.—At the lower levels of the atmosphere water vapour is always present. The amount of it varies with the season and diminishes with the increase of altitude. Under indentical conditions of temperature and pressure, the density of water vapour is only *three-fifths* of that of air and so the pressure of water vapour diminishes the pressure and density of the atmospheric air.

CHAPTER II

AIR RESISTANCE

3. Streamlines Whenever a body is moved through air (or any other fluid), or the fluid flows past a body, there is always produced a definite resistance to its motion. This resistance is usually termed **drag** in aeronautical work. The effect of this resistance in the viscous fluid is to set up displacements in the shape of eddies in the fluid.

In such cases two modes of flow are possible: (*a*) *turbulent flow*, and (*b*) *streamline flow*. In Art. 226, Part I, the natures, of both these types of flow have been described and it has been pointed out that the streamline (or laminar) flow degenerates into turbulent flow when a certain relative velocity, known as *critical velocity*, is exceeded. So when a body moves with an excessive velocity through a viscous medium, turbulent motion causing eddies and vortices results and the resistance to motion of the body increases, the magnitude of which depends also largely on the shape of the body but if a body is so shaped as to produce the least possible eddy motion and so the resistance to motion is also much reduced thereby, then it is said to have a *streamline shape*, and the lines round the body interposed in the fluid showing the directions and shapes of the disturbances are called **streamlines.** These streamlines enables us to understand the nature of the flow of the fluid past the body.

As it is difficult to investigate the disturbances on an aircraft while in actual flight, most of the aeronautical experiments for studying the phenomena of flight are carried out by scientists in the laboratory by using some **form of Wind Tunnel,*** in which air is made to flow past a model of aeroplane which remains at rest

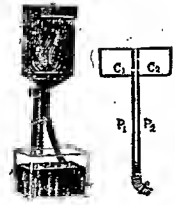
* A *wind tunnel* is nothing but a suitable chamber in which, say, a model of an aeroplane is kept and an artificial high speed air-current is produced across it by the action of an air-screw (*vide* Art. 79). The temperature of this blast is also simultaneously kept very low by means of a refrigerating plant.

relative to the tunnel. The effect is the same as if the body were made to move through *still* air, because it is the *relative motion* of air to the aircraft, or the aircraft to air, which really matters in the investigation.

Air Speed and Ground Speed.—True *air speed* of an aircraft is the speed relative to air, that is, the speed with which it would travel in the absence of wind; while *ground speed* means its speed relative to the earth, or the actual speed over the ground. For instance, if the normal speed (air speed) of an aircraft flying from A towards B be 100 m.p.h., while wind is blowing at 40 m.p.h. from B towards A, the aircraft will reach B with an actual speed (ground speed) of 60 m.p.h.

It is possible to study and photograph steamlines and eddy motions by introducing smoke into the air-flow in wind tunnels, or coloured jets into the Water Tank Experiment described below.

Water Tank Experiment.—The apparatus for demonstrating streamline flow of liquids consists of a rectangular reservoir at the top divided into two compartments C_1 and C_2 [Fig. 1(*b*)] by two glass plates P_1 and P_2 separated by a distance of about 1 mm. These plates have equidistant perforations inside the reservoir (as C_1), the perforations in P_1 being alternate to those in P_2. One of the compartments C_1 is filled up with clear water and the other C_2 with a coloured water, say water coloured with potassium permanganate. Now, the liquid flowing down between the plates from both the compartments collects at the bottom and finally flows out through a rubber tube provided with a pinch-cock. On opening the pinch-cock clear water from C_1 and coloured water from C_2 will flow down between the plates through alternate perforations. The violet-coloured tracks will show the parallel streamlines along which the water flows, and they finally curve inwards towards the end. Due to the colouring material the streamlines are made visible to an observer. The actual apparatus is shown in Fig. 1(*a*), where a thin body made of gutta-percha has been introduced in the stream between the plates to show the distortion of streamlines due to its shape. Similarly, small bodies of different shapes can be introduced to show how the streamlines are distorted in each case.



(*a*) (*b*)

Fig. 1.—Water Tank Experiment.

4. Effect of Shape:—One great object of the designer of aeroplanes is to reduce the eddy resistance to an absolute minimum, and much experimental work has been carried out with this in view. Results show that the shape of a body has a striking effect on the amount of drag produced, and that enormous advantage is gained by adopting a 'streamline' shape the example of which in nature is the outline of a fish. When air flows past a perfectly streamlined body, no eddies are created in its neighbourhood.

Fig. 2 shows some of the streamlines flowing past a few bodies of different shapes. It will be noticed in Fig. 2(*a*) that, in the case of a flat plate the airflow breaks up after passing the edge of the plate into a series of eddies and vortices, the size and nature of which will also be influenced by both the velocity of the airflow and the linear dimension of the plate. It will also depend on the inclination of the plate to the direction of air flow. Fig. 2(*b*) shows that *owing to its position both sides are* affected by the air-current. Streamlines at the bottom are deflected downwards and eddies are formed at the lower edge, whilst on the top there are similar eddies and also regions of lower pressure due to the distortion of the straight line motion of the air-current. When, however, the obstacle has got a suitably curved shape as in Fig. 2(*c*), the air or fluid passes over and behind the body in unbroken smooth lines termed streamlines, and the obstacle giving rise to a definite streamline pattern is usually called a *streamline body*.

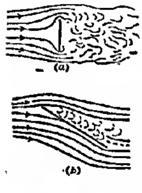


Fig. 2—Effect of shape.

On comparing flow past a rough obstacle with that past a streamline body, we notice that in the former case large portions of the fluid spin around as if they were detached portions of the fluid. These isolated portions of the fluid are called *eddies*. A ball thrown in air and moving with spin will require more energy than when it is moving without spin. An eddy differs from a fluid moving in a streamline manner in the same way as a ball moving with or without spin in air. For an aeroplane having a rough shape, the energy of the spining fluid of the eddies must ultimately be derived from the engine, and so, such bodies will tend to slow down the motion and produce inefficient flight. *Streamline shapes are, therefore necessary for the efficiency of the aircraft.*

height will be observed in the narrowest part of the tube, where the speed of the air is also greatest. But the liquid level rises in the manometers due to reduction of pressure, we have this somewhat unexpected fact that the *pressure of the air falls when its speed increases.*

As the change of potential energy is negligible, the increase of speed (and hence of kinetic energy) is obtained by losing some of its pressure energy. Hence it illustrates the Bernoulli's theorem stated above. This venturi-effect, as it is called, is employed in many scientific devices in order to produce a reduced pressure.

CHAPTER III

AEROFOILS (OR WINGS): FLAT AND CAMBERED SURFACES: LIFT AND DRAG

9. Principles of Flight:—Let us proceed now to consider the question of why it is that an aeroplane is capable of flying through air. In order that an heavier-than-air machine can fly, there must be some means of forcing the air downwards so as to provide the equal and opposite reaction which will lift the weight of the machine, and in the conventional aeroplane this is provided for by wings, which are inclined at a small angle to the direction of motion. The necessary force driving the machine forward is obtained by the thrust of an airscrew. The wings (or aerofoils) are always slightly curved, but let us consider the case of a flat plate first, as in the original attempts of flight flat surfaces were used.

10. Flat Plate inclined to Air Current:—For simplicity we suppose that a flat plate AB is at rest and that the air-current flows past the plate AB which is inclined at an angle α to the direction of the airflow (Fig. 4). In Fig. 2(*b*), it has been found that in this position both sides of the plate are affected by the air-current, due to which pressure of air on the top surface is decreased while that underneath the plate is increased. Each of these pressure-changes produces

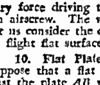


Fig. 4

forces R_1 and R_2 acting upwards on the plate giving rise to a resultant force R, which is practically normal to the surface when the angle α is small. The force R_1 arising from the decrease of pressure pulls the plate up, and the force R_2 arising from the increase of pressure pushes the plate up (*vide* Venturi-Tube expt). The force R, called the **total reaction**, can be resolved into two components at right angles—one horizontal, D, and other vertical, L, acting upwards. The component, L called the *lift*, balances the weight of the plate, and the

component D, called the *drag*, resists the motion through the air. Obviously, the L component which supplies the lifting force to the plane is of profound importance. For equilibrium the L component must equalise the weight W of the plate. If W is greater than L, the plate will fall, and if less, it will rise.

Actually in practice the flat surface is inefficient as a means of lifting because of the total resistance offered, and therefore the total engine power which has to be employed, is very high in comparison with the lift obtained, from it.

11. Aeroplane and Kite:—The flight of an aeroplane is much like that of a kite floating in air (*vide* Art. 58, Part I). In the case of the aeroplane the rush of air past the wings is due to the motion of the aeroplane itself through the air rather than to a wind, as is the case with the kite. The tension of the kite string here corresponds to the forward thrust of the propeller. The L component balances the weight of the machine, while, for equilibrium, the D component must be counterbalanced by an equal force which is obtained by the action of the screw-propeller. On increasing the propeller speed, the forward thrust and R increase. Consequently, the L component becomes greater than the weight and so the aeroplane rises. It should be noted that the air-pressure depends only on the rate and direction with which the air and the body meet, and the result is the same whether the body moves to meet the air, or the body remains still and the air flows against it. Obviously, the greater the velocity with which the aeroplane and air meet the greater will be the air-pressure.

12. Cambered Surface:—The advantage of using a suitably curved (or *cambered* as it is termed) surface, insead of a flat one, was soon discovered by which a much greater lift, especially when compared with the drag, could be produced. In this, the eddy disturbances due to the distortion of the streamlines can be minimised and so the efficiency of the system can be increased. Thus the modern aerofoil has both the top and bottom surfaces cambered. The top camber is greater than that of the bottom surface as due to this the lift component has an appreciably higher value over a wider range of the angle of incidence. The additional advantage of the curved surface is that it automatically provides a certain amount of thickness which is necessary for structural strength. The thickness is expressed as a percentage of the chord and for general use the best top surface camber is about **11** per cent. of the chord, while for high speed it should be only **7** or **8** per cent.

AIRFLOW AND PRESSURE OVER AEROFOIL

13. Some Definitions:—A transverse section of a wing (or aerofoil, as it is called) of an aircraft is shown in Fig. 5, where along the front of the aerofoil at A is the **leading edge**, and at the rear at B is the **trailing edge**.

The line AB joining the centres of curvature of the leading and trailing edges is called the **Chord.**

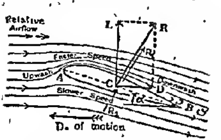


Fig. 5—Airflow over Aerofoil.

Camber is the curvature of the aerofoil of both the top and bottom surfaces. The greatest height of the top or the bottom surface, when divided by the chord length, is called the *camber* of the respective surface. Camber decides thickness of the aerofoil.

Angle of Attack is the angle between the chord line and the relative airflow, which is the direction of the airflow with reference to the aerofoil.

[**N.B.**—The angle of attack is often referred to as the angle of **incidence,** but it is better not to use this term in order to avoid confusion with the Rigger's angle of incidence, which is the angle between the chord line and some fixed horizontal data lines in the aeroplane. For a given aeroplane this angle is fixed whereas the angle of attack may alter during flight.]

The total length of the aerofoil perpendicular to the section is called the **span;** and the ratio of the span to the chord is called the **aspect ratio.**

14. Airflow past an Aerofoil:—Experiments show the following results when a typical aerofoil moves through air at a small angle of attack (*vide* Fig. 6)—(*a*) A slight upward deflection, called *upwash*, occurs in front; and (*b*) a considerable downward deflection, called *downwash*, occurs behind the wing. The downwash is important as it affects the direction of the air striking the tail plane or other parts of the aeroplane in the rear of the main plane; (*c*) A smooth streamline airflow takes place over the top and bottom surfaces; (*d*) The streamlines are closer above the top surface than over the bottom; (*e*) Above the top surface the speed of airflow is increased and below the bottom surface it is decreased; (*f*) The pressure of the air above the wing is reduced below the normal atmospheric pressure due to the increased speed of the airflow; and (*g*) the pressure below the wing is increased due to the decreased speed.

Though the facts stated in (*f*) and (*g*) appear to be puzzling at first, it can be explained by the Venturi-Tube experiment. Here the upper surface is somewhat similar in shape to the lower half of the Venturi-Tube and the closer streamlines above the highest part of the camber resemble those passing through the neck of the Venturi-Tube.

As stated in the case of a flat plate the decrease of pressure above the wing surface produces a force R_1—which is an important part of the total force—pulling the wing up and the increase of pressure below the wing gives rise to a force R_2 pushing the wing up. These two upward forces give us the resultant force R acting approximately at right angles to the chord line. But the decrease of pressure above the wing surface is more important, for to this is due the greater part of the lift force. Roughly about two-thirds of the total load on the wing may be attributed to this decrease of pressure while about one-third may account for the increase of pressure on the lower surface.

[Note.— It should be noted here that the common idea is that the airflow moving away from the upper surface of the wing causes a partial vacuum and thus provides a lift force, but this is wrong. In fact, the greater will be the increase of speed as the air is drawn closer on to the upper surface of the wing, and by the consequent reduction of pressure the upward force produced will be greater.].

15. The Centre of Pressure:—The point in the chord line through which the total force R may be considered to act is known as the centre of pressure. It has no fixed position but varies according to the speed and the angle of attack.

(a) Distribution of Pressure over an Aerofoil.—The distribution of pressure over the surface of an aerofoil has been experimentally determined, and its study is of great importance. The method consists in distributing a number of glass tubes, which are placed parallel to the direction of motion, over the upper and lower surfaces of the aerofoil. These are connected to a manometer, and different pressures are ascertained. Fig. 6 shows the pressure distribution over an aerofoil at an angle of attack of 5°, from which the following observations are made:—(a) The pressure is not evenly distributed, both the decreased pressures on the top surface and the increased pressures on the lower surface being most marked over the front portion of the aerofoil; (b) the greatest pressure-decrease (and hence the largest forces) occur on the top surface, and it is near the leading edge and over the highest part of the camber; (c) the decrease in pressure over the top surface is greater than the increase on the lower surface.

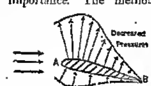


Fig. 6—Pressure Distribution over an aerofoil.

From this it is seen that the shape of the top surface is of great importance. It is the top surface, which by means of its decreased pressures, provides the greater part of the lift, and, at some angles of attack, this decrease of pressure on the top surface gives us as much as four-fifths of the lift.

(b) Movement of Centre of Pressure.—Experiments show that the distribution of pressure over the aerofoil changes considerably

with the change of the angle of attack and consequently the centre of pressure (C.P.) moves. The position of C.P. is usually defined as a certain proportion of chord behind the leading edge. The movement of C.P. is an inconvenient property of the aerofoil, for unless its centre of gravity (C.G.) and C.P. coincide, there will be a turning effect about C.G. To understand this let us suppose that for a certain angle of attack the C.G. and C.P. coincide. Now, when the angle of attack of increases there will be a forward movement of C.P., and so there will be a turning movement about C.G. equal to Rx, where R is the total wind thrust and x the distance between C.P. and C.G. This movement will rotate the aerofoil and still further increase the angle of attack and thus the equilibrium will be disturbed.

In any case, large movements of C.P. will make the aeroplane difficult to control and so in a good aeroplane the movement of C.P. should be limited which is obtained by a suitable bi-convex cross-section or by increasing the aspect-ratio, for example, by tapering the wing.

16. Lift and Drag:—In practice the direction of motion of an aeroplane, is not always horizontal and so the L component is not always vertical. It is usual to split up the *total reaction* R into two components, L and D, *relative to the airflow*—the component L which is always perpendicular to the direction of the airflow (or motion) is called **lift**, and that parallel to the direction of the airflow is called **drag**, which is always opposite to the direction of motion. Lift is used to balance the weight of the aeroplane and keep it in the air in level flight. Other parts of the aeroplane as tailplane, elevator, etc., may provide further lift forces when desired. *Drag is the enemy of flight* and every effort must be made to reduce it to a minimum. Only in normal level-flight the lift is vertical and the drag horizontal, but if, in turning, the wings of an aeroplane assume a nearly vertical position, then the lift L is nearly horizontal. Lift is always perpendicular to the direction of motion and drag is always opposite to it.

17. Lift and Drag Formulae:—In Fig. 6, R is the resultant force on a transverse section of the wing of an aircraft whose angle of attack is α and whose velocity is V. We have already seen in Art. 6 that the total reaction (or resistance) $R=\dfrac{K\rho AV^2}{g}$ lb.-wt.

We have in Fig. 6, the lift component $L=R\cos\alpha$, and the drag $D=R\sin\alpha$, whence

$$L=K\cos\alpha\frac{\rho AV^2}{g}\text{ lb-wt}\quad(1),\quad\text{and}\quad D=K\sin\alpha\frac{\rho AV^2}{g}\text{ lb-wt.}\quad(2)$$

where ρ represents the air density (in lb. per cu. ft.), A the surface or plane area of the wing projected on the plane of the chord (in sq. ft.), V the velocity of air speed (in ft. per sec.), and g the acceleration of gravity ($=32.2$ ft. per sec.^{2}).

Since K is a constant for some given conditions in a machine, we may write the symbol K_L for $K \cos \alpha$ and K_D for $K \sin \alpha$, which are spoken of as **lift and drag co-efficients** respectively.

[Note.—(*a*) These symbols should not be confused with L and D which give the actual lift and drag in pounds-weight, and K_L and K_D are constants only. (*b*) The above relations are strictly true when α is small, for we are not justified in assuming that R is at right angles to AB for large angles of attack.]

Then, we have, $L = K_L \dfrac{\rho A V^2}{g}$; and $D = K_D \dfrac{\rho A V^2}{g}$; and hence, dividing one by the other, we get the important relation, $\dfrac{L}{D} = \dfrac{K_L}{K_D}$; and L/D is known as **lift-drag ratio.**

Note that when L is exactly equal to the weight W, of the aerofoil we get, for a normal horizontal flight, $W = \dfrac{K_L \rho A V^2}{g}$.

18. Factors affecting the Lift-Drag Ratio:—The factors affecting the *Lift-Drag* ratio are

(*i*) *The angle of attack.*—We get a maximum *Lift-Drag* at an angle of attack of about 4° (see Fig. 7).

(*ii*) *The airspeed.*—Both lift and drag are directly proportional to the square of air speed. Hence increase in airspeed will increase the lift and drag, other factors remaining the same.

(*iii*) *Increase in wing surface or plane area* (*i.e*, the area projected on to the plane of the chord).—This will increase the lift and drag when the plane is flying at the same speed and the same angle of attack in air of the same density. (In practice, however, the angle of attack rarely remains constant even for a very short time.)

(*iv*) *Increase in density of the air.*—V and α remaining the same, the increase in density will increase and the decrease of the density will decrease the lift and drag.

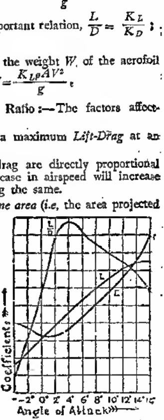


Fig. 7—Lift and Drag Curves.

19. Lift and Drag Curves:—In order to get some idea of what happens when the angles of attack of a typical aeroplane wing is gradually altered, we shall consider the lift and drag curves shown in Fig. 7. Considering the curve drawn with the lift co-efficient L and the angle of attack, it will be seen that there is a definite lift at 0°, and that the lift increases steadily between

0° and 12° where the graph is practically a straight line. The maximum value reaches at about 15° after which the lift begins to decrease rapidly. This rapid falling off is called *stalling*, and the angle of attack at which the lift reaches a maximum value is known as the **stalling angle.**

Now for the *Drag Curve* we find that its value is always positive. It is least at about 0°. The increase of drag up to the stalling angle is not very rapid, but after it the increase becomes more and more rapid.

20. Lift-Drag Ratio Curve :—We have considered lift and drag separately, but it should be realised that the ratio L/D under varying conditions is of great importance. We know that an aeroplane travelling through the air must employ power to create a propeller-thrust in order to overcome the drag of the aerofoils, and so it is desirable to require as little power as possible for a given lift or, in other words, for the sake of efficiency we want as much lift, but as little drag, as possible for our aerofoil. In fact, we want the highest possible values for the L/D ratio for any given working range. From Fig. 7, we find that the lift is highest at about 15° and the least drag we get at about 0°, so at neither of these angles we really get the drag, or the **best lift-drag ratio.** This shows the importance of the curve showing L/D ratio of the aerofoil against the angle of attack. Here we find that the greatest value of L/D occurs at about 3° or 4° at which angles the lift is about 20 times the drag. Thus, it is seen that an ideal aerofoil must be moving at an angle of attack of about 3° or 4° when it will give its best all-round result. This angle at which the best result is obtained is sometimes called the **optimum angle.**

[Note.—In Fig. 7, the values of lift and drag co-efficients are taken instead of the total lift and drag as the former will be practically independent of the air-density, the scale of the aerofoil, and the velocity employed, whereas the total lift and drag will depend on the actual conditions at the time of the experiment.]

21. Stalling :—At values greater than that corresponding to the maximum lift, the lift falls off rapidly and this rapid falling off is called **stalling**, when the aeroplane is said to be stalled. Stalling is accompanied by a loss of lift as well as much increase in drag. The airflow no longer shows a smooth streamline flow and it finally changes into a turbulent flow. It is extremely dangerous if stalling happens at the time when the aeroplane is very near the ground.

One of the devices in reducing the risk of stalling is the **Handley Page slot**, which is shaped rather like a wing and fitted on the leading edge of the main wing. On moving forward the slot at a time when the angle of attack of the aerofoil is increased, a smaller angle of attack is presented to the on-coming air causing an increasing airflow over the wing surface, and the lift is restored.

22. Aerofoil Characteristics :—The lift and drag co-efficients of an aerofoil depends on the shape of the aerofoil, and they will

change with changes in the angle of attack. The result of experiments on aerofoils can be easily demonstrated by drawing graphs to show how L co-efficient, D co-efficient, and L/D ratio alter with changes in the angle of attack. These three may be called the *characteristics of the aerofoil.*

23. The Ideal Aerofoil:—The characteristics of the ideal aerofoil are given by the curves in Fig. 7. Thus the ideal aerofoil should have

(1) *A high maximum lift co-efficient* in order to lower the landing speed for the safety of the plane. The higher the lift-co-efficient of the aeroplane, the lower will be its landing speed and greater will be the safety of the plane.

(2) *A low minimum drag co-efficient*—not only at a certain angle of attack, but it should remain low over a large range of angles. Thus the aeroplane will have a low resistance and will be able to attain high speed.

(3) *A High Lift-Drag ratio* for the sake of efficiency, good weight-carrying capacity for a small expenditure of engine-power and so less expense.

(4) *A small movement of centre of pressure* to improve stability.

(a) Compromises.—In actual practice, however, we find that no aerofoil will meet *all* the requirements. Therefore some sort of compromise is made just as in the case of a good hydrostatic balance. We cannot get an aeroplane which will serve all our different purposes and the shape of the aeroplane is the first, and perhaps the greatest compromise to be made. So different degrees of cambering is made according to the different purposes the aeroplane is desired to serve. For instance, for high speed the *top surface* camber should be about 7 or 8 per cent. of the chord while for general use it should be about 10 or 11 per cent. of the chord.

Both lift and drag are increased by increasing the camber of the upper surface. The alterations in the camber of the *bottom surface* of the aerofoil have a much smaller effect. Modern aerofoils have their lower surface flat or slightly convex.

24. Normal Horizontal Flight:—Without taking into account the forces on the tail unit, an aeroplane, when flying straight and level —which we refer as normal horizontal flight—may be said to be under the influence of the four main forces:

(1) The lift L of the main planes acting vertically upwards through the centre of pressure.

(2) The weight W of the aeroplane acting vertically downwards through the centre of gravity.

(3) The thrust T of the propeller airscrew pulling horizontally forward along the propeller shaft.

(4) The drag D acting horizontally backward. This is the total drag on the aircraft consisting of the drag of the aerofoils and also of the remaining parts of the aeroplane.

25. Conditions of Equilibrium :—In the ideal case when the aeroplane is flying level at a steady speed in a fixed direction, that is to say, the main condition of equilibrium of those four forces, which must obey the simple laws of mechanics, is that all the forces would act, through the same point. Then we have,

(i) $L = W$. This condition will keep the aeroplane at a constant height. If $L > W$ (this is secured by increasing airspeed by increasing engine power), the aeroplane will ascent, and if $L < W$ the aeroplane will descent.

(ii) $T = D$. This condition will keep the aeroplane moving at a constant speed. If $T > D$ the aeroplane will move with an acceleration and if $T < D$ there will be retardation. In practice, however, these forces are never constant owing to varying conditions, *e.g.* the weight of the aircraft does not remain constant in value, L is not constant as the angle of attack may change due to wing-thrust, the position of C.G. is not constant. Due to these difficulties the ideal arrangement of the forces is not possible.

Now when the size and position of forces change, the turning effect of the aircraft is controlled by the pilot by (i) control column movement (discussed later on) and (ii) mainly by adjustable tail plane.

CHAPTER IV

AEROPLANES AND THEIR CONTROLS : MANOEUVRES

26. Component Parts of the Aeroplane :—We have already mentioned about some parts of an aeroplane and especially have dealt with one of its main parts, *i.e.* the wings or aerofoils. Let us state here that an aeroplane mainly consists of the following parts:—

(*a*) *Fuselage* ; (*b*) *Wings* or *Aerofoils* ; (*c*) *Propeller* or *Airscrew* ; (*d*) *Tail plane* ; (*e*) *Aileron* ; (*f*) *Elevator* ; (*g*) *Rudder and Fin.*

The Fuselage.—The main body of the machine is referred to as the fuselage, which must be large enough to contain engine, tanks, pilot, bombs, goods, passengers, etc. that the machine has to carry.

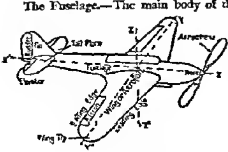


Fig. 8—Aeroplane.

Tail plane.— It is a small plant fitted at a considerable distance behind the main plane in order to provide the upward or downward forces necessary to contract the unruly action of the four main forces mentioned in Art. 24.

27. The Propeller or Airscrew:—The theory of airscrew is too advanced to be considered here, but a general idea of the work of an airscrew will be given here.

A propeller, also called an airscrew, is much like an ordinary electric fan in appearance, but while a fan sucks air from behind and throws it forward, and airscrew sucks air from the front and throws it backward. The result is that due to reaction the fan tends to move backwards, while the airscrew is thrust forward, and thus pulls the aeroplane along with it. The thrust of a propeller is the force with which it drives the air backwards or urges the aeroplane forwards. The propeller is the means by which the power of the engine, which rotates it, is transformed into a forward thrust, and thus gives the aeroplane a translational velocity. Thus, the aeroplane forces its way through the air by means of propellers rotating in a vertical plane and we may say in effect that an airscrew screws itself through the air pushing or pulling the aeroplane to which it is attached. The propellers are situated either in front of the body of the machine, when it will cause tension in the airscrew shaft and will thus pull the aeroplane forward (in which case the aeroplane is called a **tractor**); or in the rear of the body when it will push the plain forward (in which case it is called a **pusher**). Airscrews vary in the number of blades from two to four, but the two-bladed variety is the easiest to manufacture and slightly more efficient. The shape of each part of an airscrew blade, taken in a direction at right angles to its length, is found to be similar to that of an aerofoil.

The diagram (*A, B, C,* in Fig. 9) shows several cross-sections taken at various distances from the centre. The airscrew also derives similar forces from the airflow to those giving lift and drag in the case of wings but owing to variations in camber, chord and speed, the lift and drag components increase and decrease from section to section. The airscrew may be considered to be exactly like an aeroplane wings, but that, instead of moving in a straight line and supporting the aeroplane, the airscrew moves in a spiral path and produces the thrust which overcomes the drag of the aeroplane. Due to their different functions the plane form or an airscrew blade differs from that of a wing; and the airscrew blade is twisted so that the angle to the shaft of the propeller is greater at the base than at the tip, while the angle of the wing is almost the same throughout. Thus the forward thrust of the airscrew corresponds to the upward lift of the aerofoil, and drag in this case is represented by the resistance of the air to the rotatory motion of the airscrew.

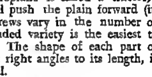


Fig. 9—Air-screw Torque.

The total airscrew thrust is the sum of the thrusts on each blade section, and it is the force which pulls the aeroplane through the air. The total drag on all the blade sections constitutes a couple—known as **airscrew torque**—which resists the rotatory motion of the airscrew (Fig. 9) and opposes the **engine torque** (or *the turning moment*) applied to the airscrew shaft by the engine. The airscrew torque has to be overcome by the engine torque. This is analogus to the thrust and drag in the case of an aeroplane.

(a) **Pitch.**—The airscrew is a screw which screws its way through the air in the same way as an ordinary screw does through wood but some important differences are to be noted. In the case of an ordinary screw the distance moved forward in one revolution is a fixed quantity and is called the *pitch* of the screw, the value of which depends on its geometric dimensions, and is usually called the *geometric pitch*. But, in the case of the airscrew, the distance moved forward in one revolution (called the *advance per revolution*) is not a fixed quantity as it depends entirely on the forward speed of the aeroplane. Another important difference between the airscrew and the ordinary screw is that the airscrew has no actual grip on the air comparable to an ordinary screw in wood and there is a certain amount of slip so that the distance moved forward is less than the geometric pitch. This distance is not also constant as it varies with the speed of the aeroplane. Thus the *slip* of a screw is the difference between the distances it should travel theoretically and its actual progress.

(b) **Pitch Angle.**—We should all know that the twisted appearance of the airscrew blades is not without any meaning—rather it is the product of highly skilful design. The sections of the blade near the tip are moving with a much greater velocity than those near the root, and so most of the thrust is produced by the portions near the tip. For this reason the *pitch* (or *blade*) *angle* is not the same throughout the airscrew blade in order that every part of the airscrew may move the same distance forward during one revolution of it. Other things being equal, a large propeller moving comparatively slowly gives more thrust than a small one driven at high speed. The *pitch* (or *blade*) *angle* is the angle which the chord of any given blade section makes with the horizontal plane when the airscrew is laid flat on this plane, its axis being vertical.

The *Experimental Mean Pitch* is the distance the airscrew moves forward in one revolution when the thrust is zero, and when the thrust and efficiency of the airscrew is a maximum, the pitch is called the *Effective Pitch*.

(c) **Efficiency**:—The efficiency of an airscrew is the ratio of the useful work done by it to the work put into it by the engine. In actual flight for the *same* rotational speed of the airscrew, a forward motion —which means some useful work done—may be attained at which each blade section meets the airflow at the angle of attack of about 3°.

which is the most efficient angle of attack for an acrofoil having its maximum lift-drag ratio. So here the ratio of the airscrew thrust to the torque is a maximum; and so at this speed the screw has maximum efficiency.

28. Fixed Pitch and Variable Pitch Airscrews:—It has been seen that only at a particular speed of the aircraft a fixed-pitch airscrew has got its maximum efficiency at a given rotational speed, but in practice, the actual speed of an aircraft varies over a more or less wide range. An airscrew whose pitch can be varied by the pilot, when in flight, is called a **variable pitch** airscrew the mechanism of which is rather complicated, though this is very effective for all conditions of flight. But whether a variable pitch airscrew is advisable or not depends on the speed-range of the aeroplane. For a high speed-range, a variable pitch airscrew is essential, and when the maximum speed is relatively low, a fixed-pitch airscrew will work quite well. With this type of airscrew an aeroplane might be brought home safely when in danger, which would have been impossible with the fixed-pitch type. So for a modern machine of high-speed range a V. P. airscrew is essential.

STABILITY AND BALANCE

29. Stability and Balance:—If an aeroplane, when disturbed, tends to return to its original position, it is said to be **stable** and the **stability** of the machine means its capacity to return to some particular condition of flight after it is slightly disturbed from that condition.

[Note.—Stability should not be confused with balance. Suppose an aeroplane flies with one wing more dipping than the other and it may, when disturbed from this state, return to its former position. Such an aeroplane is not unstable but only out of its proper balance.]

30. *Stability* :—*An aeroplane* may rotate about three axes all mutually at right angles to each other and all passing through the centre of gravity of the aircraft. These axes are as follows: *The Longitudinal* (or *rolling*) *axis XOX'* running from nose to tail; *the Lateral* (or *pitching*) *axis YOY'* in the same horizontal plane, and *the Normal* (or *yawning*) *axis ZOZ'*.

(1) The rotatory motion of the aeroplane about the lateral axis is called **pitching** caused mainly by a wind-gust resulting in the nose rising or depressing. During pitching the longitudinal axis moves in a vertical plane.

The capacity to correct pitching is defined as *Longitudinal stability.*

(2) Any rotatory motion of the aeroplane about the longitudinal axis is called rolling, resulting in one wing rising and other dropping. The lateral axis moves in a vertical plane during rolling. The ability of the aeroplane to correct rolling is called *Lateral stability.*

(3) The rotatory motion about the normal axis is called yawing. It results in the nose and tail being deflected to one side, and in this both axes move. The capacity to correct yawing is called *Directional stability.*

(a) **Longitudinal Stability.**—This is achieved by the tail plane by setting it at an angle less than that of the main plane. Suppose that due to wind-gust the nose of the machine is thrown up. The tail plane is then turned so that it presents an angle of attack less than that of the main plane and thus a force is obtained on the tail plane in such a direction as is necessary to counteract the movement of C. P. of the main plane, which is detrimental to stability, and thus to bring the machine to equilibrium position. Another condition for longitudinal stability is that the position of the centre of gravity of the aeroplane must not be too far back.

(b) **Lateral Stability.**—During normal flight the lift on the wings is vertical, and equal and opposite to the weight, but when a roll takes place one wing drops and the other goes up. In this position the lift is inclined and is no longer in the same straight line as the weight. As a result of these two non-parallel forces, the machine cannot be in equilibrium and moves bodily sideways, called side slip, in the direction of the lower wing. To overcome this lateral instability a small positive *dihedral angle* is introduced between the two wings by setting the wings to be inclined upwards by a small angle to the lateral axis. Now the vertical component of the lift on the lower wing is increased, the angle of attack being greater, and that on the other side is decreased and thus a couple is introduced which brings the aeroplane to the normal position. Lateral stability depends also on the position of the centre of gravity of the aeroplane.

[The dihedral angle is the angle between each plane and the horizontal for the normal position. It is positive when the plane is sloping upwards and negative when sloping downwards.]

(c) **Directional Stability.**—This is secured by fitting a small aerofoil vertically at the centre of the tail plane. This acts in a way similar to that of the tail plane and produces a force which opposes any tendency to spin round the normal axis. This small aerofoil is known as the fin, which is the most important factor, for, both by its surface area and position, a correcting turning moment is obtained from it.

Lateral and directional stability are inter-relative. A roll is followed by a yaw and *vice versa*, and the study of the two cannot be separated.

31. Control:—It is no doubt necessary that an aeroplane should be stable but that is not enough. It is also necessary to control the machine to force it to take any desired position, or to correct any tendency of the machine to wander from any desired path. When the pilot desires to bring about such changes he has at his disposal three

movable control surfaces which are operated from the cockpit by means of cables or rods: (*a*) *the elevator*, (*b*) *aileron*, and (*c*) *rudder*.

(a) Longitudinal Control and the Elevators.— Longitudinal control is the control of pitching and is obtained by the elevators which are flaps hinged behind the tail plane by which the angle at which the machine is flying can be altered and thus the nose of the machine can be raised or lowered as desired. Elevators are operated by means of the control column (also called *Joystick*) situated in front of the pilot's seat. By pulling the Joystick backwards, the elevators are raised by which action the aeroplane begins to ascend, and the opposite action takes place by moving the Joystick forward.

(b) Lateral Control and the Ailerons.—Lateral control is the control of rolling of the lateral axis and is obtained by the ailerons which are flaps hinged at the rear of the main wings near each wing-tip. They are connected together so that when one flap is depressed, the other on the opposite wing-tip is raised. When a machine has been tilted through an angle laterally by an wind-gust, the pilot rights the aeroplane by depressing the ailerons. Thus by the aid of the ailerons the aeroplane may be *banked*, that is, the machine may fly with one wing lower than the other. The ailerons are operated by moving the control column by the hand or sometimes by a control wheel like the steering wheel of a motor car.

The linkages of the control surfaces are so designed that the controls may be moved instinctively from the pilot's cockpit when any manoeuvre is desired.

The elevators and ailerons are moved by a single control column in the pilot's cockpit. By pushing the control column to the left, the right-hand aileron is lowered and the right-hand wing is lifted up; while at the same time the left-hand aileron is raised and the left-wing dips down. Thus the whole aeroplane is banked to the left. This control is required as instinctive.

(c) Directional Control and the Rudder.—It is the control of yawing or rotation about the normal axis, and is obtained by the rudder, which is a vertical flap hinged on to the rear of the fin. This is operated by a *rudder bar* in the cockpit and worked by the pilot's feet. On pressing the right foot forward, the rear of the rudder will be moved to the right and the aeroplane will turn to the right and so on. The function of the rudder is to keep the machine in its correct course, and it is also used in conjunction with the ailerons for *turning* the machine.

In general, the movement by the rudder will give rise to a side force on the fin, movement of the elevator will produce a force on the tail plane while the movement of the aileron increases or reduces the lift on the wing, as the aileron is pulled down or pulled up.

It should be noted that in each of the above cases the control surfaces are placed as far as possible from the centre of gravity of the machine so as to provide sufficient leverage to alter its position.

(d) **Engine.**—Besides the above control units, the engine is also considered as another unit, the primary function of which, from the point view of control, is to vary the height at which the machine is flying for a given angle of attack, speed, etc.

32. Stability and Control :—The difference between stability and control should be clearly noted. Stabilising devices, such as the tail plane and fin, restore the aeroplane to its original path of flight after a disturbance has occurred while, on the other hand, the pilot uses the control surfaces, such as the elevators, etc. to manoeuvre the machine into any desired position; but change of altitude will be resisted by the inherent stabilising devices. The control surfaces should therefore be effective enough, to overcome the action of the stabilising devices.

Stability and Trim.—When an aeroplane is in trim it will continue to fly without changes of direction or altitude, even when the pilot takes his hand off the controls provided it has the necessary stability. But if the aeroplane is not in trim it will either go slowly up or down, and this want of trim can be corrected by the use of small auxiliary flaps, called *trimming tabs*—hinged to the trailing edge.

33. Manoeuvres :—The various manoeuvres which an aeroplane may be required to perform are given below:

(1) Take-off and Landing.—In *take-off*, the throttle of the engine is opened, and the machine moves over the ground gaining speed, while the pilot depresses the elevators, thus raising the tail. The machine then rises up attaining the minimum speed to be sustained in air.

Landing is done by bringing down the speed of the aircraft until it is brought into contact with the ground. Landing may be slow or fast.

(2) Gliding.—In this the engine is throttled down until the speed of the engine is just sufficient to keep the engine going. Now the thrust T disappears and the aircraft must be kept in equilibrium by the forces of lift, drag, and weight only, *i.e.* the total reaction, or the resultant of the lift and drag, must be exactly equal and opposite to the weight. The angle between the path of the glide and the horizontal is called the *gliding angle* which is the same as the angle between the lift and the total reaction.

(3) Climbing.—In order to make a climb, the pilot holds the control column backward to have the angle of attack between the normal and stalling values.

(4) Banking.—Banking is accomplished by moving the ailerons over, so that one wing drops and the other rises. In this the lift force, in addition to lifting the machine, supplies a component towards the centre of the turn, so that a large force is obtained for pulling the machine into a circular path and settling it down to the steady condition.

Besides these, other different manoeuvres done by expert pilots are as follows: (5) *Side slip* [vide Art. 30(*b*)]; (6) *Ceiling* [vide Art. 33]; (7) *Loop*; (8) *Spin*; (9) *Roll*; (10) *Zoom*; (11) *Nose-dive.*

34. High Altitude Flying:—It has already been pointed out that with the increase of altitude the density, pressure, and temperature of the atmosphere all decrease, and these cause important modifications in the forces acting on the aircraft. The effect of decrease of the density of air may be summarised as follows: (*a*) Decrease in lift and drag; (*b*) Falling-off in the power of the engine; (*c*) Decrease in air-screw thrust.

(*a*) Lift and drag depend on the density of air. At higher altitude the density of air is considerably less than that at ground level and so the lift can no longer balance the weight of the aircraft. It is necessary, therefore, either to increase the speed of the aircraft, leaving the angle of attack the same, to obtain sufficient lift to balance the weight, or to increase the angle of attack (*vide* Art. 18), which in turn will increase the drag.

There is, however, a limit to the possible increase of speed as it depends on the power of the engine which is also limited, and further there is also a limit to the increase of the angle of attack, as, we know, when this is made too great, the lift will decrease instead of continuing to increase, or, in other words, there will be stalling of the machine.

(*b*) As the pressure of air decreases with height, the weight of petrol-air mixture taken into the cylinder of the engine for combustion is reduced and so there is a considerable falling-off in the power on the engine. This may be remedied to a certain extent by *supercharging*, i.e. by forcing the mixture into the cylinder with a pump. But ultimately the atmospheric pressure becomes so small that, with all existing engines, there is a height at which the power begins to fall off in spite of the supercharger, and we find that sooner or later a height is reached which cannot be exceeded. Thus the maximum height to which an aeroplane can fly depending on the construction, design, and, weight and engine power, is called the **ceiling** of the aeroplane.

(*c*) In rarefied air the airscrew-thrust is sufficiently reduced even when the engine and propeller revolutions per minute are sufficiently increased. In such cases variable pitch airscrew are usually employed to compensate for the loss to some extent.

In the stratosphere, the temperature is nearly—60°*F*., and at this low temperature, all metal joints become leaky, rubber becomes brittle, pipe lines freeze, and so on, unless special precautions are taken.

Again, the low temperature and low pressure at high altitudes affect the comfort of the pilot and other passengers. For the low temperature, heavy warm clothing (woolen, preferably leather cloth) garments are essential and the cabin should be electrically heated. For oxygen deficiency in the lungs, oxygen is supplied from cylinders.

But at sufficiently high altitudes, the pressure in the lungs becomes so low that the oxygen deficiency may finally endanger life. The cabin requires, therefore, to be properly sealed and pressurised to maintain the standard pressure inside.

At height more than 10,000 ft. symptoms such as drowsiness, breathlessness, muscular weakness, etc. become pronounced, and at about 25,000 ft. it become dangerous. Apparatus for the artificial administration of oxygen is always necessary for high altitude flights, and with its aid flying up to heights of about 35,000 ft. may be safely undertaken.

Besides this, discomfort or pain in the ear is often felt by the pilot due to changing atmospheric pressure. Against these disadvantages, one should remember that the weather conditions remain fairly constant at high altitudes. So high altitude flight is smooth and safe. For these reasons it is popular from the commercial point of view.

Questions

1. What are meant by 'stream-line' flow and 'stream-lined' body ? What is the importance of [illegible]

2. Write sh[illegible]
(c) Parachutes ; [illegible]
3. What is [illegible]
determine the efficiency of it ? [illegible] (Pat. 1944)
Explain fully how the wings of an aeroplane support it high up in the air. Indicate the forces that act on the machine. (Pat. 1939)
4. Write a note on the flight of an aeroplane indicating the part played by the more important portions of it (Pat. 1942)
5. Describe what happens when a flat plate moves through air, and explain why aeroplane parts are stream-line shaped. (Pat. 1939 ; *cf* '44)
6. Explain what is meant by stream-line flow. Describe an experiment to demonstrate the deformation of stream-lines by an obstacle.
Discuss the flow of air past a flat plate moving through air with a high velocity with its plane inclined at a small angle to the direction of motion. Show how a lifting force is produced on the plate and explain how it varies with the angle of incidence of the plate. (Pat. 1945)
7. What are cambered wings in an aeroplane ? Explain their action Also explain with neat diagrams the actions of the tail, elevator, fin, and rudder. (Pat. 1938, '49)

[illegible]

ailerons in an aeroplane ? Is there any limit to the height to which an aeroplane can ascend ? Give reasons. (Pat. 1942 ; *cf* Utkal, 1948)
11. What is a stream-lined body ? Describe the structure of an aeroplane wing and discuss the factors upon which the lifting efficiency depends (Utkal, 1949)
12. Write notes on : (*a*) stream-line and turbulent flow, (*b*) lift and drag, (*c*) aerofoil, and (*d*) air-screw. (Pat. 1953)
13. How do you get the 'lift' that supports an airplane in the air, and what is the corresponding 'wing-drag' ?

Define coefficients of 'lift' and 'drag,' and prove that in horizontal flight (when lift must be equal to the weight of the machine),

$$V = \sqrt{\frac{W}{dsk_L}}, \text{ where } V \text{ is the velocity of the machine,}$$

s the area of the wing, k the coefficient of lift, and d the density of air supposed uniform. (Pat. 1940)

14. Explain what you understand by 'lift' and 'drag'. Illustrate graphically how the ratio of the two varies with the angle of incidence of the aero-foil. Explain the action of an air-screw. (Pat. 1955)

15. Write short notes on the variations of 'lift' and 'drag' with the angles of incidence. (Pat. 1953)

16. A stream-line body having a frontal area of 1 sq. ft. moves through air with a speed of 180 m.p.h. Calculate the 'drag' on the body assuming the value of 'drag coefficient' to be 0·04 and density of air 0·056 lb. per cu. ft.

[*Ans.* 2·42 lb.-wt.]

17. Define 'Centre of Pressure'. How does the C.P. of an aerofoil move with the increase of the angle of attack from 0° to 20° ?

18. What are the factors on which the 'lift and drag' of an aircraft depend ?

19. Criticise the following statements :—(*a*) 'Lift' increases as the angle of attack of the wing increases ; (*b*) lift is always vertical ; (*c*) 'lift and drag' are affected only by air speed and angle of attack.

20. Draw a neat sketch of an aeroplane showing its essential parts and explain fully the control system in it. (Pat. 1943)

21. Draw a sketch showing the four principal forces acting on an aeroplane in normal horizontal flight.

22. What is an 'air-screw' ? Explain how it gives the forward motion to an aeroplane. (Bihar, 1956 ; Pat. 1939, '53)

23. Write a short note on the air-screw and explain clearly how it propels an aeroplane through air. (Utkal, 1952)

24. Describe the parts of an aeroplane which ensure its stability in all possible modes. Illustrate, by neat sketches, the mechanisms to control its motion in various directions and indicate how the pilot manipulates them in taking a turn. (Pat. 1941 ; *cf.* '44)

25. How can you distinguish the difference between stability and control ? Name the axes about which pitching, rolling and yawing of an aircraft take place. Which control is used to produce each motion ?

26. Compare the flight of an aeroplane with that of a kite in air. Explain how an aeroplane maintains its stability during flight. (Bihar, 1953)

27. At a certain speed of normal horizontal flight of an aeroplane the ratio of its lift to drag is 7·5 to 1. What are the values of 'lift, thrust, and drag' when there is no force on the tail plane ? The weight of the aeroplane is 3500 lbs.

[*Ans.* lift = 3500 lb.-wt. ; thrust = 467 lb.-wt. ; drag = 467 lb.-wt.]

28. What is the true air speed on an aeroplane at a certain height weighing 60,000 lbs. and having a wing area of 1300 sq. ft. The 'lift' coefficient is 0·5 and the density of air at that height is 0·056 lb. per cu. ft.

Calculate also 'thrust' and 'drag' when the value of L/D ratio is 8.

[*Ans.* Speed = 248 m.p.h. ; thrust = 7500 lb.-wt. ; drag = 7500 lb.-wt.]

29. Write notes on *any four* of the following :—(*a*) stream-line flow, (*b*) Bernouilli's law, (*c*) stalling, (*d*) rolling, and (*e*) pitching. (Pat. 1953)

30. Can an aeroplane fly without wings ? Can an aeroplane fly in a vacuum ? Give reasons for you answer. (Utkal, 1952)

APPENDIX (B)

TRIGONOMETRICAL RATIOS

1. **Trigonometrical Ratios**:—*Let ABC be an acute angle represented by θ* (Fig 1). From any point D in AB drop a perpendicular DE on BC. It can be shown geometrically that whatever the point D be *taken on AB*, the ratio, DE/BD, i.e. $\frac{\text{perpendicular}}{\text{hypotenuse}}$, is constant and bears a fixed relation to the magnitude of the angle θ. *This ratio is* called the *sine* of θ. Similarly, the ratio BE/BD is also constant and is called the cosine of θ. So, we have the following *trigonometrical* ratios:

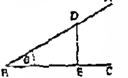


Fig 1

$$\frac{DE}{BD}=\frac{\text{perpendicular}}{\text{hypotenuse}}=\text{sine } \theta \text{ and is written, } \sin\theta;$$

$$\frac{BE}{BD}=\frac{\text{base}}{\text{hypotenuse}}=\text{cosine } \theta \text{ and is written, } \cos\theta;$$

$$\frac{DE}{BE}=\frac{\text{perpendicular}}{\text{base}}=\text{tangent } \theta \text{ and is written, } \tan\theta;$$

$$=\frac{DE/BD}{BE/BD}=\frac{\sin\theta}{\cos\theta}.$$

2. **Values of Trigonometrical Ratios**:—The values of these ratios can be geometrically deduced for angles of 0°, 30°, 45°, 60°, and 90° which are given below [*vide* Fig. 2 (*a* and *b*)]

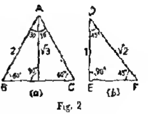


Fig. 2

The important values are tabulated below:

Angle	Sin	Cosine	Tangent
0°	0	1	0
30°	1/2	$\sqrt{3}/2$	$1/\sqrt{3}$
45°	$1/\sqrt{2}$	$1/\sqrt{2}$	1
60°	$\sqrt{3}/2$	1/2	$\sqrt{3}$
90°	1	0	∞

3. From the above table it should be noted that,

$\sin 0° = \cos 90° = 0$; $\sin 90° = \cos 0° = 1$;
$\sin 30° = \cos 60° = 1/2$; $\sin 60° = \cos 30° = \sqrt{3}/2$;
$\sin 45° = \cos 45° = 1/\sqrt{2}$.

4. The inverses of sine, cosine and tangent are cosecant, secant and cotangent respectively. This is, $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$; $\sec\theta = \frac{1}{\cos\theta}$; $\cot\theta = \frac{1}{\tan\theta}$.

5. It follows geometrically from Art. 1 that,

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sec^2\theta = 1 + \tan^2\theta$$
$$\operatorname{cosec}^2\theta = 1 + \cot^2\theta.$$

6. Signs of Trigonometrical Ratios :— According to the conventions followed:

(*i*) for all angles in the first quadrant, the signs of all ratios are positive;

(*ii*) for all angles in the second quadrant, only the sign of *sine* is positive and the signs of other ratios negative;

(*iii*) for all angles in the third quadrant, only the sign of *tan* is positive and the signs of other ratios negative;

(*iv*) for all angles in the fourth quadrant, only the sign of *cos* is positive and the signs of other ratios negative;

7. $\sin(A+B) = \sin A \cos B + \cos A \sin B.$
$\sin(A-B) = \sin A \cos B - \cos A \sin B.$
$\cos(A+B) = \cos A \cos B - \sin A \sin B.$
$\cos(A-B) = \cos A \cos B + \sin A \sin B.$

8. Solution of Triangle :—

(*i*) When two sides and the angle included between them are given, the third side and the other angles can be calculated from the *Cosine Law*.

Law of Cosines :— The square of any side of a triangle is equal to the sum of the squares of the other two sides *minus* twice their product into the cosine of the *included* angle. As for example, if A, B, C represent the three angles of a triangle and a, b, c the sides correspondingly opposite to them (Fig. 3),

$$c^2 = a^2 + b^2 - 2ab \cos C$$

or, $$\cos C = \frac{a^2+b^2-c^2}{2ab}.$$

(*ii*) $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

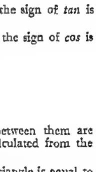


Fig. 3

APPENDIX (C)

GRAPHS

Graph:—A graph is a representation, by means of a curve, of the relation between two *variable quantities.*

Rectangular Axes of Co-ordinates.—Every point in a graph must be plotted with reference to two fixed straight lines XOX' and YOY' (Fig. 1) in the plane of the paper (*vide* Art. 27, Part I). These two straight lines are at right angles to each other, which divide the plane into four spaces XOY, YOX', $X'OY'$, $Y'OX$. These spaces are denoted by the first, second, third and fourth *quadrants* respectively.

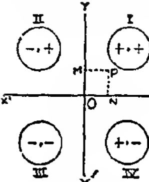


Fig. 1

The position of any point P in the plane can be located by knowing its perpendicular distances PN and PM from the two axes XOX', YOY'. These distances (PN and PM) are called the **co-ordinates** of the point P, PN being known as the ordinate, and PM the abscissa of the point P. The lines of reference XOX', YOY' are called the *rectangular axes of co-ordinates*, or simply *the axes*, the line XOX' being known as the X-axis and YOY' as the Y-axis. The point O is called the origin for which the co-ordinates for both the axes are zero, and the point is denoted as (0, 0). Thus the ordinate of a point lying on the X-axis is 0 and the abscissa of a point on the Y-axis is also 0.

It should be noted that in the *first quadrant*, both the X and Y-co-ordinates are *positive*; in the *second quadrant*, X-co-ordinate is negative but the Y-co-ordinate is *positive*; in the *third quadrant*, both the co-ordinates are *negative*; and in the *fourth quadrant*, X-co-ordinate is *positive*, but the Y-co-ordinate is *negative*. As a general rule it may be expressed thus: Ordinates *above* the X-axis are taken as *positive*, and ordinates below the X-axis are taken as *negative*. Similarly, abscissæ to the *right* of Y-axis are taken as *positive*, and abscissæ to the *left* of Y-axis are taken as *negative*.

Thus, the position of a point A $(-4, 3)$ will be in the second and that of a point B $(-3, -4)$ will be in the third quadrant.

Choice of Axes.—In all physical problems there are two variables, of which one is the independent and the other the dependent

variable. For instance, in the case of a simple pendulum, we know that time t for one complete oscillation depends upon l, the length of the pendulum. Thus, here t is the dependent and l the independent variable. As a rule **plot the independent variable along the X-axis and the dependent variable along the Y-axis.**

Choice of Units.—To choose the unit for the ordinate or the abscissa, find the difference between the highest and the lowest values of it (given in the problem) and divide this by the number of available divisions of the graph paper along the same side. Thus get the approximate value of each division and then choose the next best possible value. Since in the graph paper the tenth or the fifth lines are generally drawn thicker, attempt should always be made to choose the units in such a manner that the larger divisions are multiples or submultiples of 5. If each division represents values, which are divisible by 10, such as, 10, 100, 1000, or ·1, ·01, ·001, the plotting of points will be easier. Beginning from the origin write down the values along the X-axis and the Y-axis every 5 or 10 divisions apart.

Rule.—In drawing a graph for given physical experimental data, the following rules may generally be observed:—

(1) Obtain data for at least 6 points in the graph and tabulate the values for the X-axis (*independent variable*) and the Y-axis (*dependent variable*).

(2) If there are both positive and negative signs in the given data, then the origin, *i.e.* the point of intersection of the two axes, should be in the middle of the graph paper, but, if the signs are all positive, the origin can be shifted to the extreme lowest position on the left of the paper in order to have a graph of *larger size.*

(3) Choose the units explained before, and plot the points marking their positions in the diagram by ×or O sign.

Different suitable scales may be chosen for the two axes, but in some cases, as when area is to be calculated from the graph equal scales will be convenient.

(4) The point of intersection of the two axes need not always be the zero of each axis.

(5) From the positions of the points, judge the nature of the graph and draw a smooth curve by joining the plotted points.

(6) The curve should pass through all the points, but if it does not, keep the nature of the graph intact, it may be made to pass through *as many points as possible.* The point (or points) which does not lie on the curve is probably in *error* in the corresponding observation.

(7) The units should be so chosen that *the curve may cover as much of the graph paper as possible.*

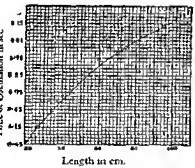
Examples

1. *The following readings were obtained with a simple pendulum :—*

Length in cms.	20	30	42	55	70	80	95	102	115	130
Time of oscillation in seconds	·45	·55	·65	·74	·835	·94	·98	1·01	1·07	1·14

Represent by a graph the relation between the length and time and find from your graph the time of oscillation of a simple pendulum of length 50 cms. (C. U. 1915)

Here we find that the time of oscillation depends upon the length of the pendulum so *time* is the *dependent variable* and should be plotted along the Y-axis, and the *length*, which is the *independent variable*, should be plotted along the *X*-axis.



Length in cm.
Fig 2

The difference between the highest value (1·14) and the lowest value (0·45) of time is 0·69 and the number of available divisions on the graph paper is 40. Therefore the approximate value of each division on the *X*-axis should be at least $\frac{0.69}{40}=0.0017$.

Take each small division on the *X*-axis to represent 0·0020, which is the next best possible value.

Take one small division to represent 4 cms. on the *X*-axis.

Since the length begins from 20, and the time from 0·45, it is necessary to start from the origin as (20, 0·45).

Write down the values of the ordinates every 5 divisions apart and being 0·45 as the zero value of the ordinates, and similarly take 20 cms. at the zero value of the abscissæ.

Now plot the points and draw the graph (*vide* Fig. 2)

To get the time of oscillation of the pendulum of length 50 cms., draw a straight (dotted) line through the point marked 50 cms. on the *X*-axis parallel to the Y-axis cutting the curve at a point, the ordinate of which has the value 0·71, which is the required time.

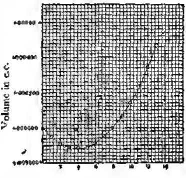
2. *From the following data plot a curve showing the variation in the volume of a mass of water with the temperature. Find graphically the two temperatures, at which the volume of 1 c.c. of water at 0°C. becomes 0·99990 c.c.* (C. U. 1909)

Temp.	Volume	Temp.	Volume
0	1·000000	7	0·999952
1	0·999948	8	1·000003
2	0·999911	9	1·000068
3	0·999889	10	1·000147
4	0·999883	11	1·000237
5	0·999891	12	1·000344
6	0·999914	13	1·000462

Here we find that on changing the temperature the volume is changed. So *temperature* is the *independent variable* and should be plotted along the *X*-axis, and *volume*, which is the *dependent variable*, should be plotted along the Y-axis.

The difference between the highest value (1·000462) and the lowest value (0·999883) of volume is 0·000579. The number of available divisions on the Y-axis is 40. Therefore, the approximate value of each division of Y-axis should be at least $\frac{0\cdot000579}{40} = 0\cdot0000144$.

Take each small division to represent 0·000020, which is the next best possible value. Take 2·5 small divisions to represent 1°*C*. on the *X*-axis.



Temperature in Centigrade
Fig. 3

Write down the values of the ordinate every 5 divisions apart taking 0·999800 as the zero reading, and also write the values of temperatures on the *X*-axis.

Plot the points and draw the graph (Fig. 3). To get the value of the temperature corresponding to 0·99990 c.c., draw a straight line through the point (0·99990) parallel to the *X*-axis cutting the curve at two points the abscissa of the first point being 2·41 and that of the second point being 5·8 nearly.

Therefore the required temperatures are 2·41° and 5·8°. Here the unknown result is determined by what is known as **Interpolation.**

3. *The battery resistance 'b' ohms for a current 'c' ampere was found in a certain test as follows :—*

b	4·2	4·8	5·0	5·8	7·6	8·5	11·0
c	0·21	0·16	0·14	0·14	0·066	0·06	0·04

Illustrate the results graphically. Are they consistent with Ohm's law ? (*Pat. 1920*)

Plot '*b*' along the *X*-axis and '*c*' along the Y-axis (Fig. 4).

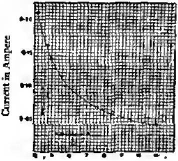
Units.—1 small division on the *X*-axis represents 2 ohms.

1 small division on the Y-axis represents 0·005 amp.

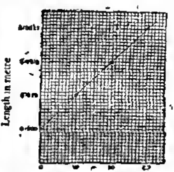
According to Ohm's law the product of current strength and the corresponding resistance should be constant, which is not the case here. Hence the results are not consistent with Ohm's law

4. *A copper rod is found to be 5 009, 5 0018, 5 0027 metres long at temperatures 10°C., 20°C., 30°C., respectively. Find by means of a graph its length at 0°C.* (C. U. 1913)

Units—Each small division on the *X*-axis represents 1°*C*. and on the Y-axis 0 00009 metre.



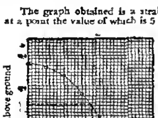
Resistance in Ohms
Fig. 4



Temperature in degrees Centigrade
Fig. 5

Here some space is left below the point 5 0009 on the Y-axis in order to allow the curve (Fig. 5) to cut the Y-axis below this point, if necessary.

The graph obtained is a straight line which being produced meets the Y-axis at a point the value of which is 5 metres from the graph. Thus the required length at 0°C. is 5 metres.



Time in sec.
Fig. 6

This method of determining the unknown result by producing the curve is known as extrapolation.

5. *Draw a curve on the squared paper supplied to indicate the height above ground, at intervals of half a second, of a body falling freely from rest at a height of 150 ft.*

Find from your graph the position of a particle after 1 76 seconds. (C. U. 1912)

The space traversed by a body falling from rest $= \frac{1}{2}gt^2$, and hence the height above the ground at any time $= (150 - \frac{1}{2}gt^2)$ ft.

Taking $g = 32$ ft. per sec^2., the distance fallen through, and so the height above the ground at intervals of half a second, is calculated and the following table is prepared :

Time in seconds	0	0·5	1	1·5	2	2·5	3
Height fallen, in feet	0	4	16	36	64	100	144
Height above ground in feet	150	146	134	114	86	50	6

Units.—1 small division on the X-axis represents 0·1 sec.

1 small division on the Y-axis represents 4 ft.

The position of the body at the end of 1·76 sec., obtained from the graph (Fig. 6), is nearly 103 ft. above the ground.

PHYSICAL TABLES

(1) UNITS

Quantity	F.P.S. Unit	C.G.S. Unit
Length	foot	centimetre
Mass	pound	gram
Force	poundal	dyne
Work	foot-poundal	erg
Power	horse-power	ergs per second

A force equal to the weight of 1 pound = 32·2 poundals. A force equal to the weight of 1 gram = 981 dynes.

(2) METRIC EQUIVALENTS

LENGTH

1 cm. = 0·3937 inch = 0·032 ft. 1 inch = 2·54 cms.

1 metre = 39·37 inches

= 3·28 feet 1 foot = 0·3048 metre

= 1·09 yards 1 yard = 0·914 metre

1 kilometre = 39370·790 in. = 3280·899 ft. = 1093·633 yd. = 0·621 mile.

AREA

1 sq. inch = 6·45 sq. cm.	...	1 sq. cm.	= 0·155 sq. in.
1 sq. foot = 0·093 sq. metre	...	1 sq. metre	= 10·764 sq. ft.
1 sq. yard = 0·836 sq. metre	...	1 sq. metre	= 1·196 sq. yds.
1 sq. mile = 2·590 sq. kilometre	...	1 sq. kilometre	= 0·386 sq. mile.

VOLUME

1 cu. in. = 16·387 c.c.	...	1 c.c.	= 0·061 cu. in.
1 cu. in. = 0·061 litre	...	1 litre	= 61·02 cu. in.
1 gallon = 4·546 litres	...	1 litre	= 1·76 pints
		1 litre	= 0·22 gallon.

1 gallon = 0·1604 cu. ft. = 10 pounds of water at 62° F.

MASS

1 grain [illegible]
1 ounce [illegible]
1 pound [illegible]
1 pound [illegible]

FORCE

1 gram-weight = 981 dynes
1 pound-weight = $4·45 \times 10^5$ dynes = 32·2 poundals.
1 poundal = 1 lb.-wt. ÷ g = 13,825 dynes

(3) MENSURATION

$\pi = 3·14159$; $\pi^2 = 9·87$; log $\pi = 0·4972$; log $\pi^2 = 0·9943$
Radius of circle = r ; Circumference of circle = $2\pi r$.
$\sqrt{2} = 1·4142$; $\sqrt{3} = 1·7321$
$e = 2·7183$ $\log_e 10 = 2·3026$.

AREA

Square (side l) = l^2
Rectangle (breadth b) = $l \times b$
Parallelogram = base × perpendicular height
Triangle = ½ base × altitude
Circle = πr^2
Surface of cube (side l) = $6l^2$
Surface of sphere (radius r) = $4\pi r^2$
Curved surface of cylinder (radius r, height h) = $2\pi r \times h$

VOLUME

Cube = l^3
[illegible]

(4) USEFUL DATA

The weight of 1 cu. ft. of water = 62·5 lbs. (approximately).

The weight of 1 cu. ft. of air at 0°C. and at 1 atmosphere = 0·0807 pound.

The weight of 1 cu. ft. of hydrogen at 0°C. and at 1 atmosphere = 0·0056 pound.

1 foot-pound = $1{\cdot}356 \times 10^7$ ergs.

1 horse-power-hour = $33{,}000 \times 60$ foot-pounds.

$$\begin{cases} 1 \text{ standard atmosphere} = 760 \text{ millimetres or } 30 \text{ inches of mercury;} \\ = 1033 \text{ grams.-wt. per sq. cm.} = (1033 \times 981) = 1{\cdot}013 \times 10^6 \text{ dynes per sq. cm.} \\ = 14{\cdot}7 \text{ pounds-wt. per sq. inch.} = 2116 \text{ pounds-wt. per sq. foot.} \end{cases}$$

Height of standard water barometer = $760 \times 13{\cdot}596$ mm. = $29{\cdot}92 \times 13{\cdot}596$ inches.

A column of water of height 2·3 feet corresponds to a pressure of 1 lb. per sq. inch.

(5) CONVERSION TABLE

To reduce	Multiply by	To reduce	Multiply by
Inch. to centimetre	2·54	Cu. ft. of water to lbs.	62·5
Sq. in. to sq. cm.	6·45	Miles per hr. to ft. per min.	88
Cu. in. to cu. cm.	16·39		
Grams to grains	15·4	lbs. per sq. in. to atmospheres	0·07
Pounds to grams	453·6		
Ounces to grams	28·35	Grams per sq. cm. to lbs. per sq. in.	0·014
Grains to grams	0·065		
Gallons of water to lbs.	10	Atmospheres to lbs. per sq. in.	14·7
Cu. ft. to gallons	6·24		
Cu. ft. to litres	28·3	H.P. to watts.	746
lbs. of water to litres	0·454	H.P. to ft.-lbs. per min.	33000

(6) DENSITY OR MASS PER UNIT VOLUME

(IN GRAMS PER C.C.)

METALS

Aluminium	2·7	Lead	11·37
Antimony	6·7	Nickel	8·9
Bismuth	9·8	Platinum	21·5
Copper	8·9	Quartz	2·65
Gold	19·3	Silver	10·5
Iron (cast)	7·2	Tin	7·3
,, (wrought)	7·8	Zinc	7·1
,, (steel)	7·7—7·9		

ALLOYS

Brass	8·4—8·7	Bronze	8·7

DENSITY OF LIQUIDS

(GRAMS PER C.C.)

Alcohol	0·79	Olive oil	0·91—0·93
Aniline	1·02	Paraffin	0·70—0·82
Benzene	0·89	Petrol	0·68—0·78
Ether	0·72	Petroleum	0·878
Glycerine	1·26	Spirit (methylated)	0·83
Kerosene	0·8	Turpentine	0·87
Mercury (0°C.)	13·596	Water (4°C) (ordinary)	1·00
Milk	1·03	„ (25°C.)	0·99708
Oil Linseed	0·94	Water (sea)	1·026

DENSITY OF COMMON SUBSTANCES

(GRAMS PER C.C.)

Chalk	1·9—2·8	Paraffin	0·9
Cork	0·22—0·25	Porcelain	2·3
Glass (Crown)	2·4—2·6	Quartz	2·6
Glass (flint)	2·9—4·6	Salt (common)	2·2
Guttapercha	0·97	Sand	2·6
Ice	0·92	Slate	2·9
India-rubber	0·9—1·3	Sugar	1·6
Ivory	1·8	Wood (teak)	0·7—0·8
Marble	2·7	Wax (Bees')	0·9

(7) ELASTICITY

[YOUNG'S MODULUS]

Aluminium	7×10^{11} dynes/cm.^{2}	Manganin	$12{\cdot}4\times10^{11}$ dynes/cm.^{2}
Constantan	$16{\cdot}2\times10^{11}$ „	Silver	$7{\cdot}9\times10^{11}$ „
Copper	$12{\cdot}3\times10^{11}$ „	Steel	$20{\cdot}9\times10^{11}$ „

(8) MELTING POINT

Bees' wax	63°C.	Tin	232°C
White wax	60°C.	Tungsten	3400°C.
Butter	28°—33°C.	Paraffin	45°—56°C.
Ice	0°C	Platinum	1773°C.
Copper	1083°C.	Sugar	160°C.
Iron	1527°C.	Sulphur	115°C.
Lead	327°C	Wax (Bees')	61° to 64°C.
Mercury	—39°C.	Wax (white)	68°C.
Napthalene	80°C.		

(9) BOILING POINT

Alcohol	78°C	Glycerine	290°C.
Aniline	182°C.	Mercury	357°C.
Chloroform	61°C.	Turpentine	158°C.
Ether	35°C.	Water	100°C.

(10) COEFFICIENT OF EXPANSION (PER °C.)

Coefficients of Linear Expansion of Solids

Aluminium	...	0·000022	Iron	...	0·0000114
Brass	...	0·000019	Lead	...	0·000029
Copper	...	0·000017	Platinum	...	0·000009
German silver	...	0·000018	Silver	...	0·000019
Glass	...	0·0000083	Tin	...	0·0000214

Coefficients of Cubical Expansion of Liquids

Alcohol (ethyl)	...	0·00122	Olive oil	...	0·0007
Aniline	...	0·00085	Sulphuric Acid	...	0·0095
Glycerine	...	0·00053	Turpentine	...	0·00094
Mercury	...	0·00018	Water (10°—30°)	...	0·000203

Coefficient of Cubical Expansion of Gases

The coefficient of increase of volume of all gases at constant pressure and the coefficient of increase of pressure of all gases at constant volume may be taken to be $= \frac{1}{273} = 0·00367$ per °C.

(11) SPECIFIC HEAT

Solids

Aluminium	...	0·21	Lead	...	0·03
Bismuth	...	0·03	Marble	...	0·22
Brass	...	0·09	Nickel	...	0·11
Charcoal	...	0·19	Paraffin	...	0·64
Copper	...	0·095	Salt (common)	...	0·20
Ice (0°C.)	...	0·50	Sand	...	0·19
India Rubber	...	0·48	Silver	...	0·056
Iron	...	0·11	Sulphur	...	0·163
Glass	...	0·16—0·19	Tin	...	0·055
Gold	...	0·03	Zinc	...	0·033

Liquids

Alcohol	...	0·62	Mustard oil	...	0·50
Aniline	...	0·50	Paraffin oil	...	0·53
Glycerine	...	0·58	Turpentine	...	0·43
Paraffin oil	...	0·53	Water	...	1·00
Mercury	...	0·033			

Gases

(At constant pressure)

Air	...	0·237	Oxygen	...	0·217
Hydrogen	...	0·41	Steam	...	0·465

(12) LATENT HEAT OF FUSION (*Calories per gram.*)

Bismuth	...	12·6	Mercury	...	2·8
Ice	...	80·0	Silver	...	21·0
Lead	...	5·4	Sulphur	...	9·4

(13) SATURATION VAPOUR PRESSURE OF WATER

(In Millimetres of Mercury)

Temperature (Centigrade)	Pressure (mm.)	Temperature (Centigrade)	Pressure (mm.)
−10°	2·1	40	55·13
0	4·57	50	92·30
2	5·29	60	149·2
5	6·54	70	233·5
8	8·01	80	355·1
10	9·20	90	525·8
12	10·51	95	634·35
15	12·78	100	760·0 = 1 atmos
18	15·46		
20	17·51	150	3569·0 = 4·7 atmos.
25	23·69		
30	31·71	200	11647 = 15·4 atmos.

(14) THERMAL CONDUCTIVITIES (in C.G.S. Units)

Air	0·00005	Iron	0·16 to 0·18
Aluminium	0·48	Lead	0·080
Brass	0·26	Mercury	0·0148
Copper	0·22	Silver	0·98
Glass	0·0005	Water (0°C)	0·0012
India-rubber	0·0004	,, (30°C)	0·001

(19) VELOCITIES OF SOUND AT 0°C.

Substances	Feet per sec.	Metres per sec.
	Gases	
Air	1090	332
Carbon dioxide	856	262
Coal gas	1609	493
Hydrogen	4163	1270
Oxygen	1041	317
	Liquids	
Water	4714	1437
	Solids	
Brass	11,480	3500
Glass	16,410	5000
Iron	16,820	5130
Marble	12,500	3810

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